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CONTENTS

MEETING AT ATLANTIC CITY, JUNE 26 TO 28, 1918

Lightning Arrester Spark Gaps—Their Relation to the Problem of Protecting Against Impulse Voltages—By Chester T. Allcutt. (*Illustrated*) 883
The Oxide Film Lightning Arrester—By Charles P. Steinmetz. (*Illustrated*) 871
The Oxide Film Lightning Arrester—By Crosby Field. (*Illustrated*). 881
The Design of Transpositions for Parallel Power and Telephone Circuits—By Harold S. Osborne. (*Illustrated*) 897
Engineers and the War—By William M. Black 945
Electric Power for Nitrogen Fixation—By E. Kilburn Scott. (*Illustrated* 957
America's Energy Supply—By Charles P. Steinmetz. (*Illustrated*) 985
Pre-Charged Condensers in Series and in Parallel—By V. Karapetoff. (*Illustrated*) 1015
Method of Symmetrical Co-Ordinates Applied to the Solution of Polyphase Networks—By C. L. Fortescue. (*Illustrated*) 1027
Sustained Short-Circuit Phenomena and Flux Distribution of Salient-Pole Alternators—By N. S. Diamant. (*Illustrated*)... 1141
Reactance of Synchronous Machines and Its Applications—By R. E. Doherty and O. E. Shirley. (*Illustrated*) 1209
Protection from Flashing for Direct Current Apparatus—By J. J. Linebaugh and J. L. Burnham. (*Illustrated*) 1341
The Automatic Hydroelectric Plant—By J. M. Drabelle and L. B. Bonnett. (*Illustrated*) 1367

PRESENTED BY PUBLICATION ONLY, AUGUST 1, 1918

Skin Effect in Tubular and Flat Conductors—By H. B. Dwight. (*Illustrated*) 1379
A Direct-Current Generator for Constant Potential at Variable Speed—By S. R. Bergman. (*Illustrated*) 1405

PRESENTED BY PUBLICATION ONLY, SEPTEMBER 1, 1918

Critical Review of the Bibliography on Unbalanced Magnetic Pull in Dynamo-Electric Machines—By Alexander Gray and J. G. Pertsch, Jr. (*Illustrated*) 1417
Magnetic Pull in Electric Machines—By E. Rosenberg. (*Illustrated*) 1425

PRESENTED BY PUBLICATION ONLY, SEPTEMBER 1, 1918

Economic Proportion of Hydroelectric and Steam Power—By Frank G. Baum. (*Illustrated*) 1471
Application of Harmonic Analysis to the Theory of Synchronous Machines—By Waldo V. Lyon. (*Illustrated*) 1477

PRESENTED AT PHILADELPHIA, SEPTEMBER 16, 1918

Electric Welding—A New Industry—By H. A. Hornor. (*Illustrated*) 1519

347959

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PRESENTED AT NEW YORK, NOVEMBER 8, 1918

Electric Motors in the Cement Industry—By R. B. Williamson. (*Illustrated*) 1531

PRESENTED AT TORONTO, CANADA, NOVEMBER 22, 1918

Electric Power Generation in Ontario on Systems of Hydro-electric Power Commission—By Arthur H. Hull. (*Illustrated*) 1607
Electrification of the Montreal Tunnel Zone—By William G. Gordon. (*Illustrated*) 1637
110,000-Volt Transmission Line Over the St. Lawrence River—By S. Svenningson. (*Illustrated*) 1653

MEETING AT PHILADELPHIA, DECEMBER 13, 1918

The Use of Electric Power in the Mining of Anthracite Coal— By J. B. Crane. (*Illustrated*) 1671
Drum Shapes as Affecting the Mine Hoist Duty Cycle and Motor Rating—By F. L. Stone. (*Illustrated*) 1677
American Engineering Research—By W. R. Whitney 1709
Research in America After the War—By R. A. Millikan 1723

APPENDIX

Standardization Rules of A. I. E. E., 1918 1747
Report of Board of Directors for Fiscal Year Ending April 30, 1918. 1867
Index to Papers and Discussions v
Index to Authors vi
Synoptical and Topical Index ..... End of Volume

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LIGHTNING ARRESTER SPARK GAPS

Their Relation to the Problem of Protecting Against Impulse Voltages

BY CHESTER T. ALLCUTT

ABSTRACT OF PAPER

This paper describes a new form of high-voltage lightning arrester gap which has been called the "impulse protective gap" because of its particular effectiveness in protecting against line disturbances of steep wave front.

The paper opens with a brief resumé of some of the results of previous investigations of the subject of impulse voltages. A discussion of the points involved in securing adequate protection against transient voltages of steep wave front follows. Particular reference is made to some of the conditions that may exist when a high-frequency impulse is superimposed on a low-frequency wave. In this connection is shown the desirability of a gap having a selective action, making it sensitive to steep wave fronts, and a number of forms of gap having this selective property are described.

Methods employed in testing these gaps are described and the results of a large number of experiments are tabulated. Tests on the action of a high-frequency impulse combined with a 60-cycle wave are included in the experimental work. From the experimental data a number of curves are plotted showing the discharge characteristics of the impulse protective gap under many different conditions.

The results of the tests are highly favorable and indicate that the new gap may have a wide application in the field of lightning protection.

IN GENERAL, a high-voltage lightning arrester consists of two distinct parts: First, a spark gap for discharging abnormal voltages; and second, means for preventing the normal line voltage from maintaining a power arc across the gap. In past years the satisfactory interruption of the power arc has been the most serious part of the lightning arrester problem, and, in consequence, the attention of manufacturers has been concentrated on the development of this feature. It is only comparatively recently that the gap itself has been receiving its due share of attention.

It is quite a simple matter to provide an electrical "safety valve" in the form of a spark gap to discharge abnormal voltages of low frequency, but when we have to deal with steep

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wave fronts, or impulse voltages, as they have been termed, there are certain complications due to dielectric spark lag. The existence of the phenomenon of dielectric spark lag was first brought to the attention of the Institute by E. E. F. Creighton[1] more than ten years ago. A later paper by Steinmetz and Hayden[2] gave the results of certain tests on the transient breakdown voltage of different gaps as a function of the energy available. Other papers[3] on impulse tests on porcelain shed some further light on the subject of dielectric spark lag but still left our knowledge of the subject in a rather incomplete state. It remained for F. W. Peek, Jr.[4] to present to the Institute the first quantitative data on the breakdown strength of different forms of gap under the action of transient voltages of carefully predetermined characteristics.

Mr. Peek's extensive investigations have definitely established the fact that some forms of spark gap require a very much higher voltage to discharge a high-frequency impulse than is required to discharge a continuously applied e.m.f. The name "impulse ratio" has been given to the ratio of the impulse breakdown voltage of a gap to the continuously applied breakdown voltage. The impulse ratio was found to vary with the shape of the gap electrodes, the length of the gap and the shape of the impulse wave applied. In case of the needle gap, for example, the impulse ratio was found to be considerably greater than two under some conditions. The sphere gap, on the other hand, was found to have an impulse ratio of substantially unity through a wide range of gap setting. The general conclusion reached by Peek was that a gap in which the discharge is preceded by corona will, in general, have an impulse ratio greater than unity, the impulse ratio increasing with the non-uniformity of the field and with the steepness of the wave front applied.

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1. E. E. F. Creighton. Methods of Testing Electrical Porcelain. A. I. E. E. TRANSACTIONS, Vol. 25, p. 365, May 1906.

2. J. L. R. Hayden and C. P. Steinmetz. Disruptive Strength with Transient Voltages. A. I. E. E. TRANSACTIONS, Vol. 29, p. 1125, June, 1910.

3. A. Chernyschoff and C. A. Butman. Different Methods of Testing Electrical Porcelain. *Electric Journal*, Vol. 12, p. 282, 1915.

L. E. Imlay and P. H. Thomas. High-Frequency Tests of Line Insulators. A. I. E. E. TRANSACTIONS, Vol. 31, p. 2233, December, 1912.

4. F. W. Peek, Jr. The Effect of Transient Voltages on Dielectrics. A. I. E. E. TRANSACTIONS, Vol. 34, p. 1695, September, 1915.

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The relation of his experimental work on transient voltages to the subject of lightning protection was discussed by Peek in a later paper[5]. In this paper he indicates the desirability of a low impulse ratio in a lightning arrester gap and shows the superiority of the sphere gap as a protective device, over the simple horn gap usually employed in connection with high-voltage arresters.

Protection Against Impulse Voltages

The papers referred to above have resulted in a great advance towards the solution of the problems involved in securing adequate protection against line disturbances of steep wave front. By the simple expedient of using a lightning arrester gap having spherical electrodes, the operating man may be perfectly sure that the voltage to ground at the point at which the arrester is installed will never appreciably exceed the 60-cycle discharge voltage of the gap. This is a great step in advance, but the question at once arises: Does it give the best possible protection against steep wave fronts? In this connection it must be pointed out that the danger to apparatus from an impulse voltage may be all out of proportion to the actual magnitude of the impulse. Consider, for example, a transformer winding designed for 50,000 volts. If 100,000 volts, at normal frequency, be applied to the winding the stresses on the insulation will only be doubled. But, on the other hand, if a high-frequency impulse of 100,000 volts be applied, the stresses on certain parts of the insulation may be many times normal because of the "piling up" of the steep wave front on the end turns of the winding. Consequently, if we are to protect the winding by means of a spark gap shunting it, the gap should be selective in its action; *i.e.*, it should discharge a high-frequency impulse at a *lower* voltage than an abnormal e.m.f. of line frequency.

The need for the selective action referred to above is emphasized by a consideration of the possible combinations of a high-frequency impulse with the normal line-voltage wave. Let us consider the three combinations shown in Fig. 1, and their effect on a winding protected by a gap.

Case A.—Impulse Occurring at Zero Point of Line E.M.F. Wave. In this case the presence of line voltage does not affect the action of the impulse. The insulation stresses in a winding

5. F. W. Peek, Jr. Lightning, *General Electric Review*, Vol. 19, p. 586, July, 1916.

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and the discharge characteristics of a gap protecting the winding will be the same as if the impulse alone were applied.

Case B.—Impulse Occurring at the Peak of the Line E.M.F. Wave and Adding Thereto. In this case the total e.m.f. impressed on the gap and tending to discharge it is equal to the sum of the line voltage and the impulse voltage. The insulation stresses between turns of the winding, on the other hand, are largely dependent on the value of the impulse alone, since line frequency does not cause a high voltage between turns. It would appear that, in this case, the gap affords a better protection

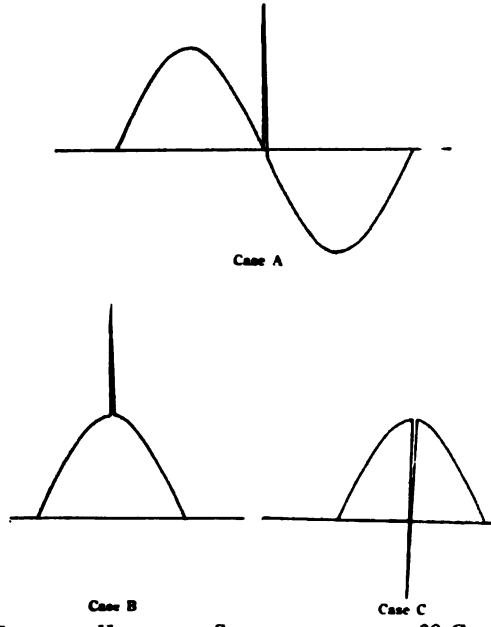

FIG. 1—IMPULSE VOLTAGES SUPERIMPOSED ON 60-CYCLE WAVES

than in Case A. For example, suppose the gap to be set at double line-voltage. In Case A, an impulse of double line-voltage may be impressed on the winding without the gap functioning, while in Case B the maximum impulse that can be applied to the winding without causing the gap to discharge is just equal to line voltage.

Case C.—Impulse Occurring at the Peak of the Line E.M.F. Wave and Subtracting Therefrom. In this case it is obvious that the gap furnishes much less effective protection than in the preceding two cases. A very considerable impulse voltage may

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be impressed on the winding without causing any rise in voltage across the gap. The presence of the line-frequency wave prevents the impulse from causing a rise in voltage across the gap but does not prevent the full destructive effect of the impulse from being felt by the winding. Assuming, once more, that the gap is set at double line voltage, it will be seen that an impulse of double line voltage may be applied without causing any rise whatever in the voltage across the gap, while it will require an impulse of *three times* line voltage to cause the gap to discharge.

The fact that such a condition is possible has not been generally appreciated, although it is clear that it may be a source of grave danger. Adequate protection against such a combination of high-frequency and line-frequency demands the use of a gap which is highly sensitive to steep wave fronts. It is true that, if the impulse is oscillatory, the second half-cycle may cause an ordinary gap to discharge, but such a discharge is too late to protect the winding against the destructive effects of the first half-cycle of the impulse.

The Impulse Protective Gap

A rather extensive experimental investigation of impulse phenomena has lead to the development of a new type of gap for lightning arrester service, which possesses in a marked degree the selective properties referred to in the preceding discussion. This gap has been termed the "impulse protective gap" because of its particular effectiveness in discharging line disturbances of steep wave front. The general principle underlying the action of the impulse gap is comparatively simple. In its most elementary form it consists of two gaps in series, each gap being shunted by a relatively high impedance. At line-frequency these impedances are proportional to the respective discharge voltages of the gaps which they shunt, but they are designed to change at different rates with changes of frequency so that, under the action of a high-frequency impulse, one of the impedances becomes much greater than the other and causes most of the high-frequency voltage to be impressed on one of the gaps. The breakdown of this gap will result in the total voltage being impressed on the remaining gap, which will break down in turn.

Fig. 2 is a diagrammatic representation of a gap constructed according to the above principle. One of the two equal gaps, g and g', is shunted by a condenser C' and the other by an equal

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condenser C and an inductance L. At line-frequency, the inductive reactance of L is inappreciable compared with the condensive reactances and, therefore, causes no unbalance in the voltage across the two gaps. To a high-frequency impulse, however, the condensers offer very little impedance and most of the high-frequency voltage appears across the inductance L and thus across the gap g. Discharge of the gap g is, in general, immediately followed by a discharge across g'.

It has been found by experiment that better results are obtained by using a single gap, having an intermediate electrode, rather than two distinct gaps. Fig. 3 shows a number of different gaps constructed in this manner. A shows a horn gap having an auxiliary electrode mounted midway between the horns. The auxiliary electrode is connected to one of the horns through a condenser C' and to the other through a condenser C and an inductance L. The action is similar to that of the gap shown in Fig. 2.

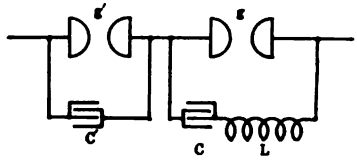

FIG. 2

In place of the reactance L, a resistance may be employed, as shown in Fig. 3B. This is the preferred form of gap, from a commercial standpoint, since the inductance used in A must necessarily be quite bulky, unless condensers of unreasonably high capacity be employed. With the structure shown in B, it has been found possible to secure excellent results with capacities as low as 10^{-11} farads.

Fig. 3C shows another form in which a reactance coil is used as an auto-transformer to produce a larger voltage between the auxiliary electrode and the main horn than is produced by the simple reactance shown in A.

Fig. 3D shows another modification, in which two intermediate electrodes are employed. R_1 is of greater resistance than R_2 so that a high-frequency impulse causes the right-hand gap to break down first, followed successively by the two others. For extremely long gaps it is probable that this form would give better results than B, but for commercial voltages it has been found possible to secure a gap of ample protective power by using but one auxiliary electrode.

Preliminary tests were made on a number of different gaps similar to those shown in Fig. 1. The early experiments included tests to determine the effect of different shapes of electrodes,

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both for the main horns and for the auxiliary electrode. The results of these preliminary experiments indicated that the scheme shown in Fig. 3B was the most promising for commercial development. A wire of small diameter (practically a pointed electrode) was found to be most satisfactory for the auxiliary electrode. The pointed auxiliary electrode gave results so much superior to a spherical or cylindrical electrode that further experiment with these latter forms was dropped.

As a result of these preliminary experiments it was decided to make complete tests on the two forms shown in the illustrations, Fig. 4 and Fig. 5. These two gaps differed only in the shape of the horn electrodes. The gap shown in Fig. 4 had

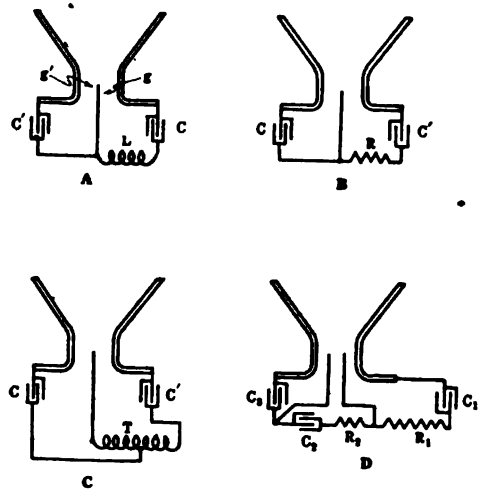

FIG. 3—IMPULSE PROTECTIVE GAPS

plain horn electrodes made of 3/8-in. (0.95-cm.) diameter brass rod, while the other gap had brass hemispheres 6.25 cm. in diameter clamped to the horns, this presenting spherical discharge surfaces. In both gaps, the auxiliary electrode consisted of a blunt point of copper wire 0.025 in. (0.064 cm.) in diameter. During the tests, each horn was supported on a pillar insulator composed of three porcelain insulator units. These pillars rested on a wooden table which brought the gap to a height of about 5 ft. (1.5 m.) above ground. The porcelain units composing the pillars were of special design, having an unusually high electrostatic capacity, approximately 2×10^{-10} farads. Two of these units in series were used as the condensers between

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the auxiliary electrode and each of the horns (corresponding to C and C' in Fig. 3B). A water tube was used for the resistance. In most of the tests its value was approximately 100,000 ohms.

Test Methods

The method employed in obtaining impulse voltages of predetermined characteristics was practically identical with that described by Peek in his 1915 paper.[6] Fig. 6 shows a diagram of connections. When the voltage of the transformer T is gradually raised, the sphere gap G_r will eventually discharge. At this instant, the condenser C is charged up to the breakdown voltage of the gap. When G_r breaks down, the condenser discharges through the inductance L and the resistance R. The wave form of this discharge current may be accurately calculated from the values of L, R and C. The shape of the voltage wave across R is, of course, identical with the shape of the current

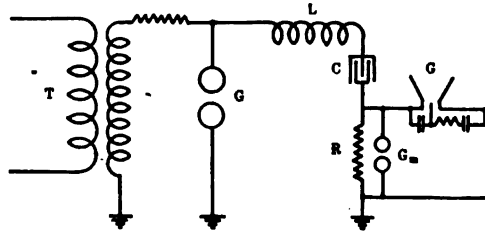

Fig. 6—Connections for Impulse Tests

wave. The gap G under test is connected across the resistance R. A sphere gap G_m is also connected across R in order to obtain a direct measurement of the impulse voltage. The voltage of the impulse applied to the gap under test may be varied by changing the setting of the gap G_r. For a more complete discussion of this method of obtaining impulses the reader is referred to Peek's paper.

The wave adopted as a standard for these tests was a critically damped impulse having a wave front corresponding to a 500,000-cycle sine wave. This wave is shown in Fig. 7. The constants of the oscillating circuit used in producing this impulse were approximately, $C = 10^{-9}$ farads, $L = 0.25$ millihenrys and $R = 1000$ ohms.

The procedure adopted in making the tests was as follows: First, the gap under test was adjusted to the desired setting.

6. Loc. cit. 4.

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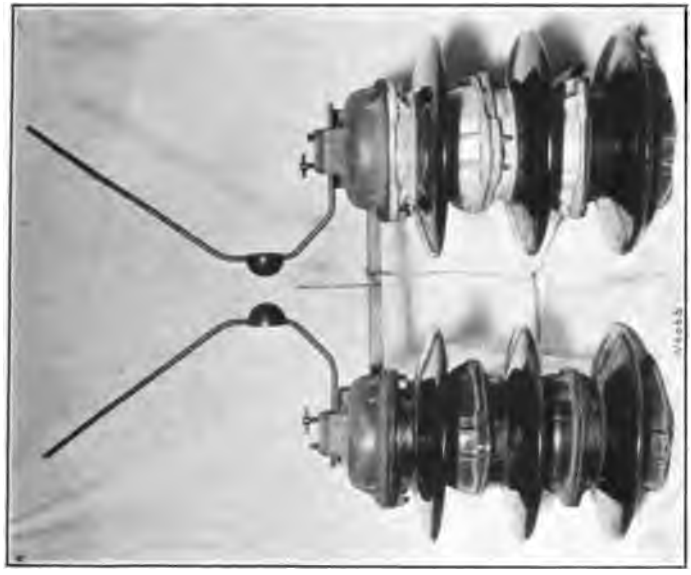

FIG. 5 [ALLCUTT]

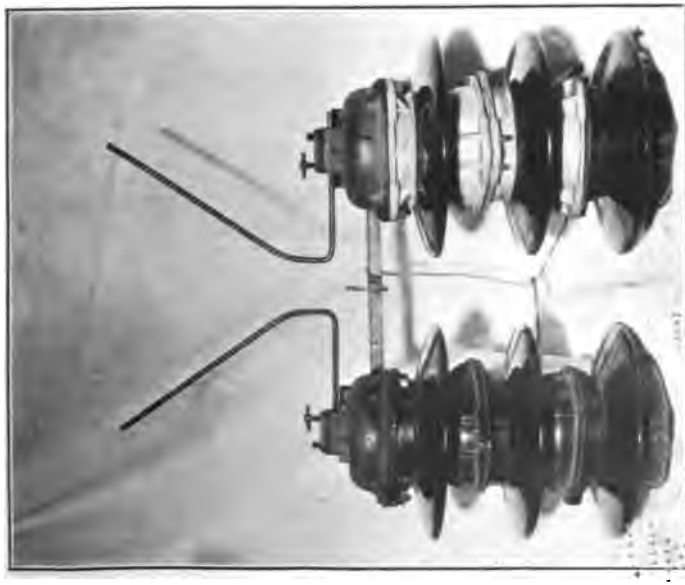

FIG 4

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Then successive impulses were applied at intervals of from 15 to 30 seconds and the gap G_r adjusted until a setting was found at which about 50 per cent of the impulses would discharge across the gap being tested. Then the measuring gap G_m was adjusted until the impulse discharges were shared equally by it and the gap under test. The setting of G_m that fulfilled this condition was taken as a measure of the impulse discharge voltage of G. Thus it will be seen that the impulse discharge voltage of the gap under test was in every case determined by direct comparison with a sphere gap.

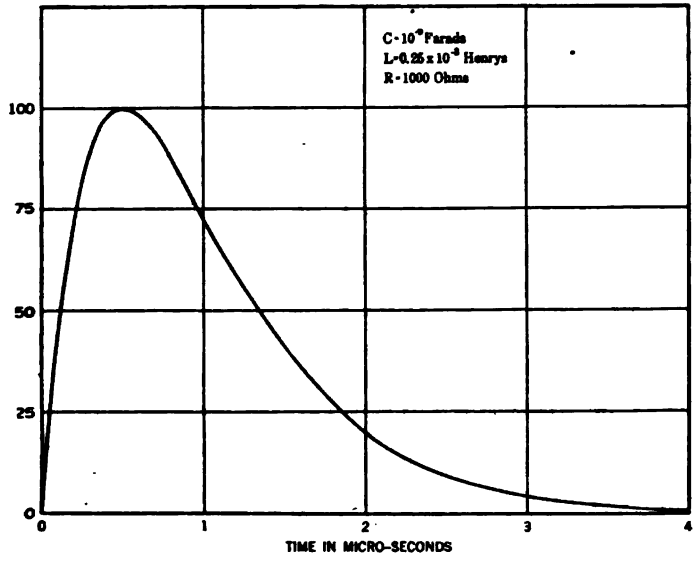

FIG. 7—500,000-CYCLE CRITICALLY DAMPED IMPULSE WAVE

The circuit used in testing the effect of an impulse voltage superimposed on a 60-cycle wave is shown in Fig. 8. Two high-tension transformers T_1 and T_2 are employed. These transformers are excited from the same 60-cycle supply circuit. T_1 applies a 60-cycle voltage to the gap while T_2 excites the impulse circuit which comprises a condenser C, an inductance L and a resistance R. As in Fig. 6, G_r is a spark gap used to regulate the impulse voltage. A condenser K of relatively large capacity is connected across the transformer T_1 external to the protective reactance L_p. This condenser presents a negligible impedance in comparison with the impedances shunting the gap G, so the

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full value of an impulse voltage across the resistance R may be regarded as being impressed on the gap under test. If the voltage of the transformer T_2 be slowly raised, the impulse resulting from the discharge of G_r will occur at the peak of the 60-cycle wave. By proper selection of the relative polarities of T_1 and T_2 this impulse may be made to add to, or subtract from, the 60-cycle wave produced by T_1, thus giving a resultant voltage wave, impressed on the gap G, similar to B or C in Fig. 1.

In making tests with the outfit described above, the voltage of T_1 was first set at the desired value. This voltage was carefully determined by means of a sphere gap, since the presence of the condenser K caused the terminal voltage of T_1 to be considerably above the value determined from transformer ratio. Then successive impulses were applied and, as before, a setting of G_r was found which would cause about 50% of the impulses

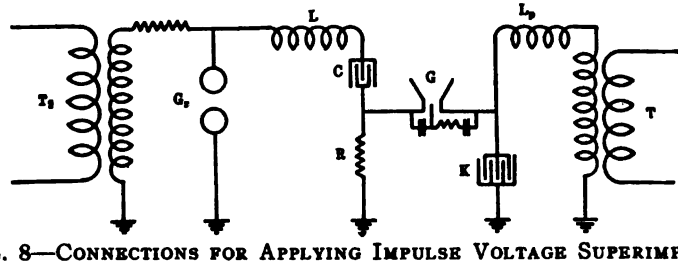

FIG. 8—CONNECTIONS FOR APPLYING IMPULSE VOLTAGE SUPERIMPOSED ON 60-CYCLE WAVE

to break down the gap G. When testing the effect of the wave shown in Fig. 1B, where the impulse adds to the 60-cycle wave, the measuring gap G_m was connected in parallel with the gap under test and a direct comparison obtained between the two. In testing case G, where the impulse subtracts from the 60-cycle wave, the value of the impulse voltage was computed from the setting of G_r and checked by an occasional measurement with a sphere gap connected across the resistance R.

The 60-cycle discharge-voltage of the gaps tested was determined in the usual way by comparison with a standard 25-cm. sphere gap.

RESULTS OF IMPULSE TESTS

The first series of experiments undertaken was for the purpose of determining the effect of the presence of the auxiliary electrode on the 60-cycle discharge voltage of the two forms of gap tested. Careful tests were made at gap settings ranging

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from ½ in. (1.27 cm.) to 3 in. (7.6 cm.). The results of these tests indicated that the presence of the auxiliary electrode does not appreciably lower the 60-cycle discharge voltage. With the auxiliary electrode in place, breaks were somewhat less consistent

TABLE 1. IMPULSE PROTECTIVE GAP
3/8-in. Diameter Horn Electrodes.

Gap setting. inches	60-Cycle discharge. kv. max.	500,000 Impulse		Impulse ratio
		Discharge one side only kv. max.	Discharge both sides kv. max.	
0.5	37	16	16	0.45
1.0	60	28	28	0.47
1.5	73	40	40	0.55
2.0	81	53	55	0.68
2.5	88	67	73	0.83
3.0	93.5	80	84	0.90

than with the plain gap, but the irregularities were not of sufficient magnitude to indicate an appreciable lowering of the breakdown voltage.

Preliminary tests with impulse voltages were then made in

TABLE II. IMPULSE PROTECTIVE GAP
6.25-Cm. Spherical Electrodes Mounted on Horns.

Gap setting. inches	60-Cycle discharge. kv. max.	500,000 Impulse		Impulse ratio
		Discharge one side only kv. max.	Discharge both sides kv. max.	
0.5	39	12.5	12.5	0.31
1.0	71	30	30	0.42
1.5	99	47	49	0.50
2.0	121	60	65	0.54
2.5	137	74	82	0.60
3.0	147	85	93.5	0.64

order to determine the best values for the capacitances and resistances used in the circuits connecting the auxiliary electrode with each horn. The results of these experiments showed that the capacitances and resistances could be varied through wide

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limits without affecting the impulse discharge voltage. The values of the capacitances employed in the succeeding experiments were, therefore, largely determined by convenience. In practically all the experiments discussed in the following pages, the condenser connected between the auxiliary electrode and each horn had a capacity of approximately 10^{-10} farads. As previously stated, each of these condensers consisted of two pillar-type porcelain insulator units in series (see Figs. 4 and 5). Using capacitances of the value given above, tests made with

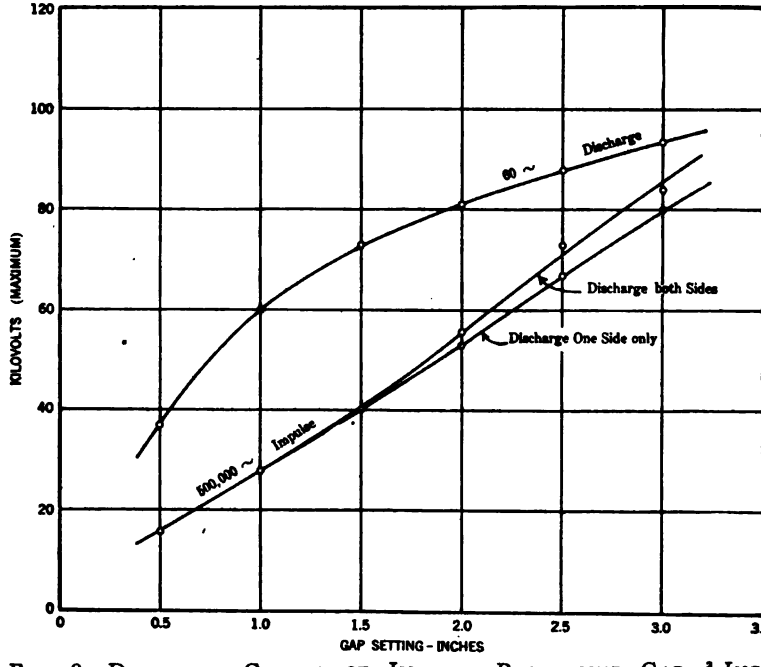

FIG. 9—DISCHARGE CURVES OF IMPULSE PROTECTIVE GAP ⅜-INCH DIAMETER HORN ELECTRODES

resistances varying from 50,000 ohms to several megohms showed no variation in the impulse discharge voltage of the gap. Accordingly, 100,000 ohms was selected as a convenient value of resistance to use in the remainder of the tests. This resistance is high enough to cause a very great unbalance in the voltage across the two sides of an impulse gap under the action of high frequency, and, on the other hand, is low enough to present a negligible impedance to 60-cycles in comparison with the capacitances employed.

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Having decided on the constants given above, the two forms of impulse protective gap shown in Figs. 4 and 5 were tested for both 60-cycle breakdown and 500,000-cycle impulse breakdown, with various gap settings ranging from ½ in. (1.27 cm.) to 3 in. (7.6 cm.). The results of these tests are given in Tables I and II.

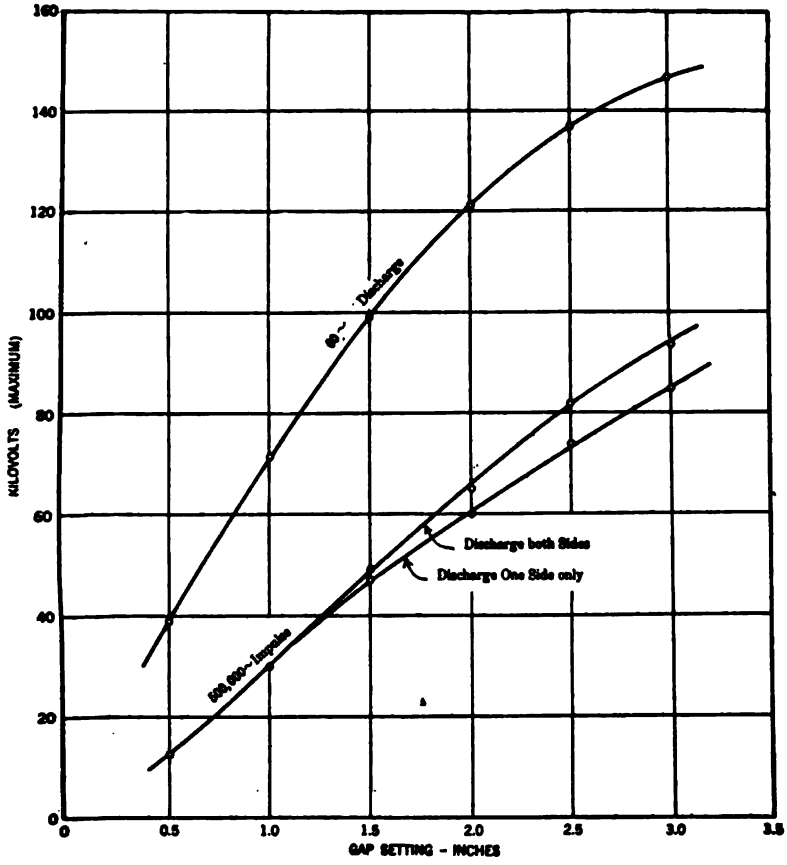

FIG. 10—DISCHARGE CURVES OF IMPULSE PROTECTIVE GAP 6.25-CM. SPHERICAL ELECTRODES MOUNTED ON HORNS

It will be noted that, under the heading "500,000 cycle impulse discharge voltage", there are two columns, one giving the impulse voltage which would discharge across one side of the gap only, and the other giving the impulse voltage required to cause a discharge across the whole gap. For small gap settings the difference between these two voltages was found to be too small

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to measure; that is to say, successive applications of an impulse of just sufficient voltage to cause an occasional discharge, resulted in part of the discharges being across the whole gap and part of them being between the auxiliary electrode and one horn only. In computing the impulse ratio, given in column 5, the value of the impulse voltage required to discharge across both sides of the gap was used. This value of the impulse discharge voltage was also used in all the curves given hereafter except where the contrary is specifically stated.

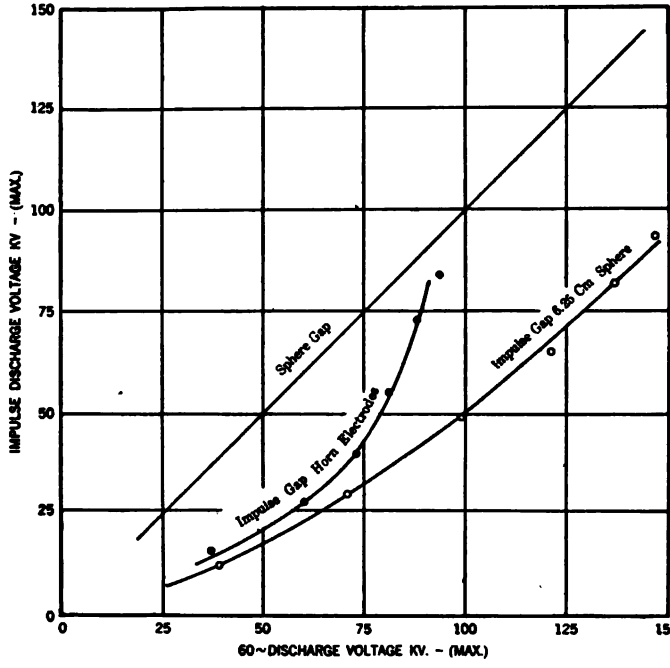

FIG. 11—CURVES SHOWING RELATION BETWEEN 60-CYCLE DISCHARGE VOLTAGE AND 500,000-CYCLE IMPULSE DISCHARGE VOLTAGE

The data given in Tables I and II are presented graphically in Figs. 9 to 12 inclusive. Figs. 9 and 10 give the 60-cycle and impulse discharge voltages as a function of gap setting for the two forms of gap shown in Figs. 4 and 5 respectively. It will be seen that in every case the impulse discharge voltage is *less* than the 60-cycle breakdown, indicating an impulse ratio of less than unity. In Fig. 11, the impulse discharge voltages are plotted against the 60-cycle discharge for the two forms of impulse gap

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and for the plain sphere gap. Fig. 12 shows the impulse ratio of the two gaps as a function of gap setting. It will be noted that with the form of gap having the plain horn electrodes, the impulse ratio increases rapidly with the larger gap settings and will probably become higher than unity for gap settings much in excess of 3 in. (7.6 cm.). Where spherical electrodes are used, it is evident from the curves that the impulse ratio will remain well under unity even for gap settings considerably greater than 3 in. The reason for the superiority of the gap equipped with

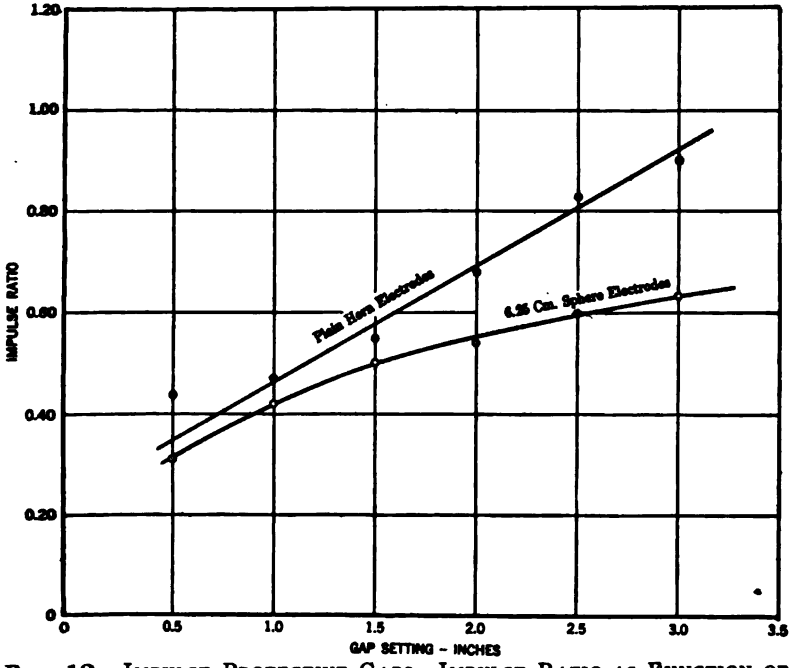

FIG. 12—IMPULSE PROTECTIVE GAPS—IMPULSE RATIO AS FUNCTION OF GAP SETTING

spherical electrodes may be made clear by reference to Figs. 9 and 10. By comparison of these curves it appears that, for a given gap setting, the impulse discharge voltages of the two gaps do not differ greatly. That is to say, a very great change in the shape of the main electrodes does not affect the impulse discharge voltage to a corresponding degree. We know, however, that the shape of the main electrodes will very materially modify the 60-cycle discharge. In view of these facts it is obvious that the lowest impulse ratio will be obtained by so shaping the main

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electrodes as to give the highest possible 60-cycle breakdown voltage. This condition is approximated with spherical electrodes.

A consideration of the foregoing results seems to lead to the conclusion that the impulse gap has great possibilities as a protective device. In themselves, however, these results are by no

TABLE III. IMPULSE PROTECTIVE GAP

3/8 in. Diameter Horn Electrodes
Gap Setting, 1½ in. (3.8 cm.); 60-cycle Discharge 73 kv. max.

Impulse applied kv. max.	Equivalent Sphere gap	
	cm.	kv. max.
40	1.30	40
47	1.45	43.5
57	1.55	46
82	1.63	48
100	1 75	51

means conclusive. The action of the new gap under more adverse conditions must be studied before its protective value can be regarded as definitely established. While it has already been shown that the impulse gap will discharge a much lower impulse voltage than a sphere gap having the same 60-cycle discharge, there remains the possibility that the gap may exhibit

TABLE IV. IMPULSE PROTECTIVE GAP

6.25 Cm. Spherical Electrodes Mounted on Horns.
Gap Setting, 1 in. (2.54 cm.); 60-cycle Discharge, 71 kv. max

Impulse applied kv. max.	Equivalent sphere gap	
	cm.	kv. max.
30	0.95	30
45	1.05	32.5
71	1.15	35
86	1.20	37
100	1.20	37

large time lag when an impulse considerably in excess of its discharge voltage is applied, thus permitting the voltage to rise to a dangerous value before discharge takes place. This condition was tested by determining the "equivalent sphere gap" of an impulse gap subjected to a transient voltage greater than its discharge voltage. The results of two series of tests made are

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given in Tables III and IV and in the curves in Figs. 13 and 14. In obtaining the above results a sphere gap was connected in parallel with the impulse gap under test. Then a setting of the

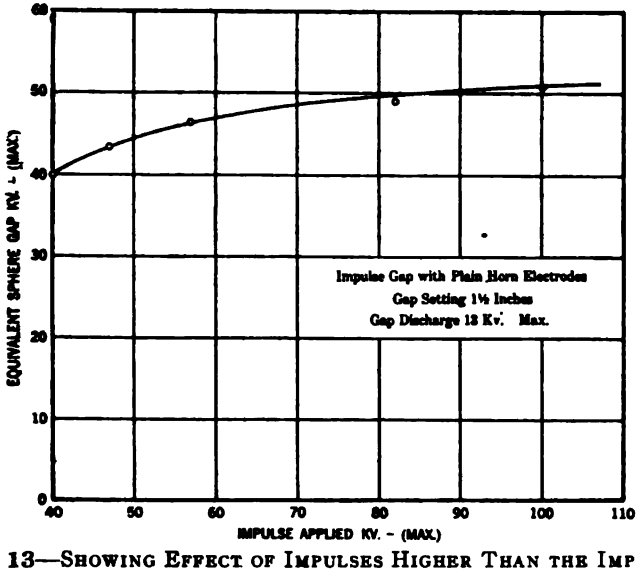

FIG. 13—SHOWING EFFECT OF IMPULSES HIGHER THAN THE IMPULSE DISCHARGE VOLTAGE OF THE GAP

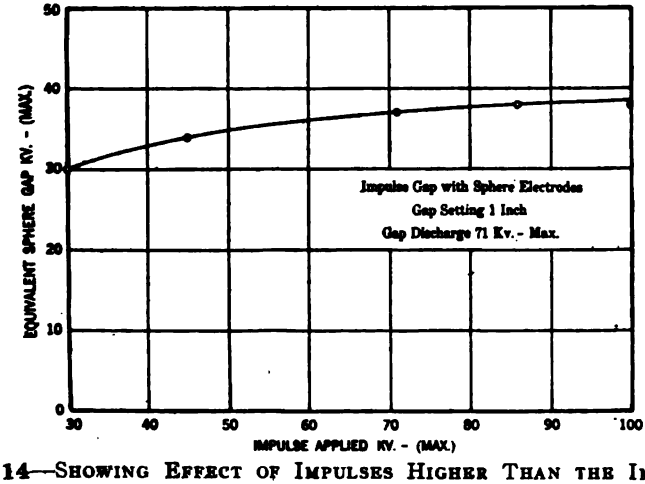

FIG. 14—SHOWING EFFECT OF IMPULSES HIGHER THAN THE IMPULSE DISCHARGE VOLTAGE OF THE GAP

sphere gap was found that would permit the two gaps to share the discharges equally when a given impulse voltage was applied. If we assume the time lag of the sphere gap to be negligible, this

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"equivalent sphere gap" setting is a measure of the actual voltage to which the impulse gap permits an impulse of super-discharge voltage to rise. Figs. 13 and 14 show that the two forms of impulse gap do permit some rise in voltage above the minimum impulse discharge value, but this rise is not of sufficient magni-

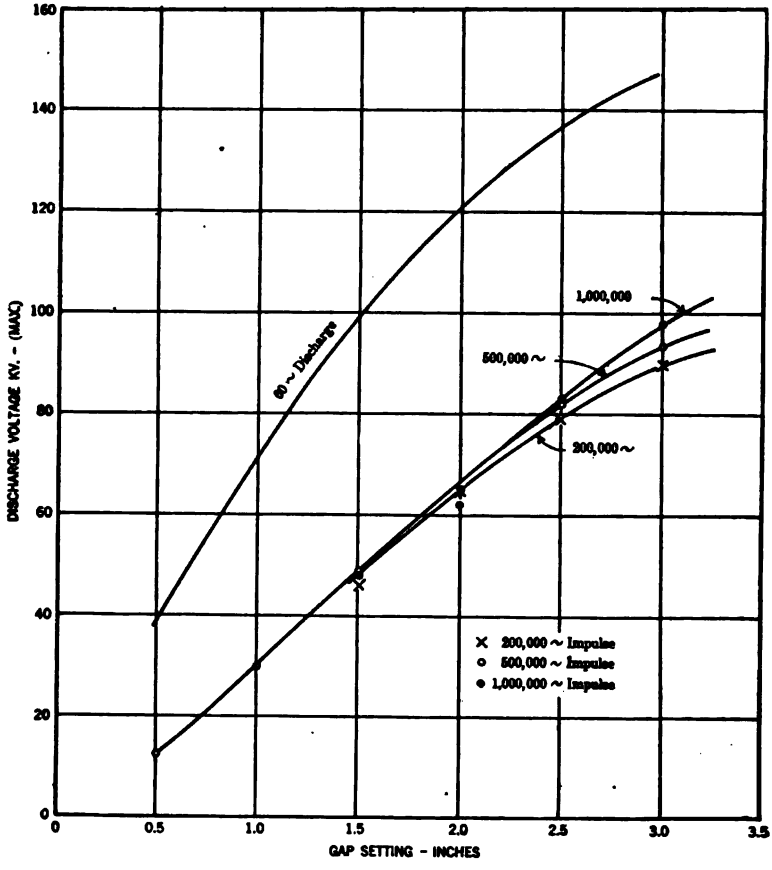

FIG. 15—TESTS ON IMPULSE PROTECTIVE GAP WITH SPHERE ELECTRODES SHOWING EFFECT OF IMPULSES OF DIFFERENT FREQUENCIES

tude to cut down seriously the degree of protection afforded. Even with an impulse applied, having a value of three times the minimum discharge voltage, breakdown occurs at very much less than the 60-cycle discharge voltage.

In addition to the above tests, it was thought desirable to undertake further experiments to determine the discharge char-

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acteristics of the impulse gap with frequencies of other than 500,000 cycles. Fig. 15 shows the results of some of these tests. It will be seen that the impulse discharge voltage increases slightly with increasing steepness of wave front applied. This

TABLE V.

Impulse Protective Gap with ⅜-in. diameter Horn Electrodes
Tests with 500,000 Impulse Superimposed on 60-Cycle Wave

Gap setting. Inches	60-Cycle discharge. kv. max.	60-cycle voltage applied. kv. max.	Impulse required to cause discharge		Equivalent line voltage kv. r. m. s.
			Case B kv. max.	Case C kv. max.	
1.0	60	30	9	30	36.5
1.5	73	36.5	16	45	44.5
2.0	81	40.5	20	52	49.5
2.5	88	44	40	71	54
3.0	93.5	47	41	90	57

increase in discharge voltage with increasing frequency is not very great, so it was not deemed necessary to make a complete series of tests at other than 500,000 cycles. It is a reasonable assumption that the tests made at this latter frequency will represent a fair average value of the discharge characteristics of

TABLE VI.

Impulse Protective Gap with 6.25 cm. Spherical Electrodes
Tests with 500,000 Impulse Superimposed on 60-Cycle Wave

Gap setting. Inches	60-cycle discharge. kv. max.	60-cycle voltage applied. kv. max.	Impulse required to cause discharge		Equivalent line voltage kv. r. m. s.
			Case B kv. max.	Case C kv. max.	
1.0	71	36	9	32	44
1.5	99	49.5	18	47	60.5
2.0	121	60	22	68	73.5
2.5	137	68	32	84	83
3.0	147	73	45	96	89

the gap under the action of high-frequency transients liable to occur in practise.

Results of Tests

Impulse Voltages Superimposed on 60-Cycle Wave

In making tests with an impulse voltage superimposed on a 60-cycle wave, the 60-cycle voltage applied was equal to one-half

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the discharge voltage of the gap. With this 60-cycle voltage applied, the value of the impulse voltage required to cause discharge was determined both for an impulse adding to the 60-cycle wave and for an impulse subtracting therefrom. This test may be regarded as closely simulating conditions that may actually occur in practise. Having the gap set for a discharge voltage equal to double the 60-cycle voltage applied, was taken as a fair approximation of the usual practise with regard to the setting of lightning arrester gaps.

The results of the tests made are given in Tables V and VI.

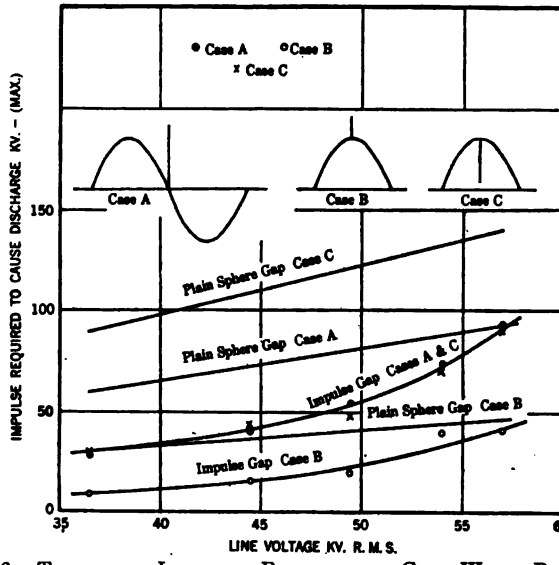

FIG. 16—TESTS ON IMPULSE PROTECTIVE GAP WITH PLAIN HORN ELECTRODES SHOWING EFFECT OF IMPULSES SUPERIMPOSED ON 60-CYCLE WAVE—GAP SET AT TWICE 60-CYCLE VOLTAGE TO GROUND

In column 3 are given the real peak values of the 60-cycle voltages applied at the different gap settings. This is, of course, equivalent to the peak value of voltage from one line to ground, which is applied to a lightning arrester gap in service. In order to form a more convenient basis for correlating the test results with service conditions, the root-mean-square value of the three-phase line voltage, equivalent to this peak voltage to ground, is given in the last column. Figs. 16 and 17 show, graphically, the impulse voltage required to cause discharge, as a function of this equivalent line voltage. The curves given for Case A are taken

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from the results, given in Tables I and II, for the impulse voltage alone. It is interesting to note that, for both forms of impulse gap, the results for Cases A and C are practically identical. In addition to the curves for the impulse gaps, the characteristics of a plain sphere gap are shown for purpose of comparison. The curves for the sphere gap are computed, assuming an impulse ratio of one. It might be stated here that actual tests on a

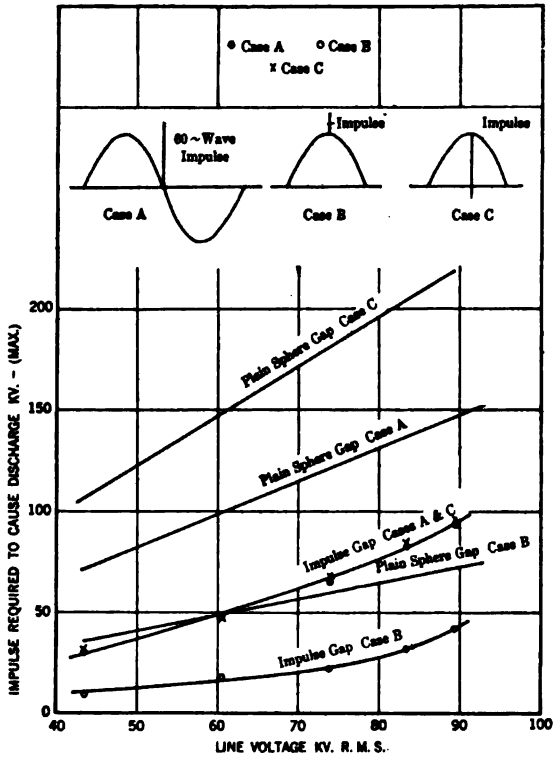

FIG. 17—TESTS ON IMPULSE PROTECTIVE GAP WITH SPHERICAL ELECTRODES SHOWING EFFECT OF IMPULSE VOLTAGE SUPERIMPOSED ON 60-CYCLE WAVE—GAP SET AT TWICE 60-CYCLE VOLTAGE TO GROUND

sphere gap subjected to an impulse subtracting from a 60-cycle wave (Case C) closely checked the computed curves given.

Figs. 16 and 17 are particularly effective in showing the very superior protection afforded by the impulse protective gap when subjected to an impulse subtracting from the 60-cycle wave (Case C). For example, on a 66,000-volt line an impulse protective gap would discharge such an impulse having a value of

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but 56 kv. max. (See Fig. 16), while a plain sphere gap would require an impulse of 161 kv. max. to cause a discharge under the same conditions.

Conclusion

The conclusions arrived at from the foregoing experiments may be summarized as follows:

1. In order to secure adequate protection against line disturbances of steep wave front, it is important that a protective gap having the lowest possible impulse ratio be used. A gap having an impulse ratio of less than unity is particularly desirable.

2. There are certain combinations of a high-frequency impulse with a wave of line frequency for which the degree of protection afforded by an ordinary spark gap is greatly lessened. The need for a protective gap having selective properties, rendering it sensitive to steep wave fronts, is emphasized by these conditions.

3. A protective gap, having a selective discharge for steep impulses, may be constructed without greatly complicating the usual horn gap structure.

4. Exhaustive tests of such a gap have shown it to be superior to the ordinary sphere gap under all conditions. Even under the most unfavorable conditions the new gap gives a high degree of protection.

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DISCUSSION ON "LIGHTNING ARRESTER SPARK GAPS—THEIR RELATION TO THE PROBLEM OF PROTECTING AGAINST IMPULSE VOLTAGES" (ALLCUTT), ATLANTIC CITY, N. J., JUNE 27, 1918.

Paul M. Lincoln: In the design of lightning arresters, we must recognize two distinct functions. First is the necessity of discharging the surges which come in from outside sources. They must be taken care of, and discharged through a path where they can do no harm instead of being allowed to go through apparatus where their discharge may result in disaster to the apparatus; that is one function. The second function is to let go of the dynamo current after it has started through the lightning arrester.

This piece of analysis apparatus proposed by Mr. Alcutt I want to point out, is of advantage on both of these scores. First, it allows a setting of the arrester gap which will permit the surges of lower value to go through and second, it allows the spacing of the horns to be increased, so that if the dynamo current of the order of 60 cycles or so starts through the lightning arrester, the dynamo current will let go easier; that is, the spacing of the horn gaps is permitted to be larger for the frequencies of the dynamo current. Therefore for both of these functions the suggestion put forth in this paper is of very material assistance.

F. W. Peek, Jr.: Before discussing directly Mr. Allcutt's results, I will briefly review some of my own work which has a bearing on this paper; this review will, I hope, make clearer what I have to say in the direct discussion.

The over-voltages that cause insulation failures in practise may be divided into three classes:

1. Gradual increase of voltage on the line, due to static or low-frequency surges. Such gradually applied voltages readily discharge over an arrester gap.

2. Very high-frequency oscillations of voltages too low for any gap arrester to discharge, but which may cause very high internal voltages in apparatus.

3. The form of over-voltage with which we are at present concerned,—lightning impulses of very steep wave front, when the voltage across the apparatus increases from normal to a very high value in perhaps a millionth of a second.

In 1913 I became interested in determining means of producing impulses of known wave shape and voltage, and in measuring impulse voltages. This work was stimulated by lightning troubles that I was asked to investigate on the lines of one of the large operating companies. In this particular case certain switch bushings with a 60-cycle arc-over voltage of about 100 kv. were protected by aluminum arresters with horn gaps set at about 50 kv., 60 cycles. Lightning always arced over the bushings, and never arced over the arrester gaps. The 60-cycle arc-over voltage of a gap or insulator was

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evidently no criterion of the lightning arc-over voltage. It was obvious that a lightning arrester gap should be designed, if possible, for a low lightning arc-over voltage, and insulators and apparatus for a high lightning arc-over voltage. This difference between the lightning and 60-cycle arc-over voltage, illustrated by the above practical case was caused by the "lag" of the gap.

It had long been known that there was some sort of a time lag between the application of voltage and the discharge of a gap, and an interesting paper by Hayden and Steinmetz showed that energy was necessary. The importance of the shape of the electrode on the arc-over voltage was however not suspected. Lightning arresters were, in fact, at that time given the E. N. G. or *equivalent needle gap* test. That the needle gap was just the wrong one to use will appear later. There were no definite

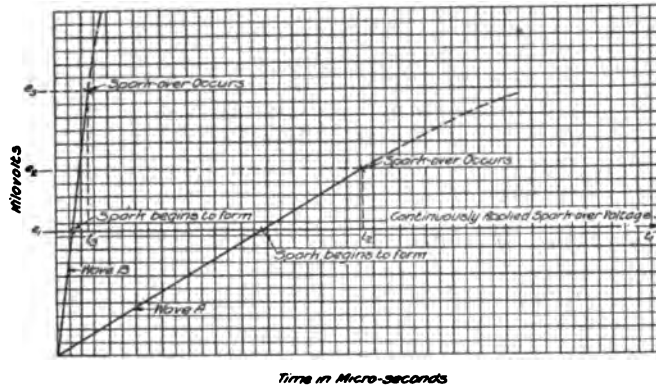

FIG. 1—DIAGRAMMATIC ILLUSTRATION OF WHY THE "LIGHTNING" OR IMPULSE SPARK-OVER VOLTAGE IS HIGHER THAN THE CONTINUOUSLY APPLIED

data in regard to time, the reason being that there was no generator for producing known impulses or a means of measuring them.

From theoretical considerations it seemed to me that the lag or time to break down a gap should depend upon the dielectric field and, therefore, upon the shape and spacing of the electrodes. A gap of a shape and spacing to cause a uniform field should be "fast"; a gap producing a non-uniform field should be slow, for reasons that will be given later. A sphere gap should, therefore, be fast, and a needle gap slow. Time lag manifests itself by an increase in the arc-over voltage as the rate of application of or steepness of the wave front is increased. The mechanism of breakdown is as follows:

When a 60-cycle voltage is slowly applied to a gap, and gradually increased, spark-over will occur at some definite

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voltage. This is the minimum voltage that will cause sufficient ionization for the gap to discharge, and it requires a relatively long time.

Lightning voltages, or voltages of steep wave front, start at zero, or line voltage, and increase at the very rapid rate of millions or billions of volts per second. When such a voltage is applied across a gap or insulator, spark-over does not occur at the instant the minimum or 60-cycle spark-over voltage is reached as a considerable time is required at this voltage. When this voltage is reached, the spark begins to form but is only completed after the rapidly rising voltage has reached some higher value. The slower the gap the higher the voltage will rise. (See Fig. 1.). In a uniform field break-down takes place over a relatively short path everywhere at the same time. In the case of a non-uniform field represented, for instance, by the needle gap, corona forms around the electrodes before arc-over. A vast amount of air must be ionized. The condition is equivalent to putting the corona or arc resistance in series with an ever increasing capacity represented by the

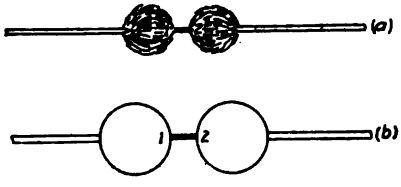

FIG. 2—DIAGRAM ILLUSTRATING SPARK FORMATION ON SPHERES AND ON POINTS

unbroken dielectric. Time is thus required to bring all of the space between the electrodes up to the breakdown gradient. (See Fig. 2).

In order to illustrate this theory experimentally, the sphere horn was devised. The original sphere horn shown in Fig. 13 was built in March 1914. The first tests were made with an oscillator, this oscillator did not give any definite or known wave shape, but illustrated the principle.

It was realized that in order to make any progress a systematic experimental investigation, to supplement the theoretical one, was necessary. It was only possible to get definite results by means of an impulse generator giving waves of known voltage, wave front, shape, duration, etc. Such a generator was designed and built, and made definite data possible. The results of this investigation were published by the A. I. E. E. in 1915.* A few of the laws resulting are as follows:

1. Two gaps, or insulators, with equal 60-cycle spark-over

*F. W. Peek, Jr. "The Effect of Transient Voltages on Dielectrics." TRANS. A. I E. E. Aug. 1915. Vol. XXXIV., Part II, p. 1857.
F. W. Peek, Jr., "Lightning," *G. E. Review*, July 1916.

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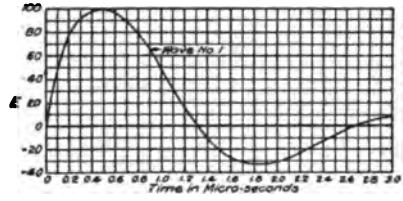
voltages, may have entirely different lightning or impulse spark-over voltages because of the time lag.

2. The time lag is greatest in a non-uniform field or for



APPROXIMATE SINGLE HALF SINE WAVE IMPULSE (ONLY FIRST HALF OF THE WAVE NEED BE CONSIDERED AS THE VOLTAGE IS COMPARATIVELY LOW AFTER THE FIRST CREST)

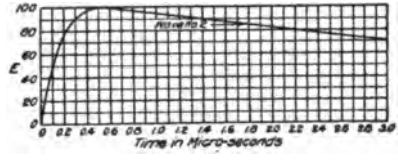


FIG. 3—IMPULSE WITH STEEP FRONT BUT OF COMPARATIVELY LONG DURATION

electrodes and a gap setting where corona precedes arc-over; it is minimum for a uniform field.

3. The time lag for any given electrodes and spacing is not constant, but depends upon the steepness of the wave or the

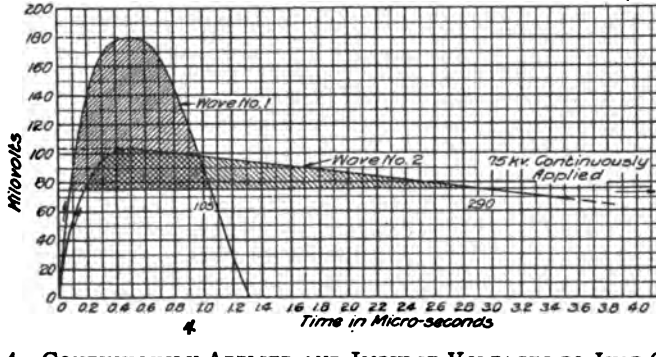


FIG. 4—CONTINUOUSLY APPLIED AND IMPULSE VOLTAGES TO JUST CAUSE SPARK-OVER OF A 10-CM. GAP BETWEEN NEEDLES

rate at which the voltage is applied. The arc-over voltage increases and the time lag decreases with increasing steepness of wave front. The ratio of the 60-cycle to the impulse spark-over voltage was termed the "impulse ratio."

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Some of the other interesting facts noted were:

The impulse spark-over voltage, unlike the 60-cycle spark-over voltage, of an insulator or gap is not greatly affected by rain. Wet and dry insulations have practically the same impulse breakdown strength. The corona starting point is practically the same for impulse voltages as it is for operating voltages. When two fast gaps are placed in series there is appreciable lag unless the arrangement is such that both gaps spark-over simultaneously. Corona produced by voltages lasting less then a micro-second can be quite readily seen. In some tests, voltages were used of such a steep wave front that a "drop" of 25 kv. was obtained over a 2-in. copper tube a yard long. Figs. 3 to 8 give some actual curves obtained in the tests.

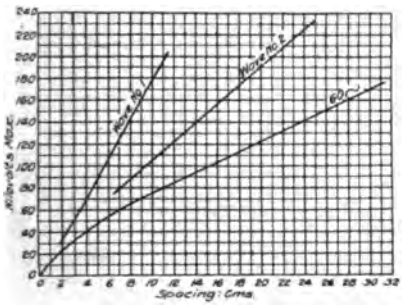

FIG. 5—NEEDLE GAP SPARK-OVER CURVES FOR 60 CYCLES AND FOR IMPULSE WAVES NOS. 1 AND 2 SHOWN IN FIGS. 3 AND 4

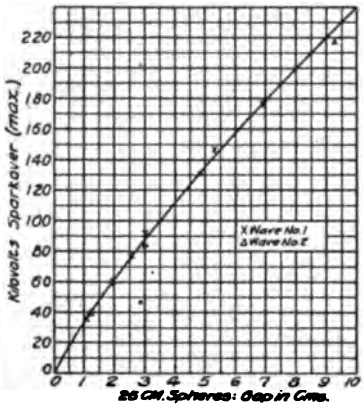

FIG. 6—SPHERE-GAP SPARK-OVER CURVES FOR 60 CYCLES AND FOR IMPULSE WAVES NOS. 1 AND 2, SHOWN IN FIGS. 3 AND 4 (DRAWN CURVE 60 CYCLES—POINTS, WAVES NOS. 1 AND 2)

It will be of interest to take a specific example to illustrate time lag:

At 10 cm. spacing between needle gaps, the 60-cycle spark-over voltage is 75 kv.; the impulse spark-over voltage for a given wave is 180 kv.; the time lag is of the order of 0.5 micro-seconds. For spheres set at 75 kv., 60 cycles, the spark-over voltage for this impulse is 75 kv. and the time lag is so small it cannot be detected, it is probably from 0.001 to 0.0001 that of a needle.

Referring to the special form of sphere-horn gap which is the subject of Mr. Allcutt's paper:

During my early work I found that while the 60-cycle and impulse spark-over voltages of a sphere gap were approximately

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equal, the impulse spark-over voltage of two spheres in series was much higher than the 60-cycle spark-over voltage.

In other words, there was no measurable lag for a single sphere gap, but considerable lag when two sphere gaps were placed in series. The reason for this is that the two gaps do not break down simultaneously unless the voltage divides between them in proportion to their respective strengths. One gap sparks over and puts the resistance of its arc in series with the other gap. The resistance of an arc depends upon the current flowing in the arc. The resistance remains high because the current is limited by the capacity of the sphere that has not yet broken down. Time is required to charge this capacity up to breakdown voltage through the resistance. This can be illustrated by experiment.

I found that this time lag could be eliminated if the gaps

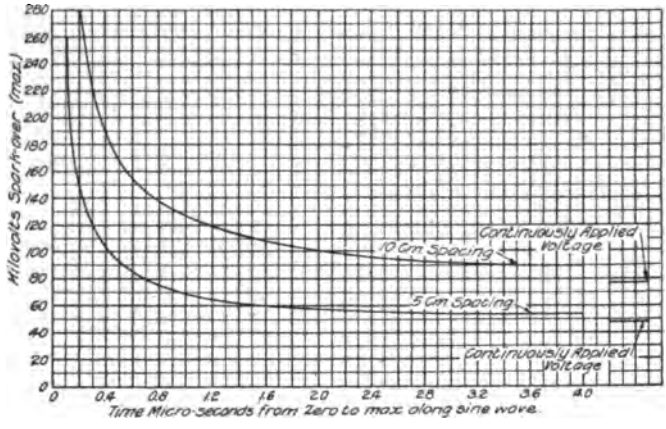

FIG. 7—TIME AND VOLTAGE TO SPARK-OVER NEEDLE GAPS (SINE WAVE IMPULSE)

were made to divide their voltages in proportion to their respective strengths so that simultaneous arcs occurred over both gaps and thus acted as a single arc, as in a uniform field.

In the gap shown by Mr. Allcutt, the division of voltage is not greatly affected under ideal conditions at 60 cycles by the auxiliary electrode. The auxiliary electrode is held at mid-potential because it is connected at the mid-point between the two equal condensers. (See Fig. 9). The capacity current is too small at 60 cycles to cause any appreciable "drop" across the resistance. If the condenser circuit were opened on one side, the gap on that side would break down at 60 cycles at about half voltage. This is exactly what happens under impulse. For steep wave fronts the resistance has the effect of opening the condenser circuit on that side. (See Fig. 10). The gap

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on that side breaks down. The voltage does not immediately disappear across the arc. The gap has lag. Corona precedes spark-over. The complete spark-over voltage is higher. Whether it is above or below the 60-cycle setting depends upon the impulse. The effect is the same as that which would result from a needle gap which could be set at, for instance,

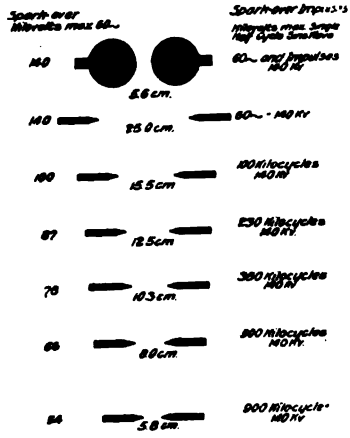

FIG. 8—SETTINGS OF NEEDLE AND 12.5 CM. SPHERE GAPS FOR EQUAL IMPULSE SPARK OVER VOLTAGES OF VARIOUS WAVE FRONTS
(IMPULSE VOLTAGE SETTING 140 KV. MAX.—WAVES SINGLE HALF SINE—100 KILOCYCLES AND 900 KILOCYCLES)

100 kv. for 60-cycle operation and instantly and automatically reduced to a 50 kv. 60-cycle setting whenever an impulse came on the line. For moderately steep wave fronts the spark-over voltage would be greater than 50 kv. and less than 100 kv., but for very steep wave fronts, the impulse ratio of the 50-kv. gap would be greater than two, or the spark-over voltage would be greater than 100 kv. This lag is shown in Mr. Allcutt's

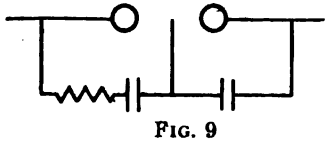

FIG. 9

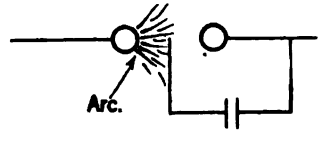

FIG. 10

data, Figs. 13 and 14. Super-voltages are equivalent to voltages of increased wave fronts and increase the spark-over voltage of the gap.

Fig. 11 gives data which I have obtained on such a gap at 3-in. and ½-in. spacings. You will note the characteristics are as discussed above. Starting with a sphere and a selective

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sphere at the same 60-cycle setting, 3 in., the sphere arc-over voltage remains practically constant for any wave front. At a moderate wave front the selective-gap has about 5 to 10 per cent lower arc-over voltage than the sphere, while for the steeper wave fronts the voltage is higher. *For steep wave fronts the sphere is the superior gap.* My tests show less advantage for the selective gap than Mr. Allcutt's. (See tests, Tables I and II. *In making such tests it is important to be sure that the impulse generator is giving a pure impulse and there are no secondary oscillations*, otherwise the results will be quite different.

Mr. Allcutt points out the important fact that the impulse voltage necessary to discharge a given gap will depend upon the point on the 60-cycle voltage wave at which the impulse reaches the gap.

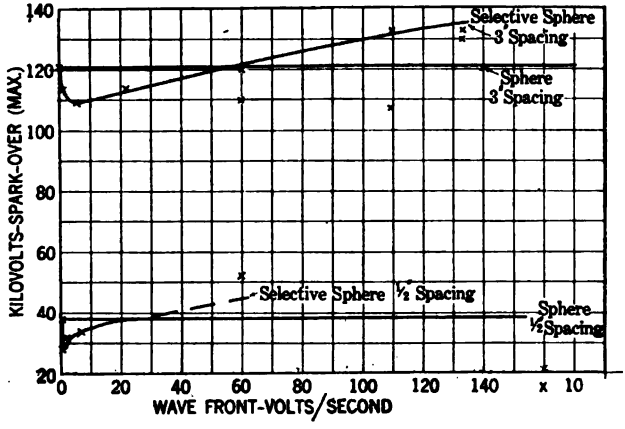

FIG. 11

There are three cases which require consideration:

Case A—where the impulse occurs at the zero point of the 60-cycle wave and is thus not affected thereby. The impulse voltage required to produce discharge may be denoted by A.

Case B—where the impulse occurs at the maximum of the 60-cycle wave and is additive. In this case it would be expected that the gap would discharge when the sum of the two voltages reached approximately the A discharge voltage of the gap. Thus, if L is the maximum of the 60-cycle line voltage, approximately,

$$B = A - L$$

Case C—where the impulse occurs at the maximum of the 60-cycle voltage wave, but in the opposite direction. In this case it would be expected that it would be necessary for the

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impulse voltage to reduce the 60-cycle voltage to zero and rise to a value A in the opposite direction, or approximately,

$$C = A + L$$

For Case C, the impulse voltage would rise to a higher value than in case A and B before the arrester discharged. End

TABLE I

SELECTIVE SPHERE-HORN
(6-¼-cm. Spheres with Auxiliary Electrode)
One side grounded.

Gap setting		60-cycle spark-over kv. max.	60-cycle spark-over ½ gap kv. max.	Impulse spark-over total gap	Apparent impulse ratio	Impulse ratio
In.	Cm.					
				83 Kilocycles (Single half cycle)		
½	1.27	38	23	28	0.74	1.21
2	5.08	105	67	81	0.77	1.20
3	7.62	121	88	113	0.94	1.29
				500 Kilocycles (Single half cycle)		
½	1.27	38	23	32	0.84	1.40
2	5.08	105	67	87	0.83	1.30
3	7.62	121	88	109	0.90	1.24
				2000 Kilocycles (Single half cycle)		
½	1.27	38	23	34	0.90	1.48
3	7.62	121	88	114	0.94	1.30
				Impulse spark-over total gap	Apparent impulse ratio	Wave front volts/sec.
3	7.62	121	88	113	0.94	0.4×10^{11}
3	7.62	121	88	109	0.90	5×10^{11}
3	7.62	121	88	114	0.94	22×10^{11}
3	7.62	121	88	110 – 120*	0.91 – 1.00	60×10^{11}
3	7.62	121	88	106 – 133*	0.88 – 110	110×10^{11}
3	7.62	121	88	130 – 132*	107 – 110	133×10^{11}

Above data for Case A—Impulse applied at zero of 60-cycle wave.
*Range of equal spark-over.

turns of transformers wou d, therefore, be subjected to the highest stresses for Case C.

Tests which I have made for both the plain sphere and the select ve sphere check approximately with the conclusions which were arrived at above from theoretical considerations.

Mr. Allcutt came to the same theoretical conclusions as

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outlined above for the sphere, for *A*, *B* and *C* and these theoretical results are plotted accordingly in his Fig. 17. He found from tests on the selective gap that for Case B the impulse voltage was reduced as would be expected, but for Case C, quite contrary to expectations, the impulse required to spark-over the gap was the same as in Case A. It is difficult to reconcile his tests *B* and *C*, since if an additive wave reduces the voltage, a subtractive wave should increase the voltage. My tests show this to be the case for both gaps—in other words, that the *relative* protective values of the sphere and the selective sphere are for an impulse of a given wave front approximately the same for all three cases.

The results of my tests are given in Table III. These tests were all made with the impulse at which the selective gap discharges at the lowest voltage as shown in Fig. 11. Attention

TABLE II

Impulse and 60-Cycle Spark-Over Voltages at the Impulse Frequency of Minimum Discharge for the Selective Gap.

Gap setting		Impulse frequency kilocycles	Sphere 60-cy. or impulse kv.max.	Selective sphere-horn					
				Pittsfield tests			Mr. Allcutt's test		
In.	Cm.			60-cy. kv. max.	Impulse kv.max	Apparent impulse ratio	60 cy. kv. max.	Impulse kv. max.	Apparent impulse ratio
½	1.27	500	38	*38	32	0.84	†39	12.5	0.31
2	5.08	500	105	105	87	0.83	121	65	0.54
3	7.62	500	121	121	109	0.90	147	93.5	0.64

*60-cycle values for one sphere grounded as in an arrester.
†60-cycle values for spheres not grounded.
Above data for Case A—impulse applied at zero of 60-cycle wave.

is called to the great difference between my tests and Mr. Allcutts. I found an average difference of only about 15 per cent between the sphere and selective gap for Case C *with the impulse giving the minimum break-down on the selective gap.* As the steepness of the impulse wave front is decreased this difference disappears and the discharge voltages become equal; as the steepness is increased the difference disappears and finally the break-down of the selective gap becomes greater than that of the sphere. (See Fig. 11.)

The impulse ratio figures given by Mr. Allcutt do not correspond to what I originally termed the impulse ratio, in my paper. The object of this ratio was to indicate the relative speed of different gaps. A gap without time lag has an impulse ratio of one; this ratio cannot be less than one. Mr. Allcutt gives values of less than one, although the gap is slow. The

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FIG. 12A—RELATIVE PROTECTIVE VALUES OF SPHERES AND POINTS

60-cycle spark-over voltage setting in kilovolts maximum.

Insulator	Spheres	Points
120	77	77

The spheres protect by discharging the impulse. The points have the same *60-cycle* voltage setting but do not discharge.

FIG. 12B—RELATIVE PROTECTIVE VALUES OF SPHERES AND POINTS

60-cycle spark-over voltage setting in kilovolts maximum.

Insulator	Spheres	Points
120	137	77

Although the sphere *60-cycle* spark-over voltage setting is, in this case, higher than that of either the insulator or the points, the spheres protect because the sphere "lightning" spark-over voltages is the lowest of the three.

[PEEK]

FIG. 12C—RELATIVE PROTECTIVE VALUES OF SPHERES AND POINTS

60-cycle spark-over voltage settings in kilovolts maximum.

Insulator	Spheres	Points
120	——	77

The points do not protect because of their very high "lightning" spark-over voltage.

The spark-overs shown are due to a succession of steep wave point impulses applied to the line.

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FIG. 13A—SPHERE AND HORN GAPS HAVING A STEEP WAVE-FRONT VOLTAGE APPLIED

60-cycle spark-over voltage settings (kv-max.)
Spheres 250 Horns 140

[PEEK]

FIG. 13B—SPHERE AND HORN GAPS HAVING A SLANTING WAVE-FRONT VOLTAGE APPLIED

60-cycle spark-over voltage settings (kv.-max.)
Spheres 250 Horns 140

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impulse arc-over voltage divided by the 60-cycle voltage of half the gap wou'd give this figure for the selective gap. See Table II. I mention this merely to prevent confusion in regard to my original use of the term. Mr. Allcutt's ratio is an *apparent impulse ratio.* As an instance, take the 3-in. setting for a 500-kc. wave; Mr. Allcutt finds an impulse spark-over voltage o' 93, while my tests give an impulse voltage of 109. My tests give an *apparent* impulse ratio of 0.90; Mr. Allcutt's 0.64. Part of this discrepancy is due to the fact that Mr. Allcutt takes the non-grounded value of 121 kv. for the 60-cycle voltage while the grounded value of 121 kv. should be used.

Any gain due to the selective sphere-horn is, in effect, that

TABLE III

Effect on the 60-Cycle Voltage on the Impulse Spark-Over Comparison of Sphere and Selective Sphere-Horn at the Impulse Frequency of Minimum Discharge for the Selective Gap.

Spacing		Selective gap impulse kv. max.			Sphere gap impulse kv. max.			*Ratio Selective / Sphere		
In.	Cm.	A	B	C	A	B	C	A	B	C
		Pittsfield Tests								
1	2.54	49	54	97	68	52	108	0.73	1.04	0.90
2	5.08	87	54	115	105	60.5	148	0.83	0.90	0.78
3	7.62	113	67	147	121	86	160	0.94	0.78	0.92
		Mr. Allcutt's Tests								
1	2.54	30	9	32	71	35	107	0.42	0.25	0.30
2	5.06	65	22	68	121	67	181	0.54	0.36	0.37
3	7.62	93.5	45	96	147	74	220	0.63	0.61	0.44

*Tests made at 500 kc., or at the wave front giving the minimum discharge on the selective gap.

Applied 60-cycle voltage = half 60-cycle discharge voltage of the gap.

Case A—Impulse applied at zero of 60-cycle wave.

Case B—Impulse applied at maximum of 60-cycle wave-additive.

Case C—Impulse applied at maximum of 60-cycle wave-subtractive.

of raising the operating voltage and not of time lag. Any apparent laboratory gain is thus lost if practise makes a large increase in spacing necessary. Greater gains of a similar nature may be made in other ways. Rain lowers the 60-cycle arc-over voltage of a gap. It is, for this reason necessary to set a gap installed outside, at about double the voltage of a gap installed inside. A fast gap protected from rain has been devised which makes a much greater practical gain for all wave fronts than that obtained by the selective gap, at its minimum discharge voltage. This is obvious since the average apparent gains due to *impulses* under the best conditions with the selec-

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tive gap, Figs. 9 and 11, are only 10 to 15 per cent, while a covered gap cuts the voltage in half. A *covered fast gap is of greatest importance.*

So far the problem has only been discussed from the arrester standpoint. While it is important to design an arrester gap of low impulse arc-over voltage, it is equally important to design leads, insulators, and insulation for high impulse puncture and arc-over voltages. Great strides have been made in this direction.

Percy H. Thomas (Read by L. W. Chubb): Mr. Allcutt has devised an apparatus which will cause a discharge over the spark gap more easily as the result of a voltage impulse of steep wave front than as a result of commercial frequency current. Furthermore, the effectiveness of the arrangement does not vary numerically with the frequency but rather may be said to be effective for all frequencies above a critical intermediate range, which is the author's intention and the desired characteristic. From a certain point of view the apparatus described by Mr. Allcutt has characteristics especially adapted to the protection of commercial electrical apparatus from high-frequency discharges. From certain other points of view, however, considerations might be urged to minimize its importance. For example, experience may show that it is not desirable to cause the spark gap to discharge with very high-frequency impulse. In commercial circuits there may be very many relatively harmless impulses of sufficient abruptness to cause Mr. Allcutt's apparatus to act when no actual protection is required.

It might happen, for example, with the device adjusted in such a way that high-frequency impulses of voltages not too high to be safe did not discharge that an adjustment would result which did not discharge commercial frequency impulses easily enough. In other words, is not the ideal condition a discharge ratio of unity? The abrupt discharge to ground of a voltage impulse on the terminal of a transformer may be far more severe on the end turns than the original impulse. This results not only because the swing of voltage is greater with the discharge to ground but because it may be of even a higher frequency than that of the original impulse.

It may also be said that it is only under the most favorable and very unusual conditions that a disturbance of a frequency as high as 500,000 cycles can be met with in transmission lines. The size of conductors and all apparatus connected with a commercial system are of such high capacity and inductance that practically all impulses will be of much lower frequency. Again, these very high-frequency discharges are of very small energy content, so that they would ordinarily be absorbed in any commercial system or any commercial apparatus without producing any considerable rise in potential. Especially if their initial voltage be so high as to break down any commercial lightning arrester.

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It is not safe, however, to object to such new devices as are proposed by Mr. Allcutt on purely theoretical grounds, for actual trial often reverses conclusions based on theoretical considerations.

It would seem to me that a particularly favorable field for the trial of this devise would be distribution systems carrying house lighting transformers from 2000 to 5000 volts. These transformers suffer frequent injury, and adequate arc suppressing devices are available in case of the use of very sensitive lightning arresters.

L. W. Chubb: This work of Mr. Allcutt's is very interesting and the tests in all cases made in shunt with the sphere-gap, leave no doubt regarding the quickness of the gap and the fact that it limits the voltage to a reasonable figure.

Some time ago we ran impulse tests on electrical porcelain and other dielectric materials, and the results show distinctly that voltage impulses of short duration far above the ordinary breakdown voltage of the insulator may be applied repeatedly without rupture. Each impulse, however, produces a permanent injury and eventually the insulation will give way under the repeated stress. High-frequency impulses coming in and striking the end turns of a transformer will not break it the first time, or for many times. Each time such an impulse comes it will cause a certain permanent injury, and repeated disturbances will eventually cause failure.

The selective properties of Mr. Allcutt's gap seem to me to be of great importance for it takes care of the first half cycle of a wave train which starts in opposition to the instantaneous value of line voltage. The ordinary gap, under the same conditions, set to double line voltage will require three times normal voltage to discharge on the first half cycle of the wave train, which will seriously injure the insulation of inductive end turns. The usual gap will, of course, discharge on the second alternation in this case, but this is too late as the voltage rise has not been restricted and the harm has been done.

C. T. Allcutt: I am very glad to hear from Mr. Peek on this subject and to find that he is conducting some experiments to check the results I have obtained. It would appear from the data and curves that Mr. Peek has included in his discussion, that there is a serious lack of agreement in our results. It is unfortunate that Mr. Peek has neglected to give any data concerning the construction of the gaps tested by him or the test methods employed. If such information had been included in his discussion I feel confident that the cause of the apparent discrepancies could be readily pointed out.

The particular forms of gap for which I have presented test data are but two of a large number of forms of gap, based on the same principle, that have been tested in our laboratory. The gaps described are, naturally, the preferred forms, *i.e.*, those that give the best results consistent with a commercially prac-

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ticable construction. Any considerable modification of the construction described may result in a large increase in the impulse discharge voltage.

I do not believe that Mr. Peek appreciated the importance of having the auxiliary electrode of such small dimensions so as

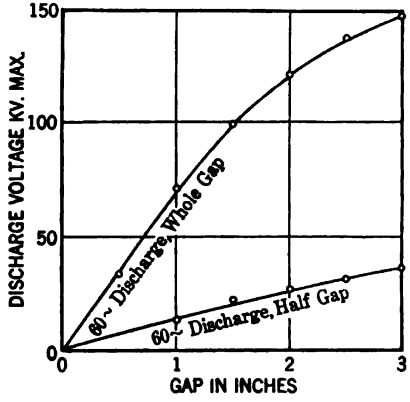

FIG. 14—IMPULSE GAP

to be, in effect, a point electrode. Perhaps the fault is mine for having insufficiently emphasized the importance of this point in my paper. For example, Mr. Peek states that the 60-cycle discharge voltage between the auxiliary electrode and one of the main electrodes is about one-half the discharge voltage of the

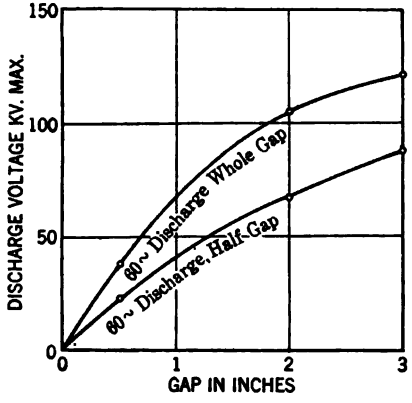

FIG. 15—PEEK'S GAP

main gap. This is clearly *not* the case where a point auxiliary electrode is used. In the sphere-horn gap tested by me, the 60-cycle discharge voltage between the auxiliary electrode and one of the main electrodes was approximately *one quarter* the discharge voltage of the whole gap. (See Fig. 14). In the gap

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tested by Mr. Peek, conditions were quite different, as is shown by Fig. 15, which was plotted from the data given in Table I of his discussion. It is evident that Mr. Peek used an auxiliary electrode of quite different dimensions from that described in my paper. This alone is sufficient to account for his failure to secure results comparable with my own, as the impulse discharge voltage is largely dependent on the 60-cycle discharge voltage of the half gap.

In the footnote to his Table II and again in the text of his discussion, Mr. Peek intimates that I used the standard ungrounded sphere gap calibration curve as representing the 60-cycle discharge characteristics of the gap under test. As I stated in my paper, the 60-cycle discharge voltage of the gap under test was obtained by comparison with a standard 25-cycle sphere gap. All tests were made with one electrode grounded. The presence of the horns and the height of the gap above ground both tended to make the 60-cycle discharge curve somewhat higher than the standard grounded calibration curve for 6.25-cm. spheres. Mr. Peek apparently was misled by the fact that the experimentally determined 60-cycle discharge curve for the sphere horn gap with one side grounded, happened to correspond quite closely with the standard *non-grounded* curve for 6.25-cm. spheres.

In another part of his discussion Mr. Peek objects to my use of his own very apt expression "impulse ratio." Mr. Peek introduced this term some three years ago with these words: "We have termed the ratio of the impulse breakdown voltage to the continuously applied breakdown voltage the "Impulse Ratio." In my paper I defined impulse ratio practically in Mr. Peek's own words and attempted to use the term in strict accordance with his own definition.

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THE OXIDE FILM LIGHTNING ARRESTER

BY CHARLES P. STEINMETZ

ABSTRACT OF PAPER

A short history of lightning protection of electric systems is given, as relating to the three successive types of electric circuits; the communication circuits, the power circuits of negligible electrostatic capacity, and the high-power circuits containing distributed capacity and inductance and capable of electric oscillation, leading to the three problems of discharging over-voltage to ground, opening the power current which follows the discharge and discharging so that no power current follows even for a fraction of a half wave. It is shown that these problems are solved by the spark gap to ground, by the use of non-arcing metals in the multigap arrester, which opens the circuit at the end of a half wave of current, and by the so-called "counter e.m.f." type of arrester, represented by the aluminum cell and the oxide film arrester.

It is shown that the necessity of taking care of recurrent discharges in high-power systems had led to the universal adoption of the aluminum cell arrester in such systems, in spite of its disadvantage of requiring daily attendance in charging, and of containing an electrolyte and oil.

In the oxide film arrester a type of arrester is presented which has the same characteristics and therefore the same advantages as the aluminum cell arrester, but does not require daily attendance and contains no liquids.

Its method of operation is explained, and its difference from the aluminum cell arrester; the dielectric film, which punctures under the discharge, and reseals after the discharge, is formed from the solid materials between the terminal plates, compressed PbO_2, and therefore no spontaneous chemical action occurs which dissolves the film, as in the aluminum cell, in which the film forms from the aluminum electrode, gradually dissolves, and therefore requires daily charging.

A short description of the construction of the oxide film arrester is given, a record of its operation in industrial service for over three years, and oscillograms showing the performance of this arrester under recurrents, oscillations and under high-power impulses.

THREE periods can be distinguished in the development of lightning arresters, corresponding to the three periods of the use of electricity:

(1) Electric circuits of negligible power, telegraph and telephone.

(2) Electric power circuits of negligible electrostatic capacity;

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d-c. lighting and railway, a-c. secondary and 2300-volt primary distribution.

(3) High-voltage electric power circuits, transmission lines, etc.

(1) In electric circuits of negligible power, such as telegraph and telephone circuits, a simple minute spark gap to ground afforded protection by discharging lightning to ground, and was sufficient until the recent years, when with the general introduction of electric power circuits the problem arose, in electric communication circuits to take care of crosses with power circuits.

(2) In electric power circuits, a simple spark gap to ground became insufficient for protection, since the power current, following the lightning discharge as arc, short-circuited the system and burned up the arrester. The problem then arose, to safely open the short circuit of the machine current to ground, through the lightning arrester, after the lightning discharge has passed, and to leave the arrester in operative condition to receive following lightning discharges.

Of the various devices developed heretofore, the magnetic blow-out lightning arrester still is used in direct-current railway circuits.

The first scientific investigation of this problem, is recorded in the paper[1] by A. J. Wurts, presented before the A. I. E. E. at the meeting of May 1894. Since that time all lightning arresters for alternating-current power circuits of negligible electrostatic capacity are based on the multigap principle between non-arcing metals, whatever constructive forms the arrester may assume—as the present compression chamber lightning arrester. The multigap arrester operates on the principle, that the lightning discharge over the multigaps closes the circuit to ground, but the power arc following the discharge extinguishes at the end of the half wave of the alternating current, as the non-arcing character of the gaps does not permit the reverse current of the next half wave to start. The multigap arrester thus short-circuits for a part of a half wave. It obviously is suited only for alternating currents.

For years difficulties were met with the question of resistance; without series resistance, in large systems the short-circuit power

1. TRANS. A. I. E. E., 1894, Vol. XI, p. 337, "*Discriminating Lightning Arresters and Recent Progress in Means for Protection Against Lightning*", by A. J. Wurts.

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even of a part of a half wave may be sufficient to disable and destroy the arrester, while the use of a series resistance, while limiting the power current and thereby protecting the arrester, also limited the discharge capacity and thereby reduced the protection. This problem was solved by the use of multigaps shunting the series resistance, so that moderate discharges passed over the resistance, while high power lightning discharges found a path without series resistance, over the shunted gaps, and at the same time, the shunting resistance made the power arc at the shunted gaps unstable and thus assisted in the extinction of the short circuit at the end of the half wave.[2]

(3) As soon however, as circuits came into use, which had considerable electrostatic capacity, such as high-voltage transmission circuits, extended underground cable systems, or lower voltage circuits (including generator circuits) inductively connected with such circuits, the multigap arrester failed by frequently, or even usually destroying itself by the discharge, burning up.

In such circuits, oscillations between capacity and inductance may occur, started by a lightning discharge or any internal disturbance such as switching etc., resulting in recurrent high-frequency oscillations, of which the arcing ground on a transmission line is typical and probably best known. With such continual discharges, often several per half wave, the multigap arrester short-circuits at the first oscillation, for the remainder of the half wave, and while the multigap functions properly and opens the short circuit at the end of the half wave, the oscillation of the next half wave again short-circuits, and so on, so that the effect is that of a continuous short circuit, and no lightning arrester, no matter how large, can dissipate the short-circuit power of a big system for any appreciable time.

For such systems, in which recurrent high frequency oscillations, as arcing grounds, may occur, a lightning arrester is necessary, which does not short-circuit the machine current even for a fraction of a half wave, but merely discharges the over voltage, the oscillation which, however high in voltage it may be, is small in energy compared with the short-circuit power of

2. See A. I. E. E. TRANS., 1907, Vol. XXVI, p. 425, "*Protection Against Lightning, and the Multigap Lightning Arrester*" by D. B. Rushmore and D. Dubois.

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the system, as it represents only the stored energy of capacity and inductance. The only arrester of this character heretofore was the electrolytic, or aluminum cell lightning arrester, developed by E. E. F. Creighton, J. L. R. Hayden, F. W. Peek and others. It acts towards an over-voltage discharge like a counter e. m. f. equal to the normal circuit voltage, and the discharge current passing through the arrester thus is the short-circuit current of the over voltage, while the normal machine voltage does not discharge, is held back and not disturbed. The aluminum cell arrester thus can discharge continual disturbances, over-voltage oscillations occurring at every half wave, for a considerable time, half an hour to several hours, before it is endangered by the temperature rise due to the accumulated energy of these discharges.

The aluminum cell arrester comprises a series of cells—usually conical and stacked into each other—of aluminum electrodes with an electrolyte, of which neither the salt nor its ions appreciably dissolve alumina. In "forming" the cell, by an alternating current passing through it, the electrodes are coated by a thin non-conducting film of alumina, which grows in thickness until it holds back the impressed voltage. Any over-voltage punctures this film, but the current passing through the puncture holes, again forms alumina and closes the holes. Thus the aluminum cell acts like a self repairing electrostatic condenser of a disruptive strength equal to the impressed voltage: about 250 to 300 volts per cell.

The practical experience of the last ten years has proven the aluminum cell arrester as the only type capable of affording protection in modern high-power circuits, and proven this so conclusively, as to lead to its universal adoption in such circuits in spite of the inconveniences incident to the need of daily attention in charging, the use of a liquid electrolyte, and the difficulty of testing the arrester without taking it apart, except by watching the appearance of the charging arc, or measuring the charging current.

These inconveniences incident to the aluminum cell arrester were well realized however, and as soon as the minor troubles met with the aluminum cell arrester in the early years had been overcome engineers went energetically to work on the problem of developing a lightning arrester of the characteristics of the aluminum cell arrester, but which does not require any attention

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beyond that given to every apparatus in a well managed system, that is, an occasional inspection, at least once or twice a year.

Numerous researches were made by the engineers whose splendid work I here acknowledge: Messrs. H. D. Brown, E. E. F. Creighton, Crosby Field, V. E. Goodwin, J. L. R. Hayden, N. A. Lougee, and G. B. Phillips. Some of these researches have not yet led to results ready for communication, but I am glad to present to you here as the result of the work of these men, and more particularly of E. E. F. Creighton, Crosby Field and N. A. Lougee, in the *Oxide Film Lightning Arrester* a new type of lightning arrester, which has all the characteristics and advantages of the aluminum cell arrester, but does not require any charging and thus requires no special attention, contains no liquid electrolyte, no inflammable material, and like the aluminum cell arrester, can be located outdoors as well as indoors.

The oxide film arrester, like the aluminum arrester, acts like a counter e. m. f. equal to the normal circuit voltage, freely discharging any over voltage, but holds back the normal machine voltage. Thus the discharge is limited to the energy of the over voltage, as in the aluminum arrester, and like the latter, the oxide film arrester can continuously discharge recurrent surges, such as arcing grounds etc., without endangering itself, for a considerable time, sufficiently long to notice and eliminate the disturbance.

Compared with the almost entire absence of knowledge of lightning phenomena in electric circuits, under which Mr. Wurts had to work in developing the non-arcing metal multigap lightning arrester, our present knowledge of lightning phenomena is very great. Nevertheless, there are so many disturbances in large electric systems, which we cannot or only incompletely reproduce in our laboratories, that the final decision on the success, that is, the effectiveness and permanence of a lightning arrester, still is best given by the experience in industrial systems.

Therefore, after extensive laboratory tests had been completed and had proven the oxide film arrester as of the same characteristic as the aluminum arrester, but requiring no special attention, a number of industrial installations were made, and more added the next year and the third year. Now, however, when a considerable number of installations of these arresters, for voltages from 110 to 33,000, have been in successful operation, some for over three years, and have proven their protective value and

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their permanence, I consider it desirable to bring the arrester to your attention.

Of the numerous tests made on the performance of the arrester, it may be sufficient here to give only two, by the oscillograms Figs. 1 to 4, showing the action on a recurrent oscillation, in Figs. 1 and 2, and on a single high-power impulse, Figs. 3 and 4.

The tests, oscillograms Figs. 1 and 2, were made in the usual manner: a surge or continual oscillation was produced by a large condenser, connected to an alternating-current supply and discharging over a spark gap through an inductance. The latter was chosen so as to give a frequency of 1200 cycles to the oscillation, and thereby bring it well within the range of the oscillograph. This surge was impressed upon the apparatus to be protected, a transformer energized by another alternating-current circuit. Fig. 1 shows the oscillogram without protection of the transformer, and Fig. 2 the oscillogram with an oxide film cell shunting the transformer and thereby protecting it.

In Fig. 1, the lowest curve shows the voltage of the 320-volt 47-cycle power supply circuit of the transformer, with the oscillations superimposed on it, rising to surge peaks of 2800 volts. The upper curve shows the oscillating currents passing through the transformer, rising to current peaks of 40 amperes. The middle curve is absent, as no arrester is used in this test. In Fig. 2 however, where an oxide film cell is shunted across the transformer, the middle oscillogram shows the current oscillations passing through the arrester, with peaks of 35 to 41 amperes. The lower curve in Fig. 2 then shows again the circuit voltage wave impressed upon the transformer, with the oscillations cut down by the oxide film cell. This voltage wave, the lower curve in oscillogram Fig. 2, well illustrates the characteristic action of this type of "counter-e. m. f. arrester"—to which the oxide film arrester and the aluminum cell belong. The oscillation peaks are sharply cut off at a maximum voltage of 60 per cent above circuit voltage: the value for which the spark gap was set. As the result, the oscillations are very greatly cut down from the high values which they have in the unprotected circuit Fig. 1 (2800 volts), and become unsymmetrical. The half waves of oscillation in the same direction as the circuit voltage are greatly reduced, by the limitation of the voltage to 60 per cent above-normal, while the reverse half waves—which lower the instantaneous circuit voltage—are less affected. Corresponding thereto the discharge current through the arrester, the middle oscil-

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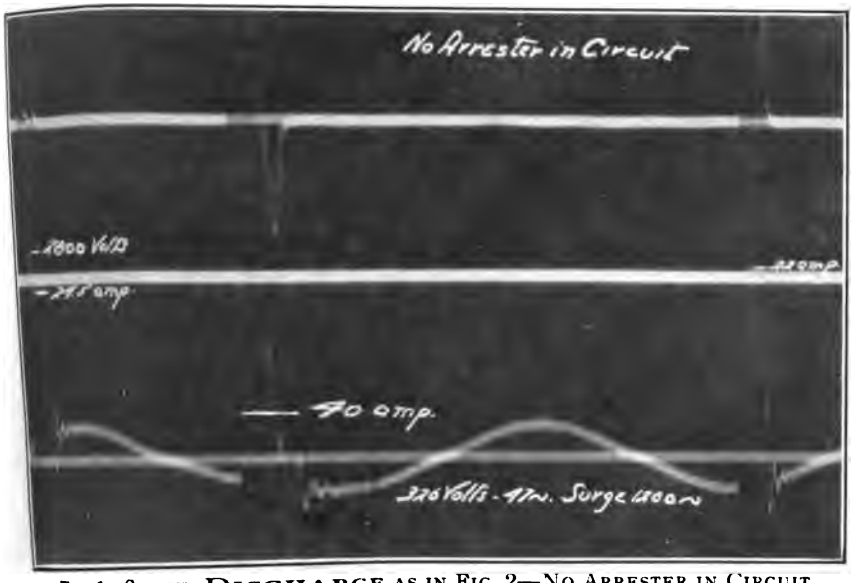
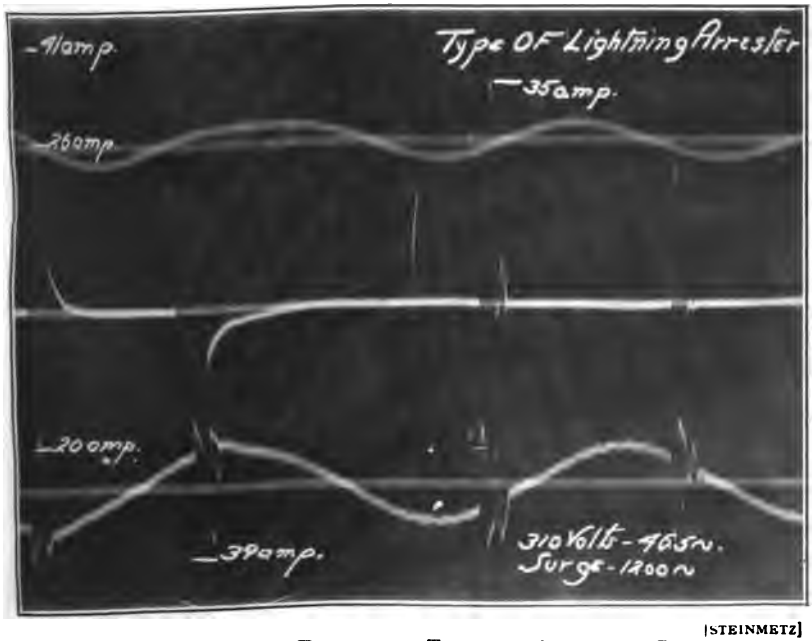


FIG. 1—SURGE DISCHARGE AS IN FIG. 2—NO ARRESTER IN CIRCUIT

Top vibrator—current through transformer.
Bottom vibrator—circuit voltage.



[STEINMETZ]

FIG. 2—SURGE DISCHARGE THROUGH ARRESTER CELL

Top vibrator—60-cycle timing wave.
Middle vibrator—current through arrester.
Bottom vibrator—voltage across arrester.

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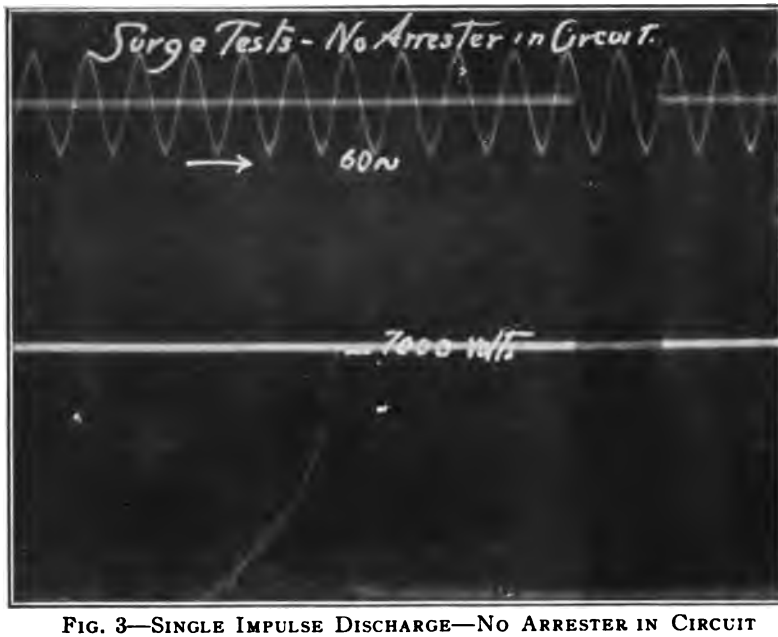
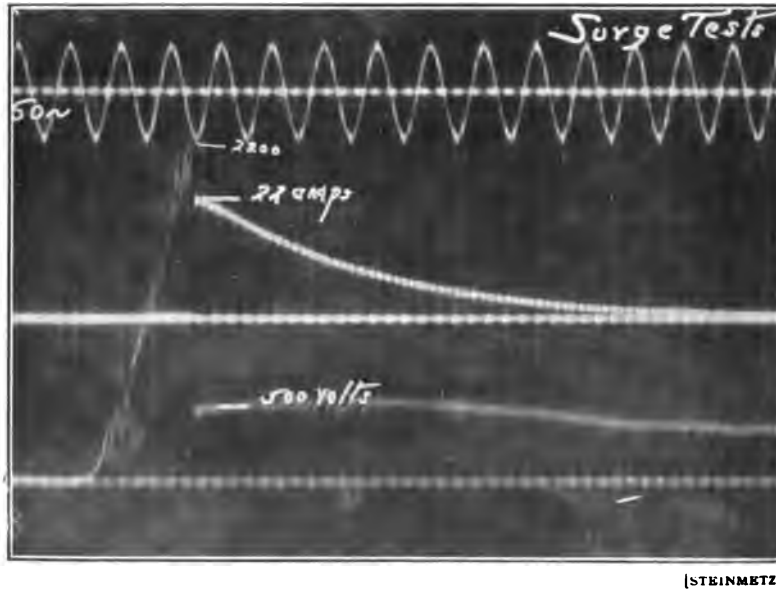


FIG. 3—SINGLE IMPULSE DISCHARGE—NO ARRESTER IN CIRCUIT
Top vibrator—60-cycle timing wave
Bottom vibrator—circuit voltage.



[STEINMETZ]

FIG. 4—SINGLE IMPULSE DISCHARGE AS IN FIG. 3—ARRESTER IN CIRCUIT
Top vibrator—60-cycle timing wave.
Middle vibrator—current through arrester.
Bottom vibrator—voltage across arrester.

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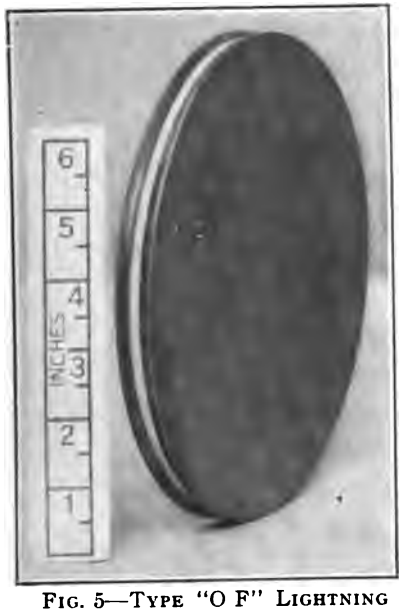
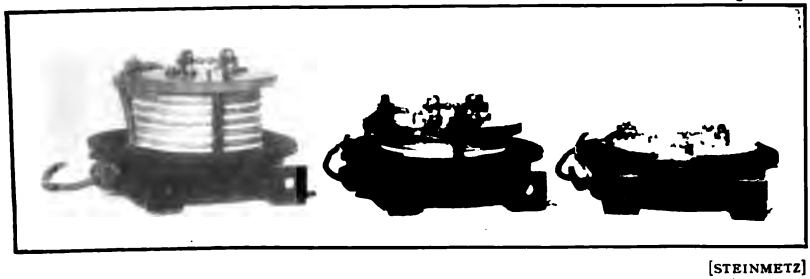


FIG. 5—TYPE "O F" LIGHTNING ARRESTER CELL



[STEINMETZ]

FIG. 6—TYPE "O F" LIGHTNING ARRESTERS WITH COVERS REMOVED RATINGS 325-650, 900-1350, 2100-2600-VOLT ALTERNATING CURRENT OR DIRECT CURRENT RESPECTIVELY

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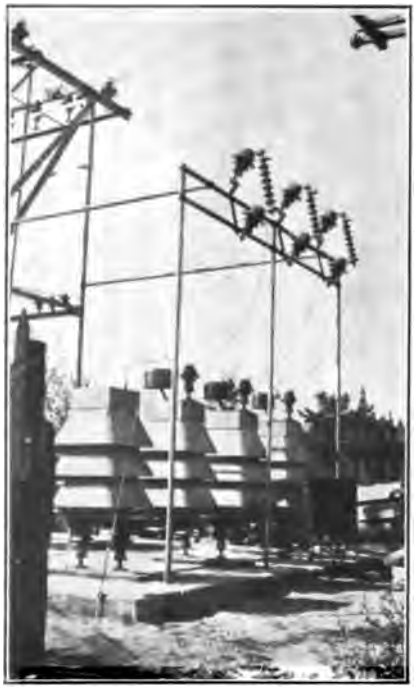
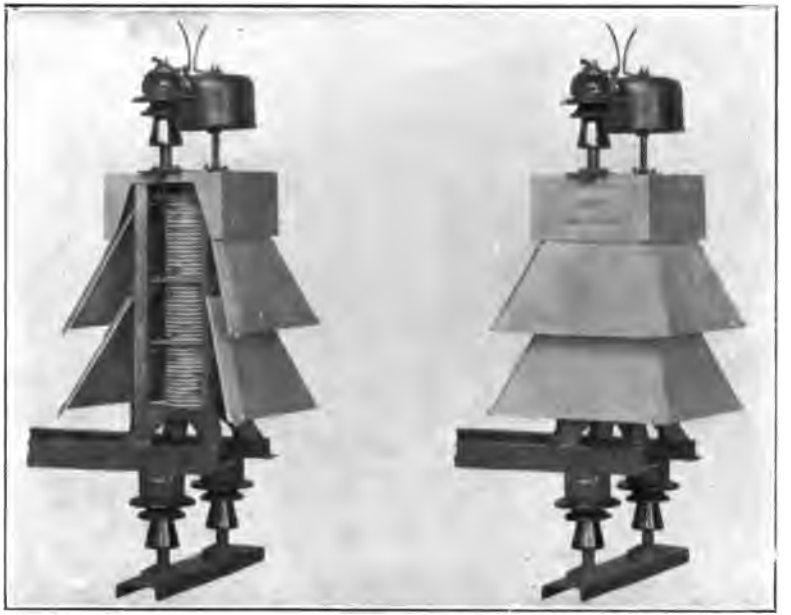


FIG. 8—TYPE "O F" LIGHTNING ARRESTER FOR OUTDOOR SERVICE INSTALLED ON A 33,000-VOLT CIRCUIT



[STEINMETZ]

FIG. 7—PHASE SECTION OF 15,000-25,000-VOLT OUTDOOR TYPE "O F" LIGHTNING ARRESTER WITH SIDE OF HOUSING REMOVED TO SHOW INTERIOR ARRANGEMENT

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logram in Fig. 2, is unsymmetrical also: in the first and second oscillation of Fig. 2, the first and third half wave of oscillating voltage is cut off, and the first and third half wave of the oscillating discharge current therefore higher than the second half wave of the oscillating discharge, which latter corresponds to a half wave of oscillating voltage in opposition to the circuit voltage, therefore not raising the circuit voltage. The third oscillation of Fig. 2 happens to start with a half wave of oscillating voltage in opposition to the circuit voltage, and the first half-wave oscillating discharge current through the arrester, in the middle curve, thus is smaller than the second half wave; the second half wave of the oscillation is cut down in the voltage and therefore gives the maximum discharge current, 35 amperes in this case.

This feature is well brought out by the oscillograms Figs. 1 and 2, due to the use of a different frequency, 60 cycles, for the power supplying the oscillator. This caused the successive oscillations to occur at different parts of the half waves of the 47-cycles circuit which was to be protected.

Fig. 3 and 4 then show the protective action of the oxide film arrester on a 550-volt direct-current circuit, against a single (non-oscillatory) impulse produced by opening a highly inductive circuit (railway motor). Fig. 3 shows on the lower curve the oscillogram of the impulse in the 550-volt circuit, rising to 7000 volts. The upper curve merely is a 60-cycle timing wave, to enable measuring the duration of the impulse. Fig. 4 shows the same circuit, with an arrester shunting it. The impulse voltage rises to the value for which the discharge gap of the arrester is set, in this case 2200 volts. Then the arrester discharges, and the voltage instantly drops back to normal, while a slowly decreasing discharge current through the arrester dissipates the magnetic energy of the impulse.

The cell of the oxide film arrester, shown in Fig. 5, consists of two circular metal plates as electrodes, which are kept apart by a porcelain ring, as shown in the figure. The space between the electrodes inside of the porcelain ring, is filled with the active material, lead peroxide PbO_2, which is put in under moderate pressure. This active material is a good conductor, but has the characteristic, that by the action of an electric discharge, it is converted in the path of the discharge, into a lower oxide, which is an insulator. Thus when an alternating current is passed through such a cell, the active material at the electrodes grad-

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ually converts into a non-conductor, and forms a thin insulating film at the electrode. This grows in thickness, until it cuts off the further flow of current and holds back the voltage, about 250 to 300 volts per cell. Then only a small leakage current, of a few milliamperes, passes at normal voltage, but if an over voltage of any kind appears at the cell, the insulating film of lead oxide punctures and freely discharges through the lead peroxide, but in doing so, converts the surface of the lead peroxide in the path of the discharge into the lower non-conducting oxide, and thereby closes the puncture holes, repairs or reseals the film.

In manufacture, naturally, just as in the aluminum cell arrester, the insulating film is not produced after assembly of the cell by the slow process of passing a current through, but the film is put on before assembly, in the oxide film arrester, by dipping the plates in a suitable insulating varnish, which gives them a coating just thick enough to hold back the circuit voltage. Then after assembly, voltage is put on the cell for testing it and sealing any holes or defects which may exist in the varnish film.

In the oxide film arrester, the electrodes have nothing to do with the arrester action, and any suitable material can be used. First we used brass, but now use sherardized iron, the latter having a higher melting point and thus standing high power discharges, which would melt holes in the brass electrodes.

In this arrester, the action, which holds back the normal voltage but passes freely an over voltage, thus resides in the active material between the electrodes, and it is this material which forms and reforms the film. As this material is a solid, no chemical action occurs such as the gradual dissolution of the alumina film in the aluminum cell arrester, but the film remains intact permanently, and thus no daily "charging", that is, repairing of the film, is required.

A number of such cells, depending on the voltage of the circuit, are piled on top of each other, with a spark gap in series, and, for low and moderate voltages, incased as shown in Fig. 6.

As the cells are hermetically sealed, by the metal of the electrodes being spun over the porcelain separating ring, the cells can be installed outdoors as well as indoors, requiring in outdoor installation merely some protection by petticoats, as shown in Fig. 7, to keep the rain from short-circuiting the cells. Fig. 8 shows such an outdoor installation of a 33-kv. arrester, with three-phase stacks and the ground stack of cells, protected against the

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weather by metal petticoats. Fig. 8 shows also the spark gaps on the line side of the arresters. They are protected sphere gaps, to give instantaneous discharge, with a horn attachment to allow the arc to flare up and thereby help in its extinction.

As well known, the plain horn gap has the disadvantage of requiring an appreciable —though a very short (microseconds) —time for discharge, and an extremely sudden high voltage, as a very steep wave front, thus may pass the horn gap and flash over elsewhere. Therefore in modern high-voltage lightning arresters the horn gap is shunted by a properly proportioned sphere gap, the latter being "instantaneous" in its action. In outdoor use however, rain lowers the discharge voltage of the sphere gap, and thus requires a setting which gives a higher discharge voltage in dry weather than necessary. Therefore a protected sphere gap has been designed, which overçomes this disadvantage in the open sphere gap, and is shown in Fig. 8.

The need of using this spark gap in series with the arrester, is the only still remaining undesirable feature which the oxide film arrester shares with the aluminum cell arrester, the multi-gap arrester and other types. While by the work of Mr. F. W. Peek, on the time lag of electric discharges[3], the means have been given to make the discharge gap "instantaneous", that is, faster than any other discharge path over gaps or through insulation in the system, so that the arrester takes care effectively of any over voltage above its discharge voltage, it does not discharge voltages lower than the discharge voltage of its spark gap, even if these lower voltages may involve some danger to the system by their high frequency. Such low voltages, while they cannot endanger the main insulation between circuit and ground, may, if of sufficiently high frequency, lead to local accumulations of voltage across inductive parts of the circuit, as regulators, current transformers, end turns and coils of generators and transformers, and there cause damage by puncturing insulation between turns and causing internal short circuits.

Against these high frequency disturbances of moderate voltage, the only existing protection is the addition to the arrester of a capacity discharge path permanently connected from the circuit to ground. Such capacity path should be without resistance to flatten steep wave fronts, and contain a moderate series

3. A. I. E. E. TRANS. 1915, Vol. XXXIV, Part II, p. 1857, *The Effect of Transient Voltages on Dielectrics* by F. W. Peek, Jr.

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resistance, to dissipate high-frequency energy and stop cumulative oscillations in their beginning. Before I leave the field of electrical engineering, I hope still to see an arrester, of the type of the oxide film or the aluminum cell, which has no spark gap, but is permanently shunted across the circuit, and thus capable of taking care not only of over voltages, but equally well of steep wave fronts and high-frequency oscillations, even if of lower than the circuit voltage. Such an arrester then would give universal protection.

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Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

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THE OXIDE FILM LIGHTNING ARRESTER

BY CROSBY FIELD

ABSTRACT OF PAPER

The oxide film arrester is a new type of lightning arrester made up of a film of insulation in contact with a conducting powder. Upon the application of over-voltage, the insulation will be pierced, but the powder will very rapidly turn into insulation and plug any holes punctured in the original insulation by the over-voltage, thus forming in substance a resealing insulation. A brief description of this arrester is given together with the principles underlying its action and a comparison with other types of lightning arresters. Mention is also made of other characteristics of this combination which are not used in the present arrester, but which are being applied in other developments. A few notes of tests on the commercial arrester complete this paper. With the exception of the basic patents issued to the author this is the first time any disclosure has been made of this arrester.

THIS paper will be confined to a brief statement of the scientific principles underlying a new type of lightning arrester called the "oxide film arrester."* The functioning of this arrester depends upon the fact that certain dry chemical compounds can be changed with extreme rapidity from very good conductors of electricity to almost perfect non-conductors by the application of a slight degree of heat. Lead peroxide is a good example of such a substance. It has a specific resistance of the order of one ohm per inch cube. The resistance varies with the pressure to which it has been compressed. At a temperature of about 150 deg. cent. the lead peroxide (PbO_2) will be reduced to red lead, commercially known as minium (Pb_3O_4). This has a specific resistance of about 24 million ohms per inch cube. At slightly higher temperatures this minium will be reduced through the sesquioxide (Pb_2O_3) to litharge (PbO), which last named is practically an insulator. [A megger reading of infinity is obtained on a column 3 millimeters long (0.11 in.) and 5 square millimeters area (0.2 sq. in.)].

Again the oxides of bismuth give similar characteristics. There

*U. S. Patent No. 1,238,660—Crosby Field.

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are, furthermore, several other compounds and mixtures of compounds that will give these same results.

Lead peroxide is normally in the physical state of a powder If this powder be placed between two electrodes and a current passed, the temperature due to the resistance at the contact of the peroxide and the metal will cause heat to be generated locally at the surface. When this heat is sufficient to create a temperature of about 150 deg. cent. a film of the lower oxides of lead

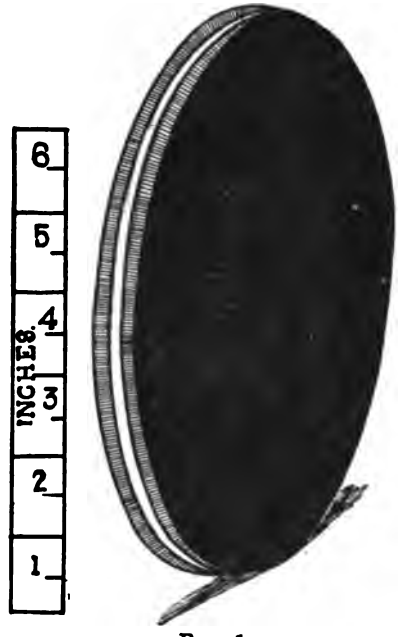

FIG. 1

A single cell of the oxide film lightning arrester consisting of a ring of porcelain with two circular steel disks, one spun on each side of the porcelain and insulated by the porcelain. The inside of the ring of porcelain between the plates is filled with peroxide of lead.

forms, producing a film of insulation which stops the current. This method of film formation over any large area is rather irregular, and of course the oxide is not used in such a fashion in the commercial arrester. Instead of this formation of litharge film any insulating film may be put on the electrodes initially. As insulating film spread on the metal plates there have been used thin layers of the following; glass, water glass, halowax, cloth, balsam, shellac, oil, paints, lead paints, varnishes, and lacquers of all available kinds. In all cases the results are sim-

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ilar, varying only with the voltage at which puncture of the film of insulation occurs.

The foregoing statements define the principle of the commer-

FIG. 2

Shows the disassembled parts of a single cell of the oxide film lightning arrester. From left to right are a steel disk spun on a ring of porcelain, a pile of brown peroxide of lead, the other steel disk and an asbestos washer.

cial oxide film arrester. Fig. 1. It comprises two sheet metal electrodes, set about 0.5 in. (12.7 mm.) apart, one or both covered with a thin insulating film and the space between the plates

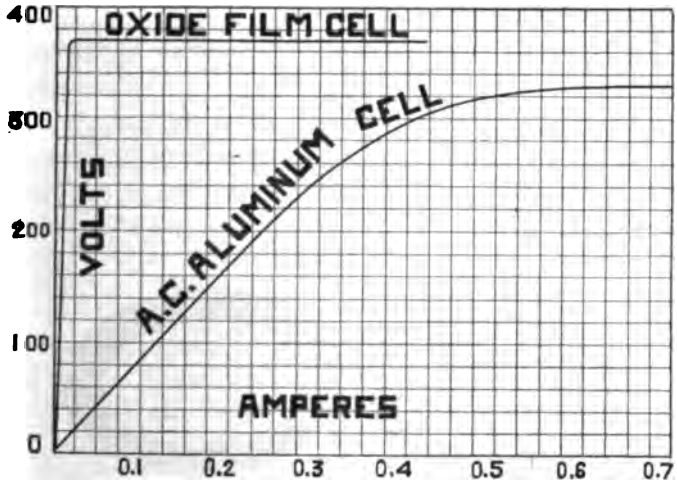

FIG. 3

The comparative volt-ampere characteristics of the oxide film cell and the a-c. aluminum arrester cell. The oxide film has only a few milliamperes of leakage current up to the critical film voltage when the film gives way more suddenly than the film in the aluminum cell. The critical voltage of the oxide film can be made approximately as low as the hydroxide film on the aluminum cell.

filled with some such substance as that described above, as, for example, lead peroxide. Fig. 2 shows the disassembled parts of a single cell. At a permissible voltage of 300 volts per cell the insulating film prevents any appreciable current flowing under

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normal conditions. As soon as the voltage rises slightly above normal the film punctures in one or more microscopic points, the lightning charge meets with practically no resistance and flows to earth. Fig. 3. The dynamic current starts to follow but because of the fact that the insulation was punctured in such

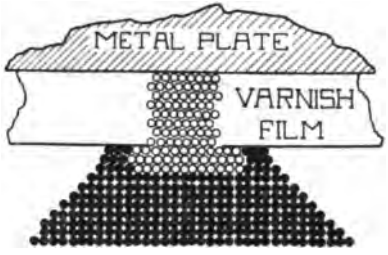

FIG. 4

A magnified, imaginary representation of one of the films on one metal plate. As shown the film is punctured by a spark and filled with a litharge plug which is represented by the open circles. The cross-section in the discharge path, a short distance away from the metal electrode, is sufficiently expanded to make the current of low enough density not to heat the peroxide to a temperature of reduction to litharge. The peroxide is represented by the solid dots and only those in the path of the discharge are shown. At the other electrode, not shown in this magnified diagram, a similar effect may be taking place although there is a difference between the positive and negative crators.

fine points, the current density near these points is exceedingly great. This results in a localized heating which speedily raises the temperature to a value sufficient to change to insulating litharge all the conducting peroxide in this minute path of the current flow in contact with the electrodes. The film conse-

FIG. 5

New electrode and also one from a cell having had passed through it several thousand discharges. The light colored plugs of litharge are plainly visible in the many spots on the surface of the plate.

quently reseals, stopping the further flow of dynamic current. This action is so rapid that its duration cannot be measured on an oscillograph giving two thousand cycles per second, that is to say, the action of resealing occurs in less than one four thousandth part of a second after the excess of lightning voltage has ceased. Fig. 5 shows the visible spots of insulating litharge plugs on the surface of an electrode.

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This film can be made of litharge itself, as well as any of the insulating materials above named. For example, metal plates may be inserted in any of the well-known lead electroplating solutions, and thus a very thin lead peroxide film (measuring a few hundred thousandths of an inch) formed. By proper heating this will be changed to litharge and this form of electrode can be used. Peroxide may also be sprinkled over any metal plate and the plate heated, which will reduce the peroxide to litharge. Again, the metal chosen for the electrode itself may be lead and if heated in the air a thin film of litharge will be formed on the surface. Again, an aluminum electrode may be put in any of the common electrolytes, and a thin aluminum film be built up. This may be used with the peroxide powder. Of these methods of forming the film the most preferable is by dipping in varnish or lacquer highly burnished surfaces of brass, steel, or copper, and is consequently used in the commercial arrester. The ohmic resistance of the arrester during discharge is quite low (less than 1 ohm per cell). Thus, when the insulating film is punctured the arrester offers very slight impedance to the flow of energy at abnormal voltages.

There is a certain range of voltage necessary to pierce any given insulation. The exact voltage depends not only upon the thickness of the insulation and its dielectric strength but also on the relation of the dielectric spark lag to the duration of the super-spark potential and the frequency of alternations of the transient surge.

If an arrester is to give protection of insulation in shunt with it, the arrester must relieve the abnormal electric pressure before damage is done to the insulation. Although tests are frequently made with the arrester and the insulation it is to protect in parallel, a more convenient method has been standardized and is known as the equivalent sphere gap test. Both the insulation and the arrester are compared by comparing each to the equivalent sphere gap.

The equivalent sphere gap of the oxide film arrester may be analyzed, as in other cases, into separate and distinct parts. First, there is the equivalent sphere gap of the main gap in series with the cells. Second, the equivalent sphere gap to initiate a discharge through the insulating film on the plate surface of the cell. Third, there is the equivalent sphere gap of the resistance drop of the current discharging through the powdered peroxide in its path. Fourth, there is the equivalent sphere gap of the inductance of the arrester.

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Commenting on these factors in their relation to this arrester, the main gap is itself a sphere gap which has the fastest spark of any practical gap. The gap setting, like that of the aluminum arrester, is only slightly above that of the normal voltage of the circuit.

The equivalent sphere gap of the film is several times greater than the thickness of the film because solid material has a greater dielectric spark lag than air, but with this multiple of the thickness of the film the equivalent sphere gap is still low. Since peroxide is a good conductor, the series resistance in the path of the discharge is insufficient to give an undesirable voltage drop. As to the inductance of the arrester, it has a minimum value due to the fact that each cell is only 0.5 in. long, as shown in Fig. 1, and these cells are placed one on top of another. In other words, the total length of the arrester (which constitutes the inductance) is short as compared to the necessary length of conductor from line to earth.

One of the obstacles that had to be overcome in the making of this arrester was the increase in the resistance after a great many heavy discharges had passed through it. The predominant reason for the increase seems to be explained by the following theory. The current passing through this small puncture in the film heats up very rapidly not only the powder but also the air contained within the interstices of the powder. The particles are thereby thrown out of contact with each other, thus producing a fluffiness. The decrease in the number of contacts decreases the actual cross sectional area of conduction, hence increases the resistance. This raises the equivalent sphere gap. This action is accelerated, of course, by the giving off of the oxygen itself evolved in the reduction from lead peroxide to the lower oxide. If, however, this same arrester be violently jarred or the filling powder be compressed, or any other method utilized to restore the particles to their previous intimate contact, the equivalent needle gap will fall again. While increased fluffiness appears to be the predominant cause of change of the equivalent sphere gap, the increased thickness of the film of litharge at the point of puncture of the film is finally a factor of moment. The total area of the film must be sufficient to give a reasonable number of years of life to the arrester. There are other factors relating to the details of manufacture which give a limited degree of control over this change in equivalent sphere gap.

In all the commercial oxide film arresters used for alternating

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current the power factor is nearly unity. For special purposes however, the power factor can be made anything desired from 10 per cent. to unity. This is obtained by combining with the conducting oxide other non-conducting materials. This principle has been made use of for condensers but it has not been found desirable to incorporate it in the arrester.

To summarize—an arrester operating under a new principle has been made which comprises in essence one or more metal electrodes covered with an insulating film, and separated by a conducting powder, which has the peculiar characteristic of becoming a non-conducting powder upon the application of heat. Voltage higher than that which can be withstood by the insulating film punctures it in one or more points of about 0.005 cm. diameter. Dynamic current flowing gives a high current density in the conducting powder adjacent to these punctures which in turn heats it up rapidly, reducing the powder to a non-conductor, and sealing the holes in the film. The powder being a poor heat conductor localizes this action, so that very little more powder is reduced than is actually necessary to seal up these minute punctures.

The critical spark voltage and that part of the equivalent sphere gap controlled thereby is a function of the thickness and kind of material used for the film.

Comparison of the "Oxide Film" with Well Known Arresters

The earliest form of non-electrolytic film arrester was known as the dry aluminum arrester.* It was a direct attempt to utilize the dry film which forms on the surfaces of pure aluminum immediately after it comes in contact with the oxygen of the air. The hydroxide film is easily formed in electrolyte and on drying becomes a dry film which gives sufficient action to prevent a discharge up to a given critical voltage, depending upon the thickness of the film. The film can also be formed by a spark or arc of a conductor in contact with a plate. Naturally this conductor should be of a non-metallic nature. In the earliest form tried powdered carbon was used mixed with dioxide of manganese which gives a liberal supply of oxygen at the heated point.

One of the objects of the development of this arrester was to decrease the cost of manufacture and it was found with the new principle involved in the oxide film arrester, where the powder

* U. S. Patent, E. E. F. Creighton.

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furnishes the film rather than the plate, that the aluminum could be replaced by a cheaper metal, such as steel, and, as already described, the initial film known in the early stages of development as the "paint skin" type could be furnished by a layer of varnish. On first sight, knowing the extreme thinness of the hydroxide film on wet aluminum cell it might not seem that the dry cell would give the same general characteristics as the wet cell. But a comparison of the volt-ampere curves shows the same general characteristics. For a-c. voltages of 300 volts average per cell the current in the dry cell is of negligibly small value up to 40 milliamperes. The power factor is nearly unity and the current flow is due to very slight leakages through the films. In the case of the aluminum electrolytic cells there is an equivalent condition, the d-c. leakage current of the order of one milliampere being due to leaks through the hydroxide film. In the a-c. aluminum arrester the leakage current on the plate area used is much greater due to the destructive action of the alternating current on the hydroxide film. Furthermore, the wet cell with its thinner film is a condenser of appreciable capacitance which takes a charging current of about 0.5 ampere at 60 cycles. When the voltage reaches a certain critical value which is between 300 and 400 volts for the wet aluminum cell and between 300 and 500 for the oxide film cell (or higher if the paint film is made thicker) the current is allowed to pass freely through the cells, limited only by the ohmic resistance of the cell independent of the film. Since the oxide film arrester has no dissolution of the film, as occurs in the wet aluminum cell, charging is not only unnecessary but undesirable. This extends the use of the oxide film arrester to localities where there are no attendants.

Although the wet aluminum plate becomes frosted to an appreciable thickness by the passage of current in long use, the actual thickness of the film, as represented by the critical voltage, is not changed. In the oxide film arrester, however, the film less than one 1 mil thick (0.025 mm.) initially thickens up by the addition of successive spots of litharge for each successive discharge. This represents the wear on the arrester and limits its total life. Fig. 3 shows comparative volt-ampere characteristics of the oxide film arrester and the a-c. aluminum arrester. Since both of these arresters have a leakage current which wears the plates of the cells when alternating current is supplied, it is necessary, as previously stated, to place a spark gap in series with the cells. This spark gap is set at a value

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slightly above the normal potential of the circuit so that nothing but abnormal voltages will cause a discharge.

The foregoing data show that the oxide film arrester has general characteristics closely like the standard aluminum electrolytic arrester. It has the obvious advantage which comes from being dry rather than wet, it will not congeal and needs no daily charging.

In making the characteristic volt-ampere discharge curves the oxide film arrester does not lend itself as readily to the test as the wet aluminum cell. While its critical film voltage is evident, the change from no conduction to full conduction is more sudden. Therefore, the discharge rate at double potential is best shown by throwing double potential on the cell and subsequently reducing the voltage to its normal value per cell.

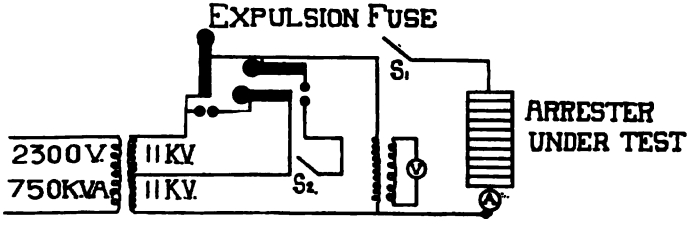

FIG. 6

Arrangement of three expulsion fuses which short-circuit gaps in the path of discharge to first throw double voltage on an arrester under test and then reduce the voltage to normal value by shifting the contacts on a transformer from full coil winding to half coil winding.

This gives a very considerable quantity of electricity through the cells and is a severe test.

In order to conveniently alter the voltage from double value to normal value the circuit is arranged as shown in Fig. 6. The object of the connections may be briefly stated: A transformer with two coils in series impresses voltage on the arrester and then the contacts of the arrester are automatically shifted to one coil giving half the voltage. A heavy pendulum closes switch *S*-1 and sets the oscillograph into operation. The full voltage on the transformer is thrown on to the oxide film arrester under test, marked *O-F*, which has a number of cells sufficient only for half the voltage of the transformer. In other words this throws double voltage on to the arrester and the heavy current passing through the cells causes fuse *F*-1 to blow. The operation of this fuse short-circuits half the transformer and throws the other half across the arrester. This is done by

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means of gaps and fuses as follows: When the expulsion fuse *F*-1 blows, the conducting gases are shot into the open gap *G*-1 which closes the circuit through fuse *F*-2 to the mid-point of the transformer. This short circuit on half the transformer causes fuse *F*-2 to blow and the hot gases discharging from fuse *F*-2 close the gap *G*-2 which throws the mid-point of the transformer on to the arrester through the switch *S*-2 which is closed just previous to starting the test. These several operations occur with a rapidity depending upon the size of fuses used. It is possible

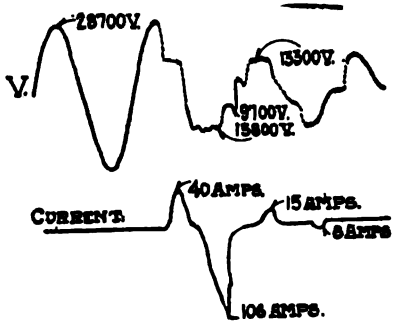

FIG. 7

Operation of an oxide film arrester on double dynamic voltage initially and its recovery on normal voltage. The upper record with a peak voltage of 28,700 volts shows the arrester connected to the circuit as the voltage wave starts to decrease on its third peak. The voltage immediately drops to the critical value of the cells—about 13,500. On the lower record the current is seen to rise to 40 amperes. On reversal of the voltage to 13,800 in the negative direction as shown by the upper record, the current as shown by the lower record rises to 106 amperes. The switching operations produce several electro-magnetic kicks as shown by the irregular voltage wave as it rises in the next cycle to 13,300 peak value. At this lower applied voltage the current in the series of cells, as shown by the lower record, is 15 amperes. In the subsequent half cycle the current rises only to 8 amperes. In the last half cycle of voltage shown, where the switching operation is complete, and the wave assumes its normal smooth form, the current in the cells is too small to be registered by the oscillograph. Its value is of the order of milliamperes. This figure is a copy of an oscillogram. The copy was made desirable by overlapping of the discharge on the two ends of the film.

by this means to throw momentarily 22,000 volts on an 11,000 volt arrester and note the character attending its discharge and recovery. Fig. 7 shows such an operation on an oxide film arrester. The initial discharge current is 40 amperes during the first half-cycle due to the point it strikes in the descending wave during the second half-cycle it is 106 amperes. After the third half-cycle the litharge film has so completely sealed up the path of discharge that the current is too small to show on the oscillogram. The leakage current with no series gap is of the order of a few milliamperes.

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DISCUSSION ON "THE OXIDE-FILM LIGHTNING ARRESTER" (STEINMETZ) AND "THE OXIDE-FILM LIGHTNING ARRESTER" (FIELD), ATLANTIC CITY, N. J., JUNE 27, 1918.

C. P. Steinmetz: As Mr. Field's paper represents the state of the work when he left this field a few years ago, I wish to bring it up to date in a few words.

On the first page of Mr. Field's paper he refers to the resistivity of the filler powder as one ohm per inch cube. At present this resistivity has been reduced to 0.1 of an ohm per centimeter, or 0.4 ohms per inch cube, that is, 25 times the conductivity, and correspondingly increased discharge rate, and as a result thereof, the resistance is materially less than 0.1 ohm per cell.

At the time that Mr. Field left it was found by laboratory experiment that there is an increase in resistance, after a great many heavy discharges made on the arrester in the laboratory, resulting in a gradual, though noticeable, increase of the equivalent sphere gap, and resulting decrease of the protective action. This, Mr. Field has explained by the production of gas by the discharge causing a fluffiness, and as a contributing cause by the reduction of the actual section, by the increase of thickness of the film and also the wearing action of the leakage current.

This feature was, indeed, the one which led me in my paper to refer to the necessity of industrial experience in existing plants, as a final criterion in lightning protective apparatus, since with all our skill and ability and thought, we cannot perfectly reproduce the actual conditions, and this was one of the reasons which led us to hold back the arrester in order to get fuller information on its life, and I am glad to say that after two to three years of industrial service, and after the arrester has received many thousands of heavy discharges, cells have been taken off and retested, and we have found no increase of resistance and no increase of the equivalent spark gap.

Although we have thus not found how long a life the arrester has, we have evidence that the life of the arrester, if it is not accidentally destroyed, is so long that in two or three years' industrial service no appreciable or measurable deterioration occurs.

After finding this out by experience, we could realize, why our tests were not representative of industrial practise. We passed continuously and in rapid succession very many thousands of heavy discharges through it, and since every discharge produces a little gas, gradually gas must accumulate, and finally result in separation of the particles, increasing the resistance and the equivalent spark gap. In nature, however, we have individual discharges, or even a large number of discharges following each other, but then followed by a period of rest, when the gas produced can be dissipated, and that is the reason why the increase of equivalent spark gap did not occur, and so far we have found in two or three years no evidence of deterioration of this arrester under industrial service.

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With regard to the wear produced by the leakage current. Such a wear exists in the aluminum arrester, where the leakage current is an electrolytic current, but it does not exist in the oxide film arrester, as the conducting material is of that class of conductors which I have called pyro-electric conductors, having a definite metallic conductivity at ordinary temperature, which, only at high temperature changes to electrolytic conductivity, and therefore this leakage current does not deteriorate.

As regards the discharge rate, I want to call attention to the oscillogram on the last page of Mr. Field's paper, which, as he states, shows the characteristic of the arrester in holding back the normal circuit voltage, and discharging with practically no resistance the over-voltage. In this oscillogram double voltage has been thrown on the arrester, taking the discharge, dropped the voltage, not to nothing, as a short-circuiting arrester would do, thus not short-circuiting the machine power, but to the "counter electromotive force" of the cell, discharging only the over voltage. As this was of limited power, the current discharge is only about 100 amperes, but this does not represent what we call the discharge rate of the arrester, which is the current which would pass at double voltage, because the current in this oscillogram is not the current taken at double voltage, but at very little above normal voltage.

The test to really find the discharge rate of the arrester is to impress double voltage with so much power back of it, that the voltage does not drop, and it is a difficult test, because it requires an enormous power supply back of the arrester, and therefore I cannot give you entirely definite values, but only the information that the discharge current at double voltage is materially above 1000 amperes, and apparently higher than that of the aluminum arrester and the protection apparently greater than or at least fully as high, as that given by the aluminum arrester.

In a future paper, Mr. Lougee, who continued the development after Capt. Field, expects to give more complete information on tests and performances of this arrester.

N. A. Lougee: The oxide-film arrester, although a new arrester, has been shown by tests and operating experience to be very reliable and to give excellent protection. Very many tests have been made on this arrester duplicating as far as possible in a laboratory all the operating conditions it might be subjected to, in order to be as certain as possible that its operation would be successful before it left the laboratory. Moreover, complete data have been obtained by means of discharge alarm recorders attached to the majority of the arresters which have been installed to date. We hope in a later paper to give an account of the extensive laboratory tests and operating results which we have obtained.

Table I, I believe, will be of interest in showing briefly how and to what extent the high-voltage units of oxide film arresters have been installed.

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TABLE I.

			1915	1916	1917	1918 up to June 1st	Total up to June 1, 1918
Number of cells installed		Indoors	200	2932	1448	1544	6124
		Outdoors	...	...	1440	2900	4340
		Total	200	2932	2888	4444	*10,464
Cells used for life runs, Tests and Development:			3488	2436	1712	465	8101
Number of Arresters installed	11—15 Kv.	Indoors	2	5	6	10	23
		outdoors	..	..	5	6	11
		total	2	5	11	16	34
	22—25 Kv.	indoors	..	7	4	2	13
		outdoors	..	..	3	4	7
		total	..	7	7	6	20
	33—36 Kv.	indoors	..	4	..	..	4
		outdoors	..	..	1	4	5
		total	..	4	1	4	9
	Total	indoors	2	16	10	12	40
		outdoors	..	..	9	14	23
		total	2	16	19	26	63
Number of recording devices installed:			1	9	16	19	45
Number of arresters destroyed:				1	1		2
Cause				Isolated transformer stations destroyed by fire.			

*In addition hereto, 10,000 cells are in production and will be installed during the present season.

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In addition to these cells, we installed in 1914 about forty 650 d-c. "O F" arresters on electric cars and car circuits. These arresters were of a preliminary type and of different design than is used at present. These first experimental units gave excellent results and many are still in operation.

As noted from the table, we installed the first high-voltage arresters in 1915 and since then we have gradually increased the number of installations and added an outdoor design. I wish also to call attention to the great number of cells we have used for development work. These 8000 are of the present cell construction—before this design of the present cell was perfected, a great number of preliminary units, of course, were used in the development. About three-quarters of the installations at present are equipped with a discharge alarm recorder so that we have definite information that the arresters are successfully taking care of all kinds of disturbances.

I think it will be of interest at this time to say a little more concerning the life of this arrester. This fact was something impossible to determine from laboratory tests alone, and consequently we were anxious to learn what effect actual operating conditions would have on the cells, before we considered it desirable to publicly announce the development of this arrester.

One cell from each of the four stacks of two different 11,000-volt "O F" arresters were returned to us for test after these arresters had been in operation about a year. Very careful tests on all these cells showed that the initial equivalent sphere gap had not increased in any case, that the 60-cycle voltage breakdown had not increased, and oscillograms showed these cells to give excellent protection against a surge voltage. The interior of the cells, which were opened, showed no deterioration of the filler, in fact the cells as a whole were exactly as good as new cells. The arresters from which these cells were taken were equipped with discharge alarm recorders and these recorders showed an advance in counter reading of about 3000 discharges in each case since the arrester was installed.

Last winter one complete phase from each of the two arresters mentioned above was returned for further tests. These arresters during the two years they had been installed showed several thousand discharges from the discharge alarm recorder, and moreover operators in each station had kept a careful record so these particular arresters were known to have had more than normal operation. Initial equivalent sphere gap measurements on a complete phase as a whole showed a value similar to what is obtained with new cells. Some of the cells were opened and no deterioration at all could be seen.

Cells from a few other installations have been returned to us and in every case to date no deteriorations in the protective value of the cells or of the interior of the cells has been found.

By having cells returned from arresters every once in awhile, we will eventually learn what the average life of an arrester

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under normal service should be, and at present there is every indication that the life of the oxide-film lightning arrester is at least as long as that of any other lightning arrester.

L. W. Chubb: I would like to know whether tests have been made to find out whether one point heals up and an entirely different point breaks down, on a heavy discharge, or whether the first place that breaks partially heals up and progressively breaks down along the edge causing the point of discharge to travel along like the wandering of an arc on an electrode. The reason I ask is because, if the original puncture heals completely there must be an interruption and rise of voltage in order to break down a new point of the film. If the electrostatic charge of the film is not sufficient to heal one spot the voltage must drop and rise with each successive puncture giving an interrupter action and dangerous conditions similar to an arcing ground. The electrolytic arrester has enough capacitance and stored energy to heat a scintillation or breakdown without an appreciable drop of voltage and thus gives the true counter e.m.f. effect.

The oscillograms Figs. 3 and 4 in Dr. Steinmetz's paper, show the discharge of an electromagnetic field, without and with the arrester in shunt. In the first case the voltage rises to 7000 volts, in the second case the voltage rises to the gap voltage of 2200 volts and then drops to 560 volts, after the arc is established. The gap, of course, has an arc drop of approximately 50 volts, leaving about 510 volts creating a current of 22 amperes. I cannot understand how the resistance per cell can be as low as one ohm or 0.1 ohm per cell in view of this result. The oscillograph is too slow to follow very high-frequency oscillations and therefore gives only a smooth mean curve.

One of the advantages of the electrolytic type of arrester is the presence of high capacitance. In many cases high-frequency and steep-wave-front impulses can be discharged across the gap and absorbed without great rise of voltage or piercing of the film. I would like to know whether the new arrester has sufficient capacitance to obtain this condensive effect.

In both papers the equivalent spark gaps of different parts of the arrester are mentioned, but no figures are given. I would like to know what the equivalent spark gap of the arrester is. It seems that a varnish film which will hold line voltage so as to extinguish the gap after discharge will require several times line voltage before it pierces on an impulse, as the impulse ratio of solid dielectric films is known to be very high.

C. T. Allcutt: About the only point I want to mention in connection with Dr. Steinmetz's paper is the matter which Mr. Chubb brought out regarding the electrostatic capacity. As far as I can figure from the dimensions given, and from certain figures regarding the charging and leakage current of the arrester given by Capt. Field, the capacity of the oxide film arrester is in the neighborhood of 0.01 of the capacity of

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the aluminum arrester, and as a condenser, its effect in discharging impulse voltages would not be very great. Apparently, it would be necessary, in discharging an impulse, to have that impulse actually pierce the dielectric film of the arrester, which might be expected to involve some time lag and some decrease in voltage. The difference in capacity is very great, and in that point, certainly the oxide film arrester does not possess all of the advantages of the electrolytic arrester, as it would seem.

C. P. Steinmetz: I will answer the three questions asked. Whether the puncture occurs at the film or a sealed-up puncture hole, is merely a question of probability. In resealing after a discharge the litharge spot thickens up so as to hold back the circuit voltage, and it is merely a question of probability whether the next discharge which punctures goes through an unbroken film or through a litharge spot, since just as in the aluminum cell, the resealed puncture hole is of the same strength as the rest of the film.

As to the capacity effect: The capacity of the cell is higher than it was in Capt. Field's time, due to the decreased resistance of the film, but it is less than that of the aluminum cell. The capacity at normal frequency is of less magnitude than the resistance, but it is, as you see from the construction, sufficiently large to act as a capacity path for those extremely steep wave fronts, and extremely high frequencies at which the time lag of the insulation comes into consideration, though at 60 cycles the capacity is practically negligible.

The oscillograms of the impulse voltage given in my paper as Figs. 3 and 4, are identically the same kind as would be given by an aluminum cell, that is, the impulse voltage rises to the breakdown voltage of the gap, and at that point the voltage drops to the circuit voltage, and the 560 volts is not the resistance drop across the arrester, but is what you might call the "counter electromotive force of the arrester", which holds back the normal circuit voltage, the voltage of the direct-current railway generator which was on the circuit. This would occur in identically the same way with the aluminum cell, and it is a characteristic of this type arrester not to discharge the normal machine voltage, but only over-voltage. But from this follows nothing regarding the discharge resistance. The discharge resistance is obtained by dividing the excess voltage over the normal circuit voltage, by the discharge current.

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THE DESIGN OF TRANSPOSITIONS FOR PARALLEL POWER AND TELEPHONE CIRCUITS

BY HAROLD S. OSBORNE

Abstract of Paper

This paper presents the results obtained in a recent design of transposition systems for telephone circuits exposed to induction from circuits of other kinds, particularly from three-phase power circuits, and the coordinate arrangements of transpositions in the power circuits. The new arrangements are the result of a systematic investigation of the degree of flexibility which could be obtained in the coordination of telephone transpositions and transpositions in outside disturbing circuits, particularly three-phase power circuits.

In presenting the results obtained, a discussion is first given of the requirements which must be met by systems of transpositions for telephone circuits in general, and by the new "exposed line" system in particular. An outline is given of the methods used in the design work and the theory upon which it is based. The diagrams in the paper show the arrangements of transpositions for all circuits on eight crossarms of telephone line. The results to be obtained from the use of these diagrams are outlined, and the suitable locations of coordinate transpositions in parallel power circuits are discussed.

The application of these and other arrangements of coordinated transpositions to specific cases must take into account the variations in separation and other changes in the power and telephone circuits. It is considered to be beyond the scope of the present paper to go into a discussion of the problems involved in these specific applications.

Introduction

IN a well-engineered telephone system the power transmitted into a long-distance telephone circuit is almost entirely absorbed in line losses. The circuit terminates in that marvelous little electric motor, the telephone receiver, which requires only a few millionths of a watt for its operation. As the power delivered to the line by the telephone transmitter may be measured in hundredths of a watt, only a fraction of one per cent of the transmitted energy need be received at the far end of the line. This is fortunate, for as the telephone currents are of relatively high frequency, the energy losses suffered by transmission over a given circuit are very much greater than would be the losses for 60-cycle current. Moreover, the telephone currents must be

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carried over very great distances. It has therefore been necessary to do an enormous amount of work in developing means for increasing the efficiency of long telephone circuits in order that country-wide service may be given with a network of conductors which should not be prohibitively expensive.

The large ratio between the energies at the transmitting end and at the receiving end of a long distance telephone line imposes very severe limitations on the amount of inductive effect which can be permitted between different telephone circuits. If the transfer of energy between two circuits is more than a millionth of the energy transmitted there is danger that one can overhear on one circuit what is said on the adjacent end of the other. This must of course be avoided, and an even stricter limitation of the transferred energy is desirable.

The method which is used to prevent excessive transfer of energy between telephone circuits on the same pole line is the installation of a systematic arrangement of transpositions of the conductors of each circuit.

Present standard telephone practise for open-wire circuits provides ten wires spaced about one foot (0.3 m.) apart on a crossarm and any number of crossarms spaced two feet (0.6 m.) apart on a pole. The currents carried by these circuits are of very high frequency compared with those used in power practise and tend to make much of small mutual impedances or admittances between the circuits. It is small wonder then that when in 1886 the first long distance telephone line was built between New York and Philadelphia it was found that, listening at the distant end, an observer could not tell which circuit was used for transmission but could hear equally well on all of them. The systematic study of telephone transpositions began at that moment and has continued until the present day.

One factor which has much complicated the study of telephone transpositions is the common use of phantom circuits. These are more accurately described by the French terms *derived* or *superposed* circuits. A phantom circuit is formed by superposing a third circuit on two 2-wire telephone circuits called the side circuits of the phantom. The phantom and its side circuits thus use the same copper conductors, and it is evident that great care in balancing the inductive relations is necessary to prevent the transfer of energy between circuits so intimately related.

Long distance telephone lines have not grown up without

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neighbors, and in the design of telephone transposition systems it has been necessary to take them into account. Telegraph systems and power transmission systems have been contemporaries with the telephone system in the process of development. These systems have not in general been obliged, for any reason of their own, to restrict their electric and magnetic fields as has been necessary with telephone circuits. From the first, however, the limitation of the effect of these outside fields has been a factor in the design of telephone transposition systems.

It is evident that when a telephone circuit is paralleled by an electric power circuit carrying hundreds or even thousands of kilowatts, the mutual inductive connection between the two circuits must be extremely minute if we are to avoid induction in the telephone circuit of power of the same magnitude as that of the voice currents or even much more. In fact, this difficult problem would probably be quite impractical of solution if we had to rely alone upon balance produced by transpositions. Fortunately, another very weighty factor works for the solution of the problem. The fundamental frequency of commercial power circuits is out of the easily audible range and quite out of the range of frequencies which the telephone line must transmit in order to reproduce intelligible speech. As a matter of fact the noise produced by power circuits in telephone circuits is practically not at all due to the fundamental but due to the harmonics or high-frequency components of the power circuit which lie within the range of telephone frequencies. The energy represented by these harmonics is represented by kilowatts or even by watts rather than by thousands of kilowatts. In spite of this fact the degree of balance required in cases of close parallel is very precise indeed.

Of all the means used to reduce the inductive effect between power circuits and telephone circuits the coordinate transposition of both classes of circuit is probably the most generally important. The application of this means to parallels between telephone and power circuits is in practical cases attended with difficulties because instead of two circuits running accurately parallel for long distances the separation usually varies at frequent intervals and very irregularly.

The usual case, in which the power circuit is three-phase, presents another complication which is fundamental to this type of circuit. The telephone circuit is a single-phase circuit and its conductors have only two relative arrangements on the pins.

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The three-phase circuit has three possible arrangements of its conductors on the pins, and three sections of line, each with a different arrangement, are necesssary for what is called a "barrel" of the power circuit. The combinations of relative pin positions of the two circuits when both are transposed are, therefore, six in number. The difficulties introduced by this relation have been discussed in an earlier paper* before the Institute.

Although the importance of neutralizing as far as practicable the effect of outside circuits has always been a factor in the design of transposition systems, the possibilities of coordinating the telephone transpositions with the transpositions in three-phase power circuits were not at first fully developed. With the great growth in both telephone circuits and three-phase power circuits this matter has been growing rapidly more important in telephone transmission.

In connection with the very extensive investigations of the Joint Committee on Inductive Interference in California it was therefore considered advisable to undertake the development of a new system of telephone transpositions which would embody the maximum possibilities of coordination of transpositions in the two classes of circuit. There has resulted from this work a system of transpositions known as the "exposed-line" transposition system.

Outline of Methods of Design

The design of a transposition system requires the mastery of a very difficult and complicated technique. It is attempted here to give only a general outline of the methods employed and some of the difficulties encountered and overcome. Some of the mathematical work is given in the appendices.

Conductive and Inductive Connections. The mutual connection between two circuits may be of any of the following characters

Resistance
Leakage
Inductance
Capacitance

Two circuits may be said to be unbalanced with respect to each other by a mutual connection of any of the four above characters when the mutual connection enters into the circuit in such a way that current in one circuit tends to cause a current in the other circuit.

**Inductive Interference as a Practical Problem.* Messrs. Griswold & Mastick, Transactions, A. I. E. E., Vol. XXXV, 1916, p. 1051.

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By resistance and leakage we denote conductive connections respectively series and shunt. Although two metallic circuits which have no resistance in common cannot have a mutual resistance unbalance, care is necessary to avoid a mutual resistance unbalance between a phantom and its side circuits. This is however a matter apart from transpositions. The leakage from telephone circuits is almost all directly to ground. This is normally kept very low because of the large effect which high leakage would have upon the efficiency of the circuits. For these reasons leakage is not an appreciable factor in the interference between telephone circuits, except under abnormal conditions, such as those produced by baling wire thrown over a telephone line.

Two circuits strung side by side have both mutual inductance and mutual capacity unbalances. The two terms represent the coefficients by which are measured the unbalanced action of the mutual electromagnetic and electrostatic fields respectively, and both represent inductive rather than conductive effects. It is the purpose of transpositions to correct for these as largely as may be.

Inductive Effects Between Telephone Circuits. The general equations for the mutual inductive effects between long parallel circuits are hopelessly complex. For the benefit of those interested a discussion is given in Appendix A of the mathematics by which these inductive effects may be computed in some simple special cases. In Appendix B are given equations for the most simple general case, that is, the case of inductive effects between two long symmetrical parallel wires with ground return. Examination of equations (7) to (10) in this appendix shows that the current or voltage in either conductor is represented by the sum of two complex exponential terms. Little consideration is required to convince one that similar general equations for forty parallel wires would be impracticable either to establish or to use.

However, in studying the inductive effects between telephone circuits (commonly called cross-talk), use can fortunately be made of a beautiful simplification. Although the circuits are very closely associated on the same pole line, the requirement for good operation after the circuits are transposed is that the transfer of energy from one circuit to another shall be exceedingly small. This being the case, the reaction of the induced current on the inducing circuit is evidently negligible. The prob-

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lem can therefore be studied by computing the current and voltage of the inducing circuit as though no currents whatever were induced in other circuits and then computing the inductive effect of these currents and voltages on each adjacent circuit under the assumption that no current flows in any of the other circuits.

In order that this computation may be carried out, it is necessary that the coefficients of induction between the circuits be determined. The mutual inductance unbalance between all pairs of circuits on a 40-wire pole lead can be readily computed. It was estimated, however, that to compute the direct capacities between circuits would require the time of one man for at least ten years. It was therefore found much cheaper and more practical to make measurements of the capacities between the wires in a short section of line under construction. The results obtained in this way are given in Table I.

Resultant Unbalance between Long Circuits. With the data and by the use of the approximations outlined above, it is possible to compute the inductive effect between any two short lengths of telephone circuit on the pole lead. In carrying out the computation of the effects for long circuits, however, a complication arises due to the fact that the current and voltage in the disturbing circuit are different both in magnitude and phase angle at different points of the line. The change in magnitude, because of attenuation effect, varies from 0.05 per cent to over 0.5 per cent per kilometer on open wire telephone circuits of different construction, and the change in phase angle varies from a small value up to nearly 10 degrees per kilometer. Also, in the propagation of induced current along the disturbed circuit to its terminal, this current undergoes a similar change in magnitude and in phase angle. Therefore, the current induced between two circuits in one kilometer length of line cannot be perfectly balanced against that induced in another kilometer length, and no matter how often or under what arrangement the transpositions are placed, they cannot serve to perfectly balance out the induced effects. There is always a resultant effect which is known as the "type unbalance". The transposition system must be so designed that this type unbalance is kept below the desired limits for all combinations of the circuits on the lead.

A brief mathematical discussion of type unbalance is given in Appendix C.

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40-WIRE POLE LEAD.

25	24	23	22
26	27	28	29
32	27	25	32
32	25	22	26
41	31	25	27
61	42	30	29
81	58	40	35
35	25	20	24
48	30	19	17
86	45	24	19
178	91	43	25
339	182	87	46
45	30	24	26
63	35	21	17
134	65	30	
336	149		
853			
			40
			39
			38
			37
			36

Wires

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The type unbalance does not measure the entire inductive effect between two telephone circuits, for this is contributed to also by another factor which is sometimes very important. It is impracticable to locate the telephone transpositions at exactly the theoretically correct points. In laying out a new line it is sometimes possible to set poles exactly at the theoretical transposition point, and when this cannot be done the nearest pole is generally chosen as the transposition pole. However, the transposition itself occupies two pole spans. Moreover, small changes in separation of wires cannot be avoided, even going round a curve introduces a certain amount of irregularity in the separation of wires. These and other irregular factors increase the inductive effects above those due to the type unbalances and must be taken into account in designing the transposition system.

Phantom Circuits. One factor which introduces a large complication into the design of transposition systems is the general

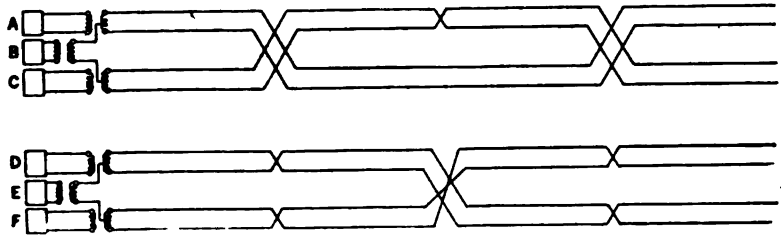

FIG. 1—PHANTOM TELEPHONE CIRCUITS

use of phantomed circuits in long-distance telephone lines. This is standard practise for long distance telephone circuits, and makes it possible to obtain three circuits from each two pairs of wires instead of two. The principle involved in phantoming is generally understood. The arrangement is indicated diagrammatically in Fig. 1.

By the transposition of a phantom circuit the pin positions of its side circuits are interchanged, and the coefficients of inductive effect between the side circuit and other circuits on the lead are changed. In order that the phantoming may be possible without interference, it is therefore necessary not only to very carefully balance the terminal transformers and other apparatus connected to the telephone lines, but also to use a transposition system that provides for a very high degree of balance of the inductive effects between the phantom and its side circuits and also between these circuits and the other circuits on the pole line.

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It is evident that in designing the transpositions in the side circuits it is necessary to take into account the transpositions in the phantom. For example, in the figure, side circuit C would be considered balanced to side circuit F if the transpositions in these two circuits only were to be considered. However, the transpositions in the phantom circuits change the coefficients of induction between circuits C and F in different parts of the section shown and therefore prevent the transpositions in the side circuits from balancing this section.

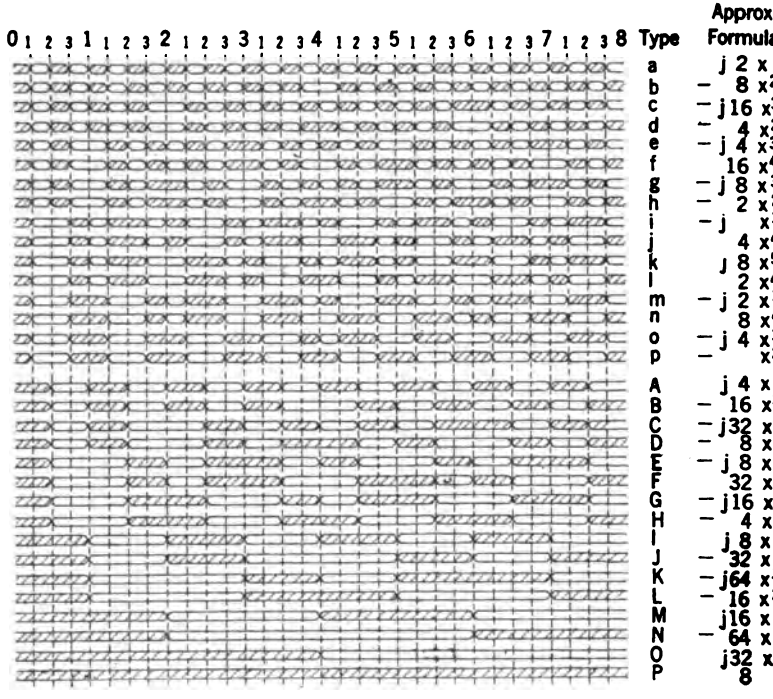

FIG. 2—TYPICAL ARRANGEMENTS OF TRANSPOSITIONS

Equations showing the method of computing the equivalent exposure and the type unbalance between circuits which are phantomed are given in Appendix D.

In order to fit in with standard telephone practises a transposition system must provide for the phantoming of any two horizontally adjacent pairs of circuits and any two vertically adjacent pole pairs. The system must also provide that any of these combinations of circuits may be phantomed and any others non-phantomed.

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Typical Arrangements of Transpositions. In carrying out the design of a transposition system it is convenient and economical to arrange the transpositions for each circuit in accordance with one or another of a list of selected typical arrangements. In Fig. 2 are shown the types, 32 in number, suitable for use in a transposition section made up of 32 parts of equal length. Reading from bottom to top, the types are arranged in the drawing so that each type has one more transposition per section than the preceding type.

The type unbalance between two telephone circuits is determined by the relative transposition of the two circuits, that is, the points at which one circuit is transposed but not both. The typical arrangements of transpositions have the characteristic that the exposure between any two circuits so transposed can be represented by some other type; that is, the exposure between any two types is the same as the exposure between some other type and a circuit without transpositions. For example, exposure f to k = exposure F to $K = A$, exposure J to $m = k$. For transposition sections divided into 64 equal parts, 32 more typical arrangements can be added to the 32 shown in the figure by adding a transposition at the midpoint of each elemental length of each type.

Typical Arrangements for Exposed Lines. The purpose of the exposed-line transposition system is particularly to provide increased facility in balancing the induction from outside circuits in so far as this can be done by transpositions. The effect of outside circuits in inducing voltages between the telephone wires and ground which may be productive of very serious interference cannot be remedied by transpositions in the telephone circuits, but can frequently be greatly reduced by transpositions in the outside circuits. The telephone circuit transpositions must be designed to equalize the voltages induced in the two sides of each telephone circuit with the greatest possible flexibility in the location of the transpositions in the outside circuits. This imposes great limitations on the typical arrangements of transpositions which can be used.

The outside disturbing circuits can be classified as follows:

1. Non-transposed circuits such as telegraph circuits. This applies also to the residual components of current and voltage in power circuits.

2. Two-wire transposed circuits such as single-phase power circuits.

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3. Three-phase power circuits.

In order that the disturbing effect of circuits of the first class may be reduced as far as possible, no typical arrangement of telephone transpositions can be used whose type unbalance factor to non-transposed circuits exceeds a certain value. This eliminates a few of the types having the least numbers of transpositions.

Two-wire outside circuits can best be transposed in accordance with some of the typical arrangements for telephone circuit transpositions. A balance to these circuits requires then that certain types be set aside for their use and that the exposures between the transposed telephone circuits and the outside circuits transposed in accordance with these types be kept below the limiting values set for type unbalance for these cases.

The inherent complication in balancing the inductive effects

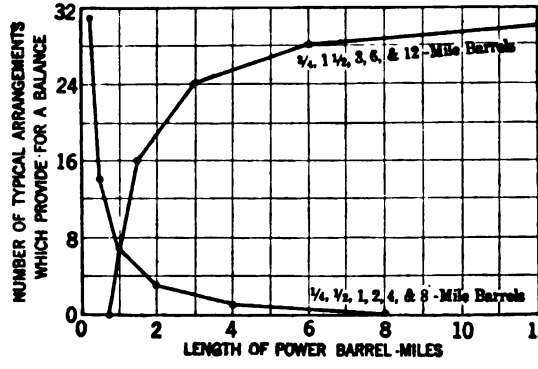

FIG. 3—NUMBER OF TYPICAL ARRANGEMENTS OF TELEPHONE TRANSPOSITIONS WHICH COORDINATE WITH THREE-PHASE POWER LINES TRANSPOSED WITH DIFFERENT LENGTHS OF BARREL

between telephone circuits and three-phase power circuits arises, as has been noted, from the fact that a three-phase circuit has three different arrangements of conductors on the pin positions and a two wire telephone circuit has but two. There are, therefore, six combinations of relative pin positions of the two circuits, all to be used for equal lengths of line.

In considering the severe restrictions placed on the selection of telephone transposition types by the necessity of balancing to transposed three-phase power circuits, the following conditions may be noted:

1. If the barrels of the power line are very long, they can be made to coordinate with almost all of the typical arrangements by having each

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non-transposed section of power circuit, that is, each third of a barrel, equal in length to one-half or one-fourth of the telephone transposition section, for most of the typical arrangements balance within this length. This is shown in Fig. 3, assuming 12.8 km. (8 miles) to be the length of the telephone transposition section. Thus with a 19.2-km. (12-mile) power barrel the telephone circuits may be balanced for each 6.4-km. (4-mile) non-transposed length of the power circuit. However, as the length of the power barrel is progressively decreased to 9.6 km. (6 miles), 4.8 km. (3 miles) 2.4 km. (1 ½ miles) etc., the number of typical arrangements of telephone transposition which can be used very rapidly decreases.

2. If the barrels of the power line are very short, they may be made to coordinate with all the typical arrangements by having a complete barrel between telephone transpositions. In the case shown in Fig. 3 this requires a 0.4-km. (¼ mile) barrel. As the length of barrel is increased, however, to 0.8 km. (½ mile) 1.6 km. or 3.2 km. (1 to 2 miles) etc., the number of typical arrangements of telephone transposition which can be used, very rapidly decreases.

3. Barrels in the power line of relative lengths other than those mentioned do not in general coordinate with telephone transposition arrangements.

As a chief requirement of the exposed-line system was that it should provide a maximum possible flexibility in the location of transpositions in parallel three-phase power circuits, this was made a matter of most careful study. The results obtained are outlined in the description of the system given in the next section of this paper.

Determination of Limiting Permissible Exposures. Before the attempt can be made to select, from among the typical arrangements of transpositions which can be used, a type for each telephone circuit on the lead, a table is made up of the limiting type unbalances which can be permitted between pairs of circuits, assuming that a given limit of the induced current between circuits cannot be exceeded. The procedure is as follows:

1. A computation is made of the total amount of unbalance which could be permitted between each combination of circuits on the lead, taken two at a time. This unbalance is expressed in terms of the length of untransposed exposure between the circuits which would give the limiting value of induced current, no current being induced in the rest of the circuit.

2. The total allowable unbalance for each pair of circuits is reduced by a figure representing the unbalance which must be expected to arise from irregularities with the type of construction to be employed.

3. The remaining unbalance represents the limiting permissible effect of type unbalance in a long length of line.

4. From data regarding the attenuation of currents along the telephone circuits, and from the total limiting effect of type unbalance, a com-

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putation is made of the limiting type unbalance per transposition section. This sets a direct limit on the relative exposure type between each pair of circuits.

The Design of a Transposition System. With the information regarding the requirements of the proposed transposition system prepared as indicated above, the creation of a design which shall meet the requirements of maximum economy and efficiency is a problem requiring a great deal of skillful and conscientious application on the part of a trained expert. The problem is of course one of selecting from the typical arrangements which can be used, the best type for each circuit such that the type unbalance between no two circuits will exceed the established limit.

In order that a design may be complete it must provide for any number of crossarms on the telephone lead. However, it is generally desirable to take into consideration only four crossarms at the beginning of the design. Considering all the circuits, phantom and two-wire, which may be placed on the lead, taken in combinations of two there are about 2500 such combinations on a standard 40-wire pole line.

In carrying out the design of the exposed-line transposition system it was desired to make the requirements both of efficiency and economy as exacting as could possibly be met. These requirements were set, therefore, at the beginning of the work, so high that no design could meet them, and then gradually reduced until it was possible to complete the design. As the design depends on a number of interdependent requirements, this process not only requires a very large amount of work but demands the continual exercise of trained judgment on the part of the designer to bring about the mutual adjustment of requirements so as to produce the best final result.

The process of determining whether or not a design can be established to meet a given set of requirements is very technical and will not be described here in detail. It involves a systematic consideration of all the possible arrangements. These possibilities are very great in number—about seven million in the case of the *E* section design—and are, of course, not considered individually, but in groups, and successive groups of possibilities are rejected when it has been proved that they could not meet the requirements.

The systematic grouping of the possibilities, and the means adopted for the examination and elimination of the groups depends a great deal upon the experience and skill of the designer. The general method which has been successfully employed can

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be roughly described as a consideration of the design in steps. First the attempt is made to carry through a preliminary design in which pairs of circuits which cannot be exposed to each other untransposed for the entire length of the transposition section (*P* exposures) are provided with different typical arrangements of transpositions, but in which no other requirements are considered. If this design is possible, the other balance limitations of the types selected are added, one by one.

When a design is finally reached it is usually found that any one of a considerable number of arrangements will meet the balance requirements almost equally well. The most economical of these is chosen for use. In determining upon the relative economy of different designs, account is taken of the extent to which circuits on different pin positions will exist in the telephone plant and the extent to which it might be possible, in retransposing existing lines, to use the present transpositions as a part of the transpositions called for in accordance with the new system.

Exposed-Line Transposition System

Special Electrical Requirements. The exposed-line transposition system was designed to meet the following special electrical requirements:

1. The greatest practicable flexibility for the arrangement of transpositions in power circuits paralleling the telephone circuits, the power circuit transpositions co-ordinating with those in the telephone circuits so as to produce a balance of the inductive effects. This requirement to be met for transpositions both in three-phase circuits and in single-phase circuits.

2. A suitable balance against the inductive effects of telegraph circuits or other ground return circuits paralleling the telephone circuits.

3. A degree of freedom from interference between the telephone circuits (crosstalk) suitable for modern high grade telephone circuits used in connection with loading and with telephone repeaters.

General Requirements. In addition to these special requirements, it must meet the following requirements for any satisfactory transposition system:

1. *Neutral Points.* The system must be arranged so that neutral points, that is, points at which a maximum degree of balance is obtained between all circuits on the telephone line are

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provided where there are discontinuities in line construction or in the parallel. In the case of open wire toll lines of the Bell telephone system, there must be neutral points at regular intervals of about 12.8 km. (actually 7.88 miles). In addition there must be neutral points wherever there is an important junction between telephone lines and at other points of discontinuity. It is, therefore, desirable that the system include a design intended for a maximum length of 12.8 km. and another design or designs for shorter maximum distances.

2. *Cost.* The completed section must be designed for as low a cost as practicable without sacrifice of the electrical requirements. This means that as far as other conditions of

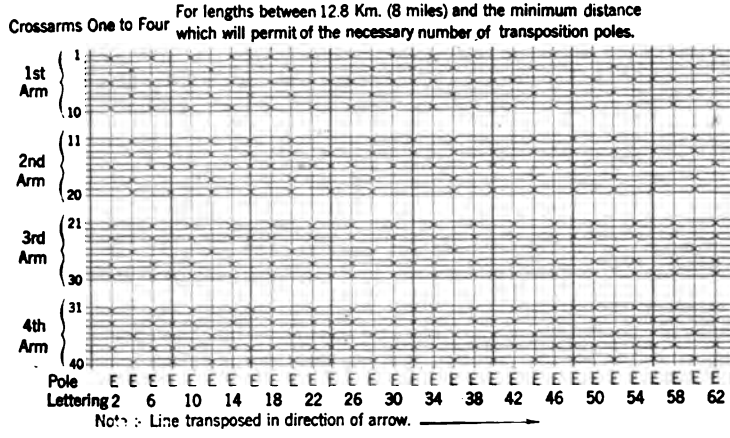

FIG. 4—*E* SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

design permit, the typical arrangements having the least numbers of transpositions should be placed on the circuits most often used. In general, it is desirable that phantom circuits have fewer transpositions than two-wire circuits as phantom circuit transpositions are the more expensive.

3. *Transposition Poles.* It is desirable that the transposition poles be as few in number as practicable not only on account of expense but because the use of large numbers of transposition poles limits the minimum length of a transposition section. The use of many poles may also tend to increase the induction between telephone circuits due to irregularities.

E Section and L Section. As a result of the study, two designs have been completed which are called respectively the *E*

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section and the L section. Diagrams showing transpositions according to the E section for crossarms 1 to 8 of a pole line are shown in Figs. 4 to 9, inclusive, and according to the L section in Figs. 10 to 15, inclusive. It will be noted by examination of the figures that the designs provide for the phantoming of any two horizontally adjacent pairs or of any two vertically adjacent pole pairs on the lead. In carrying out the designs provision has been made for extension if necessary to any number of crossarms.

The E section is designed for a maximum length of approximately 12.8 km. It was found impossible to meet the electrical

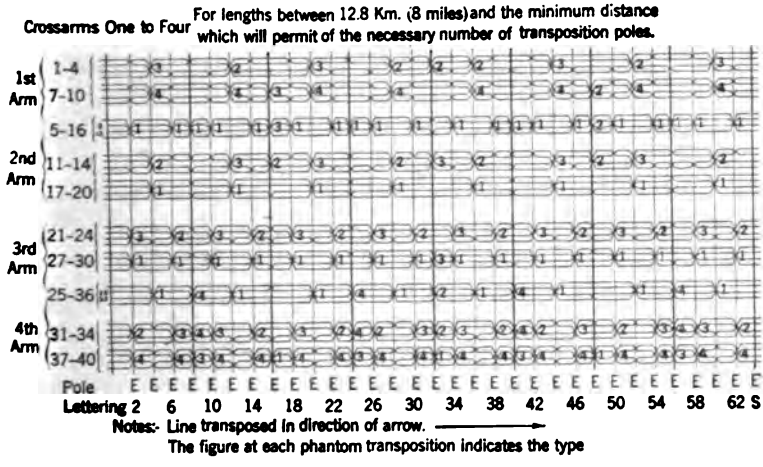

FIG. 5—E SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

requirements for this case with the use of 32 transposition poles per section. It was possible, however, to design the E section so that in the first four crossarms the non-phantom circuits and the phantom circuits which are most commonly used require only 32 transposition poles, so that in some toll lines not exceeding four crossarms, the larger number of transposition poles will not be necessary. For circuits on crossarms below the fourth and for some infrequently used phantomed circuits on the upper crossarms, 64 transposition poles are required.

The E section may be used for as short a length of line as supplies the required number of transposition poles. For distances less than 6.4 km., however, it is economical to use the

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L section. It has just half the number of transposition poles for the corresponding circuits.

When two or more L sections are used consecutively in the same loading section, junction transpositions should be installed at the S poles between them in order to reduce the induction between the telephone circuits. These junction transpositions are indicated in the drawings for as many as four consecutive L sections.

Use of E and L Sections on Exposed Lines. The results obtained in meeting the special electrical requirements regarding balance to the inductive effects of outside circuits are indicated in the following table:

	E Section	*L Section*
1. Element of section in which each circuit is transposed at least once, *i. e.* balances to a parallel non-transposed disturbing circuit.	8th Section	Quarter-Section
2. Neutral points suitable for the location of transpositions in paralleling three-phase power circuits, separation of approximate centers of disturbing and disturbed groups of wires *more* than 6 meters.	8th Section points; *i. e.*, barrels having nominal length of 4.8 km. (3 miles) or 9.6 km. (6 miles) can be used. Five points dividing distance between any two 8th-section points into six equal parts, all five transpositions to be rotated in the same direction; *i. e.*, nominal 0.8 km. (0.5 mile) barrels can be used also.	Quarter-section points; *i.e.* nominal 4.8 km. (3 mile) barrels can be used.
3. Neutral points suitable for the location of transpositions in paralleling metallic single-phase power circuits—separation as defined above *more* than 6 meters.	8th-section points. 3 points dividing distance between any two 8th-section points into four equal parts.	Quarter-section points.
4. Neutral points suitable for the occurrence of discontinuities—separation as defined above *more* than six meters.	8th-section points	Quarter-section points
5. Neutral points suitable for the location of transpositions in disturbing power circuits or the occurrence of discontinuities in disturbing power or telegraph circuits—separation as defined above *less* than 6 meters.	Mid-section point	Mid-section point
6. Neutral points suitable for the occurrence of discontinuities in telephone circuits.	S-poles	S-poles

The results given in the table are further illustrated in Figs. 16 and 17 which show premissible relative locations of the tele-

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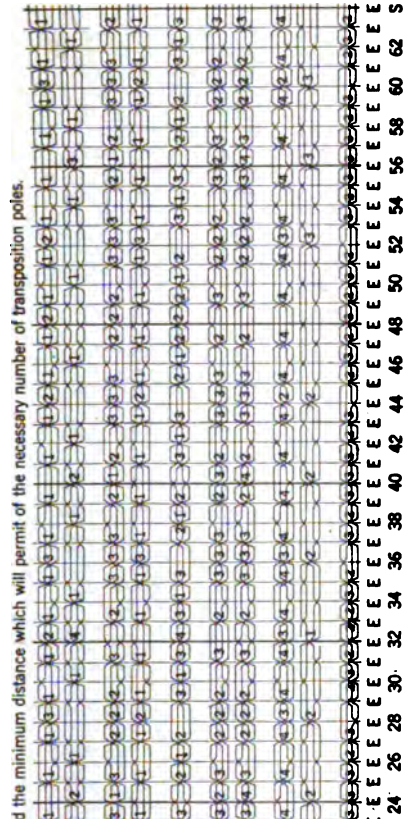

FIG. 6—*E* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR HORIZONTAL POLE PAIR PHANTOMS AND ODD VERTICAL PHANTOMS

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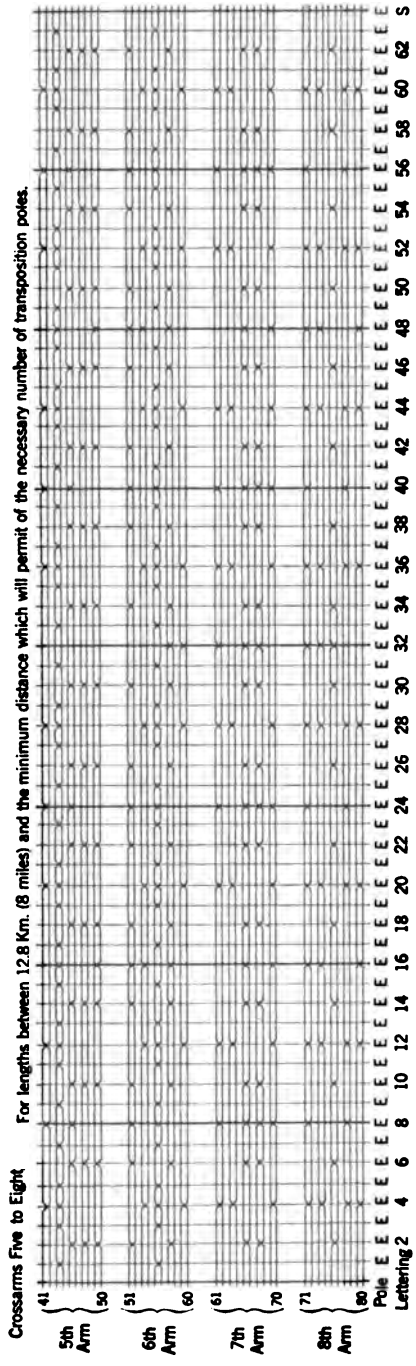

FIG. 7—*E* SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

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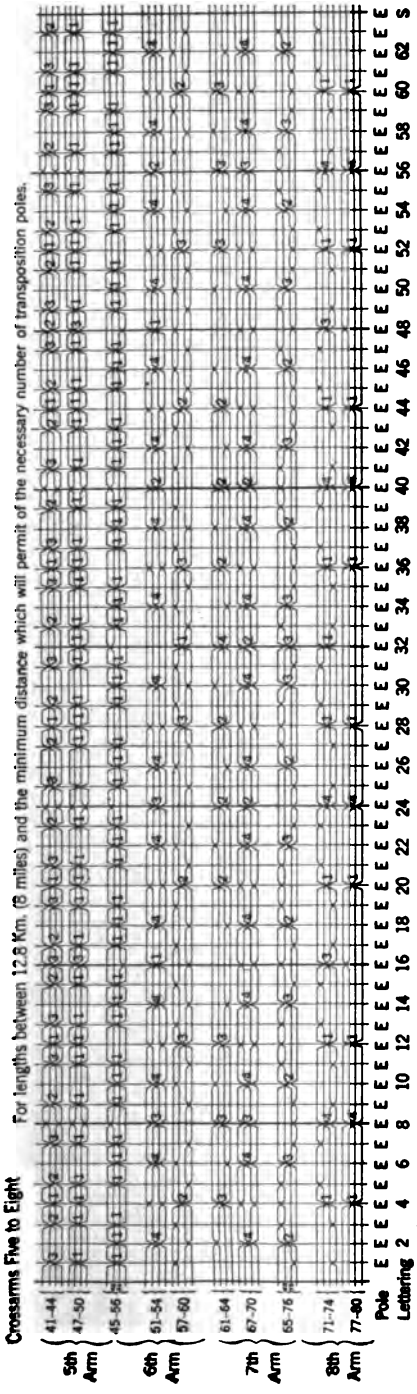

FIG. 8—E SECTION—TRANSPOSITION SYSTEM FOR PHANTOM CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

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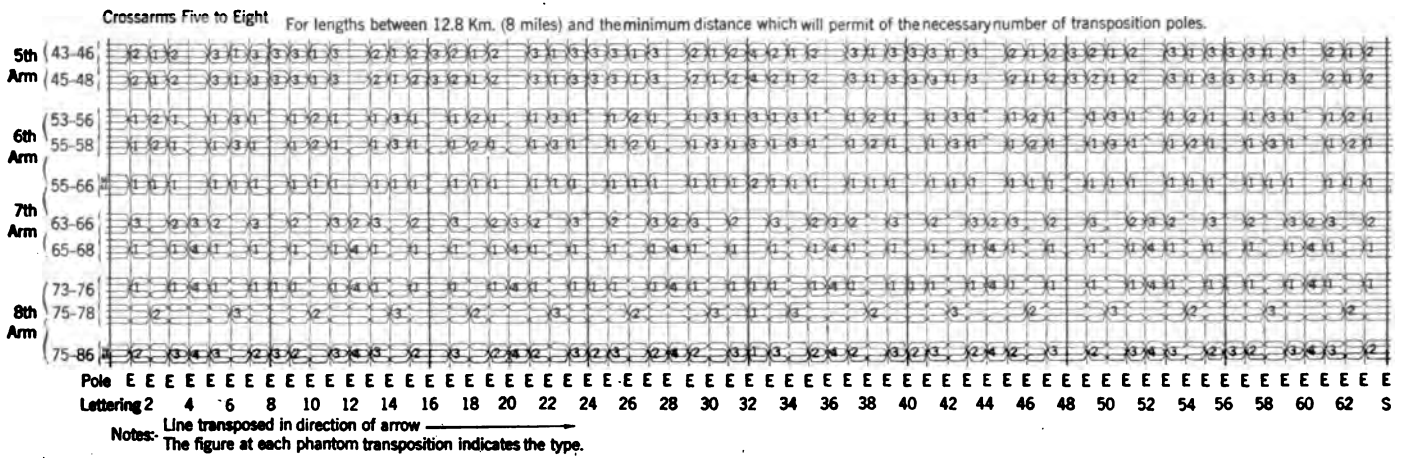
Crossarms Five to Eight
For lengths between 12.8 Km. (8 miles) and the minimum distance which will permit of the necessary number of transposition poles.
5th Arm
43-46
45-48
6th Arm
53-56
55-58
7th Arm
55-66
63-66
65-68
8th Arm
73-76
75-78
75-86
Pole E E E

phone transposition poles and of transpositions and discontinuities in the outside parallel circuits or changes in separation between the two classes of circuit.

It will be noted that the E section can be used to coordinate with transpositions in three-phase power circuits which create for the maximum length of E section 0.8 km. (½ mile), 4.8 km. 3 miles) or 9.6 km. (6 miles) barrels in the power circuit. As it was considered very important to obtain the maximum practicable degree of flexibility in this respect, this matter was given very thorough study and a large amount of work was required to arrive at this result. It should be noted, moreover, that in the use of a 0.8-km. barrel, two barrels occupying a 1.6-km. section

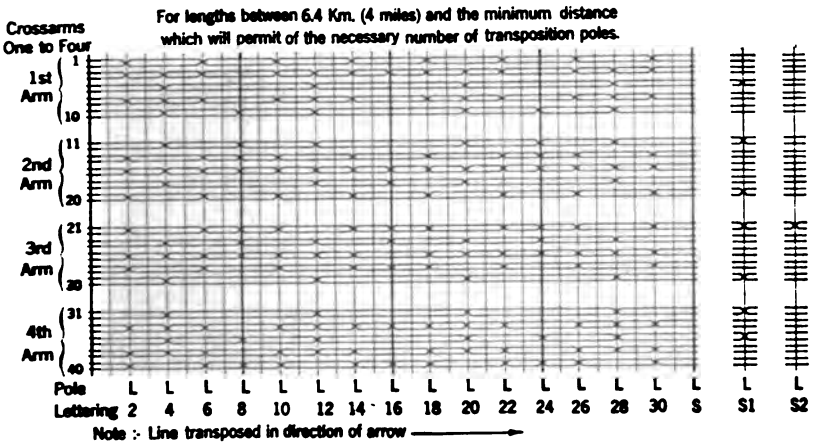

FIG. 10—L SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

of the transposition system are required to obtain a balanced condition.

In the L section the requirements for the avoidance of excessive interference between the telephone circuits are more exacting than in the E section because the use of a smaller number of transposition poles makes available a correspondingly smaller number of typical arrangements of transpositions. It was found possible, however, to design this section so that it balances to a three-phase power line transposed at the quarter section points.

The exposed-line transposition system has been designed primarily for use on toll lines and not for use on joint lines with

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power distribution circuits. It is, however, somewhat better for this case than the other transposition systems in use.

When large inductive effects are experienced from circuits less than 20 ft. (6 m.) away from a side circuit, the variation in separation between the telephone circuit and the exposing circuits as the side circuits shifts from one pair of pin positions to another must be taken into consideration. The *E* section is not designed to give a high degree of balance under this condition for all circuits in each eighth-section, but does give balance to uniform outside induction effects for most circuits in each eighth-section and with one exception, for all circuits in each

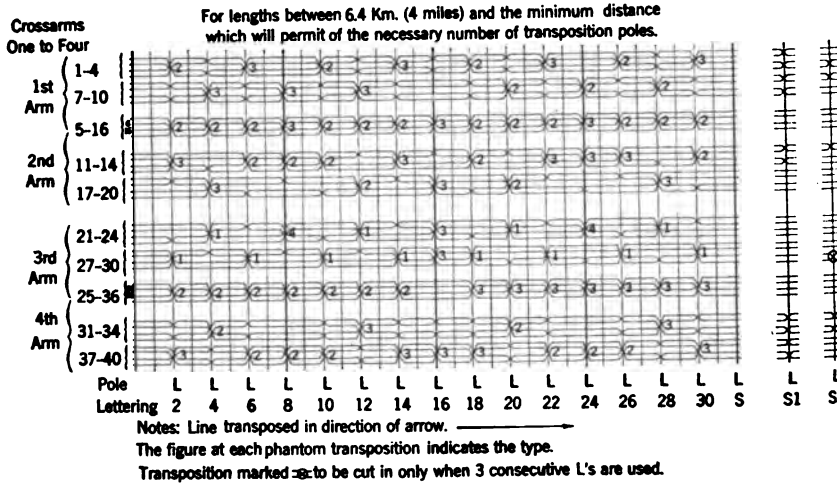

FIG. 11—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUIT ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

half-section. Under similar conditions the *L* section gives a balance for most circuits in each quarter section and for all circuits in each half section.

It should be clearly pointed out that in the use of the telephone transposition sections to coordinate with transpositions in power circuits, it is important that the transpositions in the two lines be coordinated both as regards relative length and as regards relative location of telephone transposition section and transpositions in the power circuits. The locations of the transpositions in either line must, therefore, be made with reference to the requirements of the other. It is frequently the case that

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the locations of the transpositions in the power circuit are to a large degree determined by the changes in separation and other characteristics of the parallel.

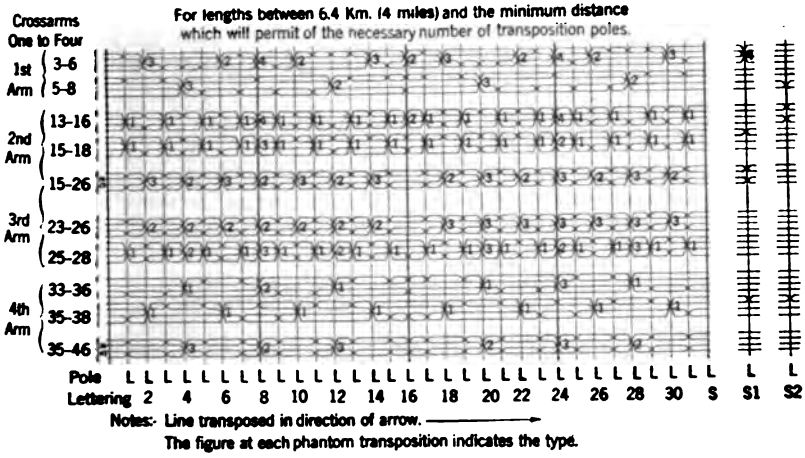

FIG. 12—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR HORIZONTAL POLE PAIR PHANTOMS AND ODD VERTICAL PHANTOMS

The *E* and the *L* section are both designed to give a high degree of balance of the inductive effects between the telephone

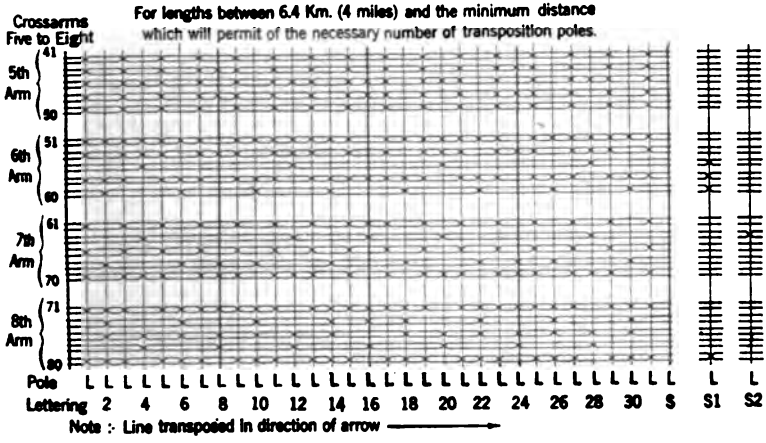

FIG. 13—*L* SECTION—TRANSPOSITION SYSTEM FOR NON-PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES

circuits. In using them, therefore, it is important that the transposition poles be spaced as accurately as practicable in

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order to avoid diminishing the effectiveness of the system in this respect because of irregularities.

The design work led to a large number of possible arrangements of approximately equal electrical efficiency. In choosing from these the most economical, account was taken both of the cost of transposing when stringing new circuits and of the cost of retransposing to the new system telephone lines now transposed according to other arrangements standard with the Bell Telephone Companies. Although a most careful study was made of economy it was necessary because of the special characteristics required in this system to allow in the final sections a cost of

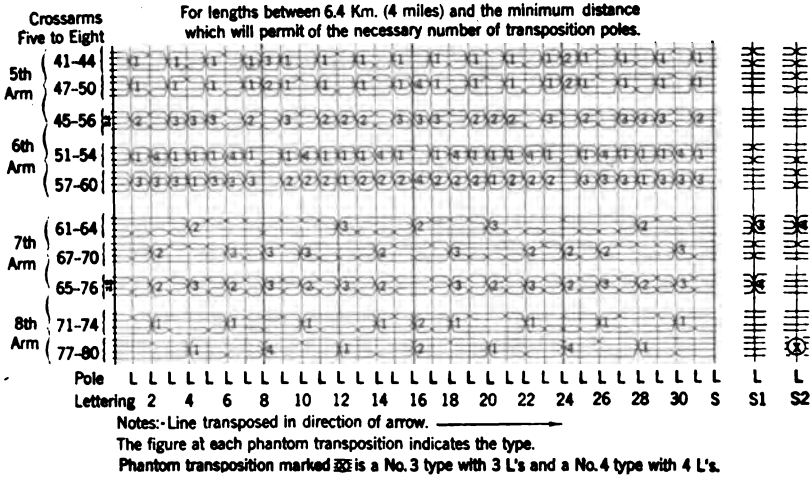

FIG. 14—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR PHANTOMING ALL CIRCUITS

telephone transpositions 25 to 50 per cent greater than that of the section which is standard in the Bell Telephone System for use on non-exposed lines.

CONCLUSION

The results obtained in the design of the *E* and *L* sections were possible only through a systematic study of the problem. The work directed particularly to the design of these two sections occupied about two man-years' time of the specially trained experts who carried out the work. The investigation has been so thorough that the conclusion seems warranted that the *E* section and the *L* section are substantially the best arrangement

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of transpositions which could be devised to meet the requirements established for them.

The work of many members of the engineering department of the American Telephone and Telegraph Company has contributed to the results. The theory of induction between telephone circuits and the special analytical methods of design have been developed in the department during a considerable number of years. Special mention should be made of the work of Mr. G. A. Campbell who, years ago, laid the foundations for the theory of induction between telephone circuits, and of

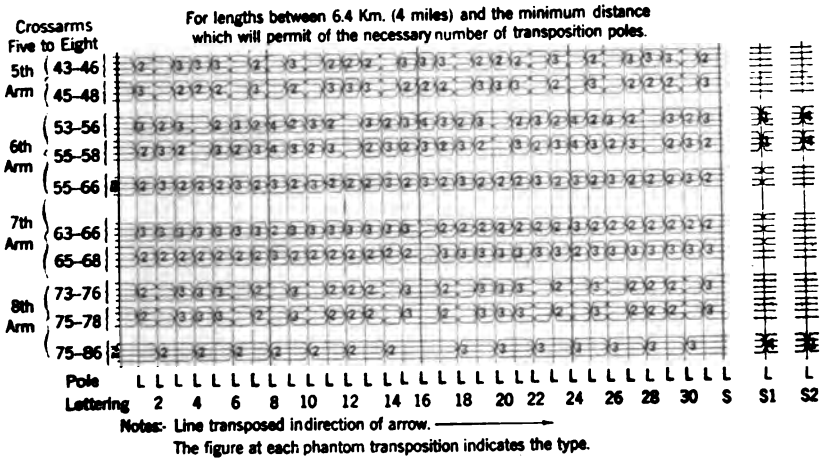

FIG. 15—*L* SECTION—TRANSPOSITION SYSTEM FOR PHANTOMED CIRCUITS ON EXPOSED TELEPHONE LINES—ARRANGEMENTS FOR HORIZONTAL POLE PAIR PHANTOMS AND ODD VERTICAL PHANTOMS

Messrs. A. G. Chapman and R. E. Leonard, who perfected the methods of systematic search and carried out the design of the *E* and *L* sections.

APPENDIX A. INDUCTION BETWEEN PARALLEL WIRES

1. *Two Wires with Ground Return—Short Length of Exposure.* If the current and voltage in a short element of circuit No. 1 (Fig. 18) are represented by I and E, the inductive effect which this element of circuit No. 1 exercises on parallel circuit No. 2 can be represented as follows:

The voltage induced in circuit No. 2 because of current I is

$$V_2 = -j\,\omega\,I\,M\,l \tag{1}$$

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where ω equals 2π times the frequency, l the length of element of circuit considered and $M\omega$ the mutual impedance between the circuits per unit length. In the case of a perfectly conducting earth surface, M is made up entirely of the mutual inductance caused by the linkage of flux from circuit No. 1 with circuit No. 2. With a perfectly conducting earth surface this mutual inductance can be readily computed by the method of images. Under actual conditions the mutual inductance is affected by the resistance of the earth, which increases the linkages between the two grounded circuits because it causes the return current from circuit No. 1 to distribute more or less widely throughout the

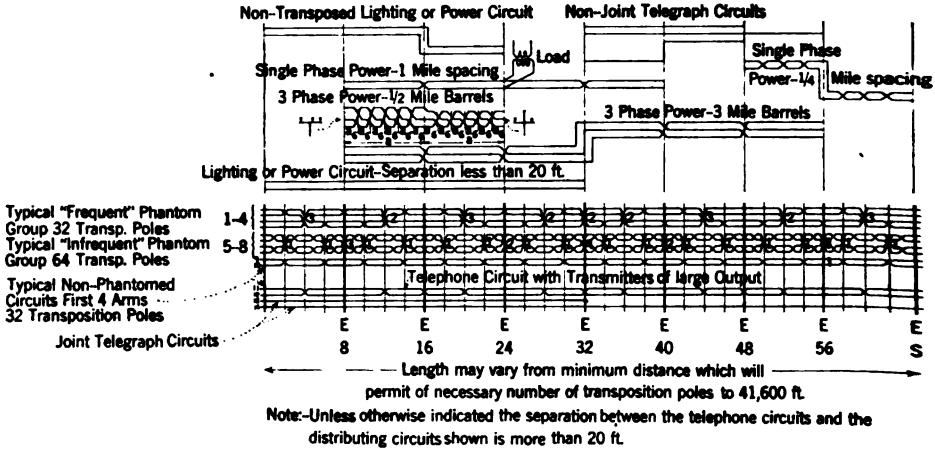

FIG. 16—CHARACTERISTICS OF EXPOSED LINE E SECTION —DIAGRAM ILLUSTRATES LENGTH, NUMBER OF TRANSMISSION POLES AND PERMISSIBLE LOCATIONS OF TRANSPOSITIONS AND DISCONTINUITIES IN EXTRANEOUS DISTURBING CIRCUITS

earth rather than to flow in a current sheet on the earth surface. In many cases, also, the fact that the two circuits both use the earth return having a finite resistance introduces an appreciable resistance term into the mutual impedance. Sometimes this term is very important.

The voltage induced in a short section of wire 2 because of the voltage between wire 1 and ground is

$$e_2 = E\frac{C_M}{C_2 + C_M} \tag{2}$$

In this equation C_M and C_2 are the direct capacities from wire 2 to wire 1 and to the ground, respectively; that is, they are the

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capacities measured by the charges on wire 1 and on the earth when wire 1 is connected to the earth and unit potential is applied between wire 2 and the combination of wire 1 and earth.

If the short section of wire 2 is connected to the earth, the charging current flowing to ground is

$$i_2 = j\,\omega\,l\,E\,C_M \tag{3}$$

That is the induced voltage acts like a generator having a voltage e_2 connected between wire 2 and ground through a condenser whose capacity is equal to the total grounded capacity of the exposed section of wire 2. This is indicated in Fig. 19.

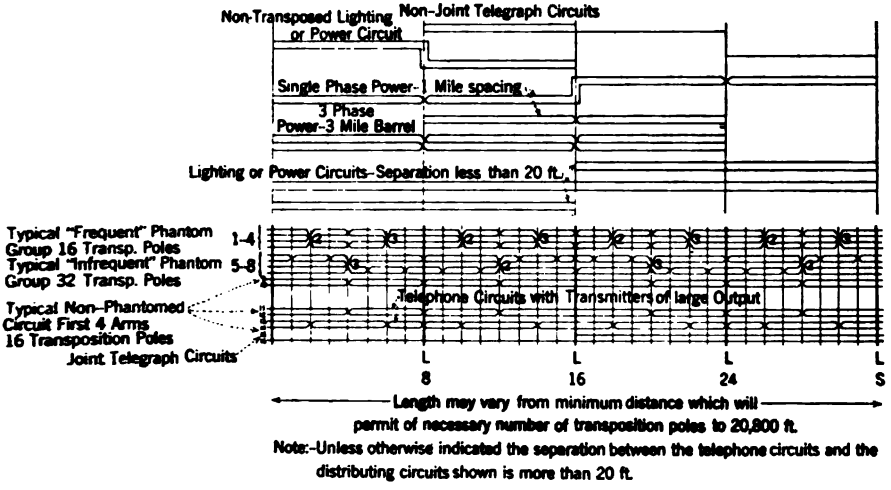

FIG. 17—CHARACTERISTICS OF EXPOSED LINE L SECTION—DIAGRAM ILLUSTRATES LENGTH, NUMBER OF TRANSPOSITION POLES, AND PERMISSIBLE LOCATIONS OF TRANSPOSITIONS AND DISCONTINUITIES IN EXTRANEOUS DISTURBING CIRCUITS

2. *Small Induced Effects.* In cases where the induced currents and voltages are small compared with the inducing currents and voltages so that their reaction on the inducing circuit can be neglected, the magnitude of the induced effects can be obtained by solving for the values of current and voltage in the disturbing circuit, neglecting the presence of the other circuit, and then applying equations similar to those given above for each element of the parallel between the two circuits.

3. *A number of Parallel Wires.* The computation of induced currents is much complicated when there are several disturbing wires or several disturbed wires or both.

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The computation of the electromagnetic induction may be complicated by the reaction between the induced currents in different disturbed circuits. In cases where the induced effect is small, however, the electromagnetic induction can be computed for this case as for a single circuit.

In determining electrostatic induction with a large number of parallel circuits, it is evident that it becomes exceedingly complicated to compute the direct capacities which are used. It is possible to greatly simplify this work in cases in which the disturbed and disturbing circuits are separated so that each group has little effect on the direct capacities of the other group. Under these conditions the direct capacities within the disturbing circuits can be computed and from this computation a determination can be made of the charge of electricity on each of the disturbing wires taking account of the voltages on all wires. From these charges it is possible to compute the potentials between the disturbed wires and ground if isolated in space.

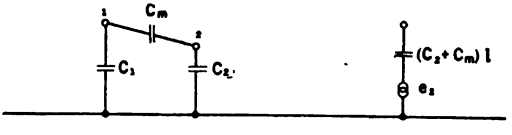

FIG. 18 FIG. 19

These potentials may then be assumed to regulate through the direct capacities of the disturbed circuits as indicated in the above discussion of induction between two parallel wires. In computing the flow of current with many disturbed wires the effect on one wire of the other wires must, of course, be considered.

This outlines briefly the method which is in use for computing the induction between three-phase power circuits and telephone circuits. The method sometimes described in the text books is incorrect in that it does not take account of the mutual reactions between the three conductors of the three-phase circuit which materially modify their resultant electrostatic field.

4. *Long Parallel Circuits.* In the case of two long circuits which closely parallel each other without means for neutralizing inductive effects, the reaction of the induced currents on the disturbing circuit cannot be neglected. If the circuits are very long there tends to be the same distribution of current and potential at the receiving ends of the disturbing and disturbed

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circuits as there would be if equal voltages were applied to the two circuits at the transmitting end. The simplest case of this sort is when the two circuits are symmetrical. The equations for this case are given in Appendix B. It may be seen that even in this case the equations are so complicated that their use in practise would be burdensome. Where there are many long parallel wires the problem becomes one of utmost complexity.

5. *Parallel Telephone Circuits.* In the case of telephone circuits many conductors are run very closely parallel for long distances. It would seem, then, that the general solution of the inductive effect between telephone conductors would be so complicated as to be hopeless for practical use.

In attacking this problem, however, we fortunately are not interested in the general case. If telephone circuits are suitable for commercial service they must be so arranged that the inductive effects between any two circuits are exceedingly small.

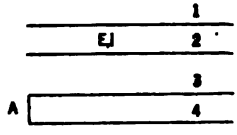

FIG. 20

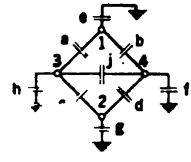

FIG. 21

This is, therefore, a case in which the reaction of the induced currents on the disturbing circuit can be neglected.

If one considers then a short section of close parallel between a disturbing circuit 1-2 carrying a voltage E and current I and a disturbed circuit 3-4 as indicated in Fig. 20 the voltage due to electromagnetic induction can be expressed as in equation (4)

$$e = j\,\omega\,I\,M\,l \tag{4}$$

In computing the current flowing at A due to electrostatic induction one may represent the direct capacities in the form of a Wheatstone bridge as in Fig. 21. In this figure the direct capacities a, b, c and d represent the Maxwell capacity coefficients. Capacities e, f, g and h represent the sum of the direct capacities between the wires, 1, 4, 2 and 3 respectively and all other wires and ground. This grouping is permissible because the induced effects all being small, the other circuits on the pole line are practically at ground potential, and have the same effect on the mutual capacity unbalance between the circuits 1, 2 and 3, 4 as they would if actually connected to earth.

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The induced current is represented approximately by equation (5)

$$i = j\,\omega\,E\,\frac{a'\,d' - b'\,c'}{a' + b' + c' + d'} \tag{5}$$

$a' = a + \frac{h\,e}{s}$, $b' = b + \frac{e\,f}{s}$, etc., s being equal to $e + f + g + h$.

When the circuits are transposed so that the electrostatically induced current is small it results that in a balanced transposition section a', b', c' and d' are all very nearly equal. Under these conditions the magnitude of the electrostatically induced current in one small section of line is represented with a sufficient degree of accuracy by equation (6).

$$i = j\,\omega\,E\,\frac{a' - b' - c' + d'}{4} \tag{6}$$

The equations given above indicate the coefficients of induction which must be determined in order to make possible computations of the induced currents between telephone circuits. The mutual inductance unbalance between any two circuits can be computed from the geometrical positions of the conductors. The mutual capacity unbalance can be computed readily from the table of measured values of direct capacity for all wires on a 40-wire pole lead. (Table I.) The capacities of course vary with the number and to a small extent with the size of conductors on the lead. Forty wires is a typical condition and has been used as representative. When desirable, correction has been made for the use of wires of larger size.

The above equations therefore present a thoroughly practical method of computing the current induced between short sections of two telephone circuits which are well transposed against each other. The correctness of the results obtained in this way has been checked by experiment.

APPENDIX B—THE INDUCTION BETWEEN TWO LONG PARALLEL SYMMETRICAL CIRCUITS

Assume two long symmetrical wires, A and B, as shown in Fig. 22, operating with ground return. Assume the linear self impedances of the circuits to be z_1 and z_2 respectively, the linear mutual impedance to be m, and the linear direct admittances to

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ground and linear mutual admittance to be y_1', y_2', and y_m respectively.

The differential equations for inductive effect between the two circuits are as follows:

$$\frac{-d v_1}{d x} = i_1 z_1 + i_2 m \qquad \frac{-d v_2}{d x} = i_2 z_2 + i_1 m$$

$$\frac{-d i_1}{d x} = v_1 y_1 - v_2 y_m \qquad \frac{-d i_2}{d x} = v_2 y_2 - v_1 y_m$$

In the above formulas, $y = y_1' + y_m$, $y_2 = y_2' + y_m$, and m and y_m are both written as positive quantities.

For the case of two symmetrical wires let

$$z_1 = z_2 = z$$

$$y_1 = y_2 = y$$

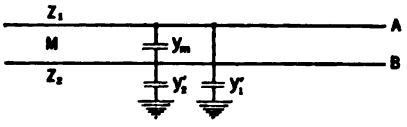

FIG. 22—INDUCTION BETWEEN TWO SYMMETRICAL GROUNDED CIRCUITS

Use is made of the following auxiliary equations:

$$A = 2\sqrt{\frac{z - m}{y + y_m}} \qquad B = \frac{1}{2}\sqrt{\frac{z + m}{y - y_m}}$$

$$\gamma_s = \sqrt{(z - m)(y + y_m)} \qquad \gamma_p = \sqrt{(z + m)(y - y_m)}$$

Equations are given below for a number of conditions which are of interest. In these equations, v_1 and i_1 represent the voltage and current on the disturbing circuit A at any distance x from the transmitting end, and v_2 and i_2 represent the voltage and current on the disturbed circuit B.

Case I. Voltage impressed on circuit A at $x = O$ is E_1
Circuit B connected to earth at $x = O$ through impedance W
Both circuits infinite in length.

$$v_1 = E_1 \frac{2 B (A + 2 W) \epsilon^{-\gamma_p x} + A (2 B + W) \epsilon^{-\gamma_s x}}{W (A + 4 B) + 4 A B} \tag{7}$$

$$i_1 = E_1 \frac{(A + 2 W) \epsilon^{-\gamma_p x} + (4 B + 2 W) \epsilon^{-\gamma_s x}}{W (A + 4 B) + 4 A B} \tag{8}$$

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$$v_2 = E_1 \frac{2\,B\,(A + 2\,W)\,\epsilon^{-\gamma_p x} - A\,(2\,B + W)\,\epsilon^{-\gamma_s x}}{W\,(A + 4\,B) + 4\,A\,B} \tag{9}$$

$$i_2 = E_1 \frac{(A + 2\,W)\,\epsilon^{-\gamma_p x} - (4\,B + 2\,W)\,\epsilon^{-\gamma_s x}}{W\,(A + 4\,B) + 4\,A\,B} \tag{10}$$

Case II. If the resistance, leakance, and internal reactance are negligible and the equivalent height of the wires above ground is the same for static and magnetic induction, $\gamma_s = \gamma_p$ and

$$\frac{A}{4\,B} = \frac{z - m}{z + m} = \frac{y - y_m}{y + y_m} \tag{11}$$

Then, if $W = \infty$, it follows that

$$v_2 = v_1 \frac{y_m}{y} \tag{12}$$

Under these conditions, ($\gamma_s = \gamma_p$ and $W = \infty$) the sending end impedance of the disturbing circuit is

$$z_0 = \sqrt{\frac{z\,y}{y^2 - y_m^2}} \tag{13}$$

This equation shows that under these conditions there is no reaction from circuit B at the terminal of circuit A.

Case III. If $z\,y_m + y\,m = 0$ (14)

Then $B = A/4$ and it follows that

$$v_2 = \frac{E_1}{2}\,(\epsilon^{-\gamma_p x} - \epsilon^{-\gamma_s x})$$

$$i_2 = \frac{E_1}{A}\,(\epsilon^{-\gamma_p x} - \epsilon^{-\gamma_s x}) \tag{16}$$

that is, at $x = 0$, the induced voltage and current are 0.

If y_m is negligible, equation (14) requires that $m = 0$. This may be approximated by the use of compensating transformers. The above relation suggests, however, that by properly adjusting the value of m an approximate neutralization of induced voltage might be obtained in some special cases in which the electrostatic induction is considerable.

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Case IV. When $W = \infty$, then at $x = O$,

$$v_2 = E_1 \frac{y\,(D-1) + y_m\,(D+1)}{y\,(D+1) + y_m\,(D-1)} \tag{17}$$

where $\frac{\gamma_p}{\gamma_s} = D$.

The ratio of this voltage to that computed by equation (**12**) gives an idea of the error involved in the assumption which leads to equation (**12**).

Case V. If at $x = O$,

$$W = \frac{v_1}{i_1},$$

then

$$W = \sqrt{A\,B}$$

and

$$v_2 = E_1 \frac{2\sqrt{B} - \sqrt{A}}{2\sqrt{B} + \sqrt{A}} \tag{19}$$

This impedance, $\sqrt{A\,B}$, is

$$\sqrt{A\,B} = Z_0 = z_0 \sqrt[4]{\left[1 - \left(\frac{m}{z}\right)^2\right]\left[1 - \left(\frac{y_m}{y}\right)^2\right]} \tag{20}$$

Where $z_0 = \sqrt{\frac{z\,y}{y^2 - y_m^2}}$ = the infinite line impedance for a

(**13**) single wire.

Case VI. At a long distance from the origin,

$$v_1 = v_2 = \frac{E_1\,e^{-\gamma_p x}}{2} \text{ if } W = O. \tag{21}$$

$$\text{Also } v_1 = v_2 = E_1 \frac{4\,B}{A + 4\,B}\,e^{-\gamma_p x} \text{ if } W = \infty. \tag{22}$$

It will be noted that the voltages on the disturbed and disturbing wires tend to become equal at a considerable distance from the origin, and that the two expressions for v_1 and v_2 with maximum and with minimum shielding are nearly the same.

APPENDIX C—TYPE UNBALANCE BETWEEN TRANSPOSED TELEPHONE CIRCUITS

The induction between long transposed telephone circuits on the same lead is the vector sum of the induction between all

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short elements of the circuits, taking into account the attenuation and phase change of the inducing currents and voltages from the transmitting point, and the attenuation and phase change of the induced currents in their propagation to the end of the circuit from the point at which the inductive effect occurs.

A simple typical arrangement of transpositions between two telephone circuits in a short balanced section is indicated in Fig. 23 in which similar telephone circuits P and Q are divided into four sections of lengths A, B, C and D by the transpositions in the two circuits. The resulting induction between the two circuits in this section is the sum of two terms:

a. Unbalance Due to Irregularity in Length. As this is a short section of line, one may consider as a first approximation that the current and voltage in the inducing circuit P are the same throughout this section. Currents induced in sections B and D,

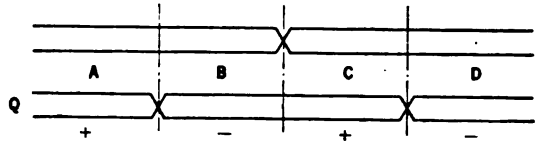

FIG. 23

therefore, tend to neutralize currents induced in sections A and C, provided the length $A + C$ is equal to the length $B + D$. When this is not the case there is a resultant induced current in circuit Q due to irregularities approximately proportional to the resultant unbalanced length $A - B + C - D$.

b. Type Unbalance. If lengths A, B, C and D are all exactly equal there will nevertheless be a resultant induced current in circuit Q because of the fact that the currents and voltages in either circuit P or Q change in magnitude and phase as they are propagated along the circuit.

In this appendix are given equations illustrating the method of computing type unbalance.

If P is a section of a long telephone circuit the current and voltage in it can be represented by equations (**23**) and (**24**).

$$I = I_0 e^{-\alpha l} \sin (\omega t - \beta l) \tag{23}$$

$$E = E_0 e^{-\alpha l} \sin (\omega t - \beta l) \tag{24}$$

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In these equations α and β are the attenuation constant and the phase-change constant of the telephone circuit and of course vary with the frequency of the current.

The induced current in a short element of circuit a distance L from the end of the circuit, can be represented as equation (25).

$$d\,X = K\,d\,l\,e^{-\alpha l} \sin(\omega\,t - \beta\,l) \tag{25}$$

where $K\,d\,l$ is the induction between short elements at the end of the circuits and is computed with the help of equations (4) and (6) for the electromagnetic and electrostatic components. (Appendix A.) This induced current will be propagated on circuit Q back to the beginning of the circuit and under the condition for transmission without reflection, that is, terminal impedance equal to line impedance, will have when it reaches the beginning of the circuit, the value given in equation (26)

$$d\,X_0 = K\,d\,l\,e^{-2\alpha l} \sin(\omega\,t - 2\,\beta\,l) \tag{26}$$

A similar equation can be written for each element of the circuit. The induction due to length A is obtained by integrating expression (26) for the length between zero and A as shown in equation (27)

$$X_A = \int_0^A K\,d\,l\,e^{-2\alpha l} \sin(\omega\,t - 2\,\beta\,1) \tag{27}$$

The general expression for this integral is rather complicated. In practical telephone circuits, however, the effect of the attenuation in the short section of circuit between successive transpositions is very much less than the effect of the change in phase angle. Under these conditions the attenuation can be neglected for short lengths and the effective value of the expression (27) reduces to that given in equation (28)

$$X_A = K\,A\,\frac{\sin\beta\,A}{\beta\,A}\,\angle\,\beta\,A \tag{28}$$

Similarly, the crosstalk at the end of the circuit due to section B of the exposure between the lines can if B is equal in length to A be represented as in equation (29).

$$X_B = -\,K\,A\,\frac{\sin\beta\,A}{\beta\,A}\,e^{-2\alpha A}\,\angle\,3\,\beta\,A \tag{29}$$

In practical cases the ratio $\dfrac{\sin\beta\,A}{\beta\,A}$ is practically unity

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and the exponential expression $e^{-2\alpha_A}$ is also practically unity. Under these conditions the induced current at the end of the circuit caused by sections A and B of the parallel can be represented approximately as in equation (**30**).

$$X_{AB} = 2\,K\,A \sin \beta A \,\underline{/\pi/2 - 2\,\beta A} \tag{30}$$

Similarly, the resultant induction in circuit Q from sections A, B, C and D can be represented as in equation (**31**)

$$X_{AD} = 4\,K\,A \sin \beta A \cos 2\,\beta A \,\underline{/\pi/2 - 4\,\beta A} \tag{31}$$

This simple example corresponds to one of the typical arrangements of transpositions, namely, that denoted by M in Fig. 2. A similar equation can be worked out for each typical arrangement, and numerical values worked out for any given values of β and A.

For much work a sufficiently good approximation can be made in the formulas for type unbalance by assuming that

$$\sin n\,\beta A = n\,\beta A$$

$$\cos n\,\beta A = 1$$

The resulting simple formulas give an easy means of judging of the approximate relative effectiveness of different arrangements. They are given for different typical arrangements under the heading *Approximate Formulas* in Fig. 2. When X represents the phase change per mile these formulas represent numerically the type unbalance for a 12.8 km. (8 mile) section. The angles of the type unbalance for different types of the same length differ only by 90 deg. or multiples of 90 deg., and these differences only are indicated in the approximate formulas.

The type unbalance varies, of course, with the frequency. In the design of telephone transpositions type unbalances are computed for a frequency found to represent the average crosstalk effect.

It remains to point out the relation between resultant unbalance due to type for a long length of line and type unbalance for one transposition section. Considering current of a single frequency, the ratio between these two for N sections is

$$\frac{1 - e^{-2N\gamma L_0}}{1 - e^{-2\gamma L_0}} \tag{32}$$

where L_0 is the length of the section.

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Equation (**32**) assumes that the wires have the same relative pin positions at the beginning of each section If the pin positions are the same in alternate sections and the opposite in successive sections formula (**32**) becomes

$$2 \sin \beta L_0 \frac{1 - e^{2N\gamma L_0}}{1 - e^{-4\gamma L_0}} \tag{33}$$

For a large number of sections, and for the attenuations prevailing on good telephone toll lines equations (**32**) and (**33**) become approximately

$$\frac{1}{2 \gamma L_0} \tag{34}$$

and

$$\frac{\sin \beta L_0}{2 \gamma L_0} \tag{35}$$

Actually, results obtained by these simple formulas must be corrected for the fact that the telephone current is a compound of currents of different frequencies, for which the values of β and γ are different. In the case of loaded lines a correction can be made rather simply. The change in phase angle from one loading section to another is large and varies widely for different components of the telephone current. Good results are therefore obtained by assuming that the effects of successive sections add together by a root-sum-square law. The resulting ratio between long line unbalance and type unbalance for a single section is

$$\sqrt{\frac{1 - e^{-4N\alpha L_0}}{1 - e^{-4\alpha L_0}}} \tag{36}$$

Since N is large and αL_0 small, this becomes approximately

$$\frac{1}{2 \sqrt{\alpha L_0}} \tag{37}$$

APPENDIX D, FORMULAS FOR TYPE UNBALANCE IN PHANTOMED CIRCUITS

Under the heading, *Phantom Circuits*, in the paper it is pointed out that the use of phantoms in telephone line construction complicates the computation of type unbalance because each transposition of a phantom circuit interchanges the pin positions

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of its two side circuits and, therefore, changes the coefficients of induction between these and other circuits.

The equations for computing the resultant induction per section due to type unbalance are given below. These equations are given for the case in which a typical arrangement of transpositions is associated with each side circuit of the phantom independent of its interchange of pin positions with the other side circuit of the phantom. This is the arrangement which has been found most suitable for use in the exposed line transposition system. Transposition systems can be designed with a typical arrangement associated with the wires on a given pair of pin positions. With this arrangement, each side circuit of the phantom is transposed partly in accordance with one typical arrangement and partly in accordance with another as it changes from one pair of pin positions to the other with transpositions of the phantom circuit. Under some conditions, this arrangement has advantages in the economy of retransposing non-phantomed circuits to form phantoms. It is used in the transposition system which is standard with the Bell Telephone Companies for non-exposed lines.

The formulas for the resultant induction per section due to type unbalance are as follows:

Suppose two circuits, 1 and 2, to be combined in a phantom R and two circuits, 3 and 4, to be combined in a circuit Q.

Denote by T_1, T_2, T_R, etc., the type unbalances corresponding to the typical arrangements of transpositions on circuits 1, 2, R, etc. Further, let K_{1R}, K_{2R}, etc. represent the induction per unit untransposed length between circuits 1 and R, between circuits 2 and R, etc.

For Side Circuit to Phantom of Which it is a Part.

$$X_{1R} = S\,T_{1R} + D\,T_1 \tag{38}$$

$$X_{2R} = S\,T_{2R} - D\,T_2 \tag{39}$$

$$\text{Where } S = \frac{K_{1R} + K_{2R}}{2}$$

$$D = \frac{K_{1R} - K_{2R}}{2}$$

T_{1R} represents the type unbalance for the typical arrangement corresponding to the relative exposure between type 1 and type R.

T_R is similarly defined.

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For Side Circuit to a Non-phantomed Circuit or a Phantom Other Than its Own.

$$X_{1Q} = S\,T_{1Q} + D\,T_{1RQ} \tag{40}$$

$$X_{2Q} = S\,T_{2Q} - D\,T_{2RQ} \tag{41}$$

When
$$S = \frac{K_{1Q} + K_{2Q}}{2}$$

$$D = \frac{K_{1Q} - K_{2Q}}{2}$$

The nomenclature is the same as before.

T_{1RQ} represents the type unbalance corresponding to the relative exposure between the arrangement giving unbalance T_{1R} and typical arrangement Q.

For example, if circuit 1 is transposed to Type A
" R " " F
" Q " " L

$$T_{1RQ} = T_{(AF)L} = T_{KL} = T_O$$

That is, T_{1RQ} is in this case the type factor for arrangement O, which is obtained by combining successively types A, F, and L.

For the Two Side Circuits of a Phantom

$$X_{12} = K_{12}\,T_{12} \tag{42}$$

For Side Circuits in Two Different Phantoms

$$X_{13} = S\,T_{13} + D_a\,T_{13R} + D_b\,T_{13Q} + D_c\,T_{13RQ} \tag{43}$$

$$X_{14} = S\,T_{14} + D_a\,T_{14R} - D_b\,T_{14Q} - D_c\,T_{14RQ} \tag{44}$$

$$X_{24} = S\,T_{24} - D_a\,T_{24R} - D_b\,T_{24Q} + D_c\,T_{24RQ} \tag{45}$$

$$X_{23} = S\,T_{23} - D_a\,T_{23R} + D_b\,T_{23Q} - D_c\,T_{23RQ} \tag{46}$$

Where

$$S = \frac{K_{13} + K_{14} + K_{24} + K_{23}}{4}$$

$$D_a = \frac{K_{13} + K_{14} - K_{24} - K_{23}}{4}$$

$$D_b = \frac{K_{13} - K_{14} - K_{24} + K_{23}}{4}$$

$$D_c = \frac{K_{13} - K_{14} + K_{24} - K_{23}}{4}$$

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In the above equations the nomenclature is as before.

T_{13RQ} denotes the type unbalance corresponding to the relative exposure between the types giving type unbalances of T_{13} and T_{RQ}.

The D terms in the above equations are generally not negligible compared with the S terms. This fact requires that in designing, for example, transpositions between side circuits in two different phantoms, it is necessary to take account not only of the type unbalance of the relative exposure between the two circuits, but also of type unbalances formed by building up combinations of the typical arrangements of transpositions on the two side circuits and on the phantoms of which they form a part.

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DISCUSSION ON "DESIGN OF TRANSPOSITIONS FOR PARALLEL TELEPHONE AND POWER CIRCUITS" (OSBORNE), ATLANTIC CITY, N. J., JUNE 27, 1918.

V. Karapetoff: In the appendix, Mr. Osborne uses the expression "mutual impedance." This term is new to me, and I should like to have a definition of it. If this quantity is equal to $2\pi f$ times the mutual inductance, then it would seem proper to call it "mutual reactance," in the same way in which the usual reactance is equal to $2\pi f$ times the coefficient of self induction.

It is also pointed out that the ground may modify this coefficient of mutual impedance and this may be a justification for using the term "impedance" instead of "reactance," to indicate that it is composed in a quadratic relation of resistance and reactance. But then the difficulty arises that the mutual impedance seems to be a coefficient which characterizes the two circuits, and if the resistance is to be taken into account, it is not clear the resistance of which of the two circuits is to be used in the expression for mutual impedance. Our Standardization Rules define "mutual impedance" in application to telephone work but I am not sure that it is applicable here. I also notice that another author (Mr. Fortescue) in a paper presented at this convention uses the expression "mutual impedance." It seems to me that we ought to get a general definition of this quantity, and introduce it into the Standardization Rules so that there may be no doubt about its meaning and use.

Charles Fortescue: In the event of a power company having installed a transmission line without transpositions, what would be considered the ethical arrangement between the power company and the telephone company, in case of inductive interference? If transpositions of the power lines are determined on before the installation is made, I can easily see that the cost would be a very small item, but after the power line has been installed the cost of transpositions might be quite a large item, and I should think it would be a case for a compromise between the telephone company and the power company under such circumstances.

Another condition that one may run across is, quite frequently the power transmission line is not symmetrically designed. Of course, a perfectly symmetrical three-phase transmission line would have lines at the apexes of an equilateral triangle, but quite frequently the lines are equally spaced in the horizontal plane, and quite frequently they are unequally spaced, depending on conditions which may have existed before the line was installed. I am very much interested in this, as it happens to be part of my duty to look into these cases of trouble which arise, from such causes as this, and they are sometimes very difficult to adjust. There is always a great deal of controversy as to who is to blame, and who should be the one to correct the trouble.

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I might refer to the question that Prof. Karapetoff brings up. I have myself used the expression "mutual impedance" in some cases. In alternating-current circuits where we have currents which have a common part in a part of the circuit, and are purely mutually inductive in other parts of the circuit. and which it is convenient to consider separately, the expression "mutual impedance" comes in very handily, and is a very useful quantity to use. As far as I know, it has never been defined, but I think it is a perfectly definite and well understood quantity, and it is commonly used among telephone engineers.

A. G. Chapman: Certain experimental data relating to mutual interference between telephone circuits may be of interest. Mr. Osborne shows in Fig. 2 of this paper typical arrangements of transpositions for telephone circuits and in Appendix C discusses the method of computing approximately the inductive effects between two telephone circuits transposed in accordance with any two of these arrangements. It is interesting to note that increasing the number of points at which one circuit and not the other is transposed may increase rather than decrease the inductive effects between the two circuits. For example, if the method of computation outlined in Appendix C is followed, the *N* exposure shown on Fig. 2 will be found to be much more effective against the mutual inductive effects commonly termed crosstalk than the *M* exposure, although the latter type of transposition arrangement has three transpositions while the former has but two. The reason for this is clear if it is noted that the second half of the *M* exposure starts out with the wires in the same position as in the first half and, therefore, the inductive effects into the two halves tend to add. With the *N* exposure the second half starts out with the wires reversed and, therefore, the inductive effects into the two halves tend to subtract. Extending this reasoning further, it may be shown that the *A* exposure having 15 transpositions is less effective than the *N* exposure since in the case of the former the small inductive effects in each successive eighth of the length, all tend to add.

In order to check the theory of the effect of different arrangements of transpositions roughly outlined above, tests were made of the relative effectiveness of the *M*, *N*, and *O* exposure types. Fig. 1 shows the method of making the measurements. A single-frequency source of alternating current was used in order to facilitate the comparison with computed values. In order to get the effect of a short exposure in a long telephone line, the distant ends of both circuits were terminated with 600-ohm non-inductive resistances, approximately the impedance of a long circuit of the construction used in the tests. The test end of the disturbed circuit was also terminated with 600 ohms resistance. A measured amount of single-frequency alternating current was sent out on the disturbing telephone circuit. A sensitive receiving circuit of high impedance was then alternately

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bridged across the 600-ohm termination at the test end of the disturbed circuit and across a comparison circuit which impressed at the terminals of the detector a known voltage of the same frequency as that impressed on the disturbing circuit. By adjusting the voltage of the comparison circuit and listening to the sounds heard in the receiver in its two positions, it was possible to measure the voltage impressed by the disturbing circuit across the 600-ohm termination of the disturbed circuit. The current through this termination could then be readily computed.

The single-frequency tests were made on two adjacent specially transposed No. 12 N. B. S. gauge pairs in a 40-wire toll line. The inductive effects were measured in a transposition section about 7 miles (11.2 km.) long. The transpositions were carefully located so that the crosstalk would be due principally to type unbalance and not due to irregularities in wire and pole

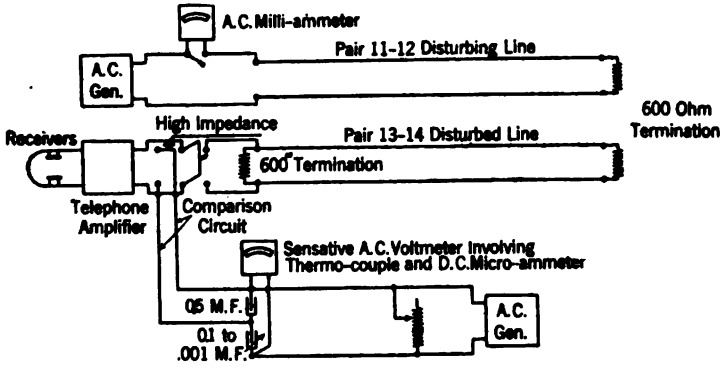

FIG. 1—METHOD OF MEASURING SINGLE-FREQUENCY CROSS-TALK

spacing except for combinations of circuits having very highly efficient transposition types.

The crosstalk in a short relatively untransposed length was first tested and then the crosstalk in a length of about 7 miles for the three exposure types was measured. By comparing the latter measurements with the former it was possible to express the exposure of any type in terms of an equivalent untransposed length. The following table shows a comparison between the equivalent untransposed lengths obtained in this manner and corresponding values computed theoretically from the equations of propagation of current on the circuits. From a comparison of the readings of different observers and of successive readings of the same observer, it was estimated that the precision of observations was ordinarily about 10 per cent. In the case of the *N* exposure, the precision was much less since the method was not suitable for accurately measuring the small amount of crosstalk involved. In addition the effect of the

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irregularities would, of course, be much greater in connection with this exposure since the crosstalk due to the type unbalance is very small.

M, N AND O EXPOSURE TYPES

MEASURED VERSUS COMPUTED VALUES OF EQUIVALENT UNTRANSPOSED LENGTHS

Type exposure	Frequencies tested	No. of readings	Average ratio measured to computed	Computed length 1000 cycles
O	840, 1,000 1,100	12	0.88	0.75 mi.
M	840, 1,000 1,100	12	0.99	0.38 mi.
N*	1,000 2,000	6	0.42	0.044 mi. or 230 feet

*Measured values too small to be read accurately.

Fig. 2 shows the arrangements of transpositions tested and a comparison of computed and measured values of crosstalk for 1000 cycles. It should be noted that the circuits were tested for impedance, insulation and direct-current resistance before making any measurements to insure that they were in good condition. The measured values were obtained from the averages of two sets of readings each involving two observers and taken on different days. The crosstalk diagrams on Fig. 2 show the effect of the transpositions in combining crosstalk

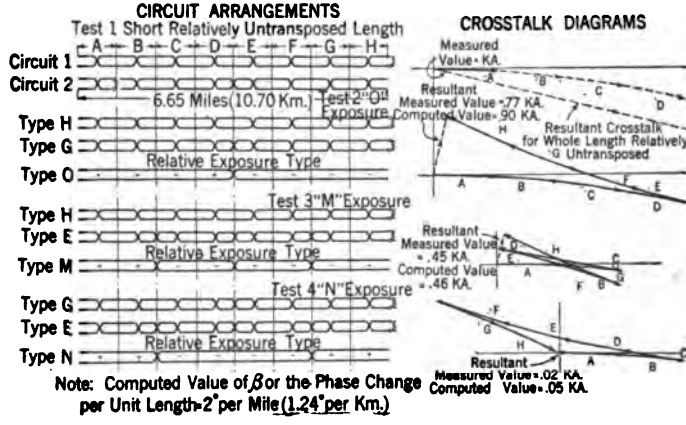

FIG. 2—SINGLE-FREQUENCY (100-CYCLE) "CROSSTALK" TESTS

at the terminal due to the various eighths of the transposition section. The average measured valve of the ratio of received to disturbing current for the test on the short untransposed length, that is $K\ A$ of the diagram, was 725 millionths while the computed value was 770 millionths.

Certain talking tests made later may be of interest since they show absence of crosstalk between circuits having an N exposure which, as previously discussed, is a relatively efficient

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transposition type compared with the other types here discussed. These tests were made in a manner similar to that shown in Fig. 1 for the single-frequency test, with the difference that the ratio between the received and disturbing currents was measured by shunting off a known fraction of the disturbing current and comparing it with the received current.

Fig. 3 gives data on the telephone lines employed in these talking tests. As indicated about 28 miles (45 km.) of a toll line was transposed by means of four *E* sections. The circuits tested had an *N* exposure in each transposition section and had comparatively large coefficients of induction between them, thus requiring an effective transposition arrangement. Tests made on two successive days indicated that the energy transferred from one circuit to the other was about one hundredth

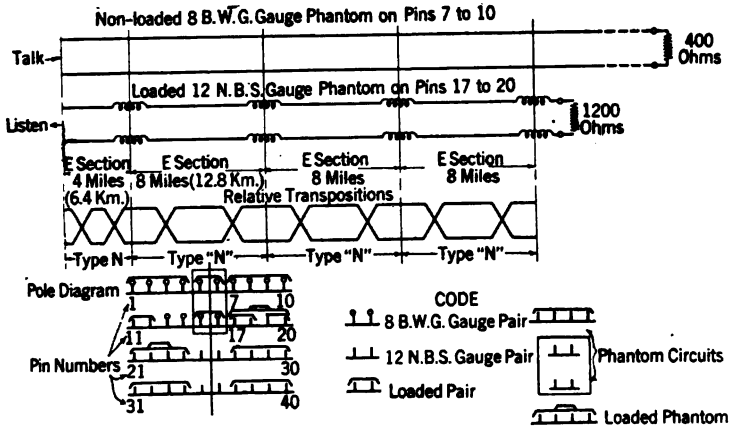

FIG. 3—"CROSSTALK" BETWEEN ADJACENT PHANTOM CIRCUITS HAVING TYPE "N" RELATIVE EXPOSURES

of a millionth of that transmitted into the disturbing circuit. About this amount of energy transfer was estimated due to the measured small irregularities in the locations of the transposition poles. The energy transfer estimated due to type unbalance was negligible in comparison.

In design of telephone transposition systems, a study must be made of the minimum separation which is permissible between circuits non-transposed to each other. It is, of course, desirable to design arrangements of transpositions for but a few crossarms and then repeat these arrangements on the lower arms. The minimum separation between similarly located relatively untransposed circuits on the first and lower arms is, therefore, of interest.

In Table I of his paper, Mr. Osborne gives a table of the capacities between various combinations of wires and between the wires and ground for a 40-wire pole lead, and in Appendix A

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discusses the method of computing from these measured values, the coefficients of electric induction between the various circuit combinations. In order to study the question of the repetition of a transposition system on the lower crossarms, it was necessary to make similar capacity measurements on an 80-wire pole lead. The lower curve of Fig. 4 is obtained from the data of these tests and shows the variation with separation of the coefficients of electric induction between a phantom on the end of the first arm and similarly located phantom circuits on lower arms. The magnetic coefficients of induction were readily computed from the known conductor spacings.

H. Mouradian: It may possibly further the understanding of the practical significance of the paper if I should state, briefly,

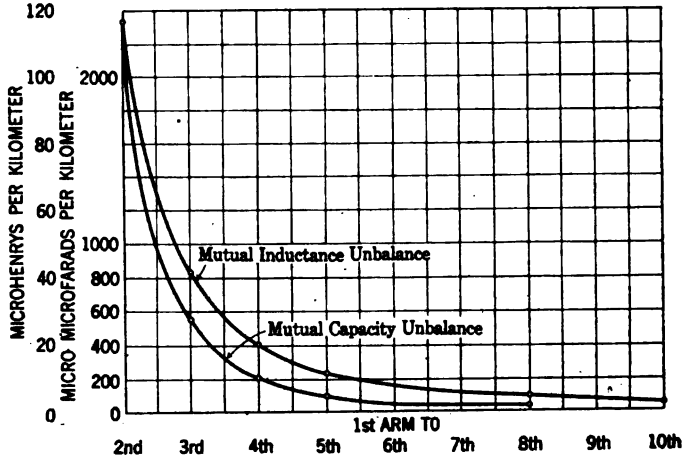

FIG. 4—ELECTRIC AND MAGNETIC COEFFICIENTS OF INDUCTION BETWEEN PHANTOM CIRCUITS ON PINS 1 TO 4 AND PHANTOMS ON CORRESPONDING PINS ON LOWER ARMS WITH STANDARD 10-PIN ARMS SPACED 2 FEET APART

some of my own experiences in the matter of the inductive interference on telephone lines in connection with two specific cases of exposure:

(1) In the first case of exposure, of approximately eight miles (12.8 km.) the power circuit was 33,000-volt, three-phase, star grounded neutral. The power line was built in 1905 or 1906. The power company installed on this eight-mile stretch two complete barrels,—a barrel every four miles. Special transpositions were also installed at the time, one special transposition on each telephone toll circuit midway between every two successive cross-talk transposition points on the telephone line throughout the length of the exposure. These two systems of transpositions, one on the power circuits and the other on the telephone circuits, were not, however, co-ordinated. The

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result was that the telephone circuits, exposed as described above, were very "noisy", in fact, so noisy that the main industrial company in one of the towns served by the telephone lines actually made a complaint that, if the telephone company did not improve its service, it did not want the service. The next phase in the history of this case brought about a coordination of the telephone transpositions. The situation was further improved by the installation of two special telephone transpositions, instead of one, between every two successive cross-talk transposition points. The noise on the telephone circuits diminished and was found not to exceed 200 to 300 units of noise. Then, by further work on the special telephone transpositions, with a view to locating them as advantageously as possible with due regard to the irregularities in the exposure, the noise on the telephone circuits was finally reduced to approximately 60 to 70 units. At the present time this condition is considered satisfactory from the standpoint of commercial operation.

(2) In the second case I have in mind a direct application was made of one of the systems described in Mr. Osborne's paper. In this case the exposure was for six miles (9.6 km.), the separation fairly uniform at 50 ft. (15.2 m.). The power circuits were three-phase, 23,000-volts, delta. The telephone circuits have just been transposed in accordance with the system described on Fig. 9 of Mr. Osborne's paper. Determinations of the amount of noise were made before and after the installation of the special transpositions with the results summarized in the following table:

	Noise Readings			
	Distant end of telephone line open		Telephone station at distant end of telephone line	
	Before	After	Before	After
Side circuits.....	175	37	172	17
Phantom circuits.....	775	112	887	37

It will be noted from the above table that a material reduction has been obtained through the use of the system of transpositions above referred to. I think I might say, in closing, that the paper discloses several important practical solutions of the rather complex problem of inductive interference.

H. S. Osborne: Professor Karapetoff raises a question about the term "mutual impedance." This term is defined in Rule 916 in the Telephone Section of the Standardization Rules. The definition given there is entirely general, and as Mr. Fortescue points out, the term is used in other than telephone work. It seems that the term is a valuable one and that it should be changed in the Standardization Rules so as to be given there its more general application.

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In response to Mr. Fortescue's question regarding the ethical arrangements between power and telephone companies in the case of inductive interference, I would say that my own belief is that, in general, the best practise requires that when either of the two or more schemes for reducing the interference is possible that one should be selected which gives the least total cost, counting up costs of changes in both the power circuits and the telephone circuits. Both systems are run for the benefit of the public and the interest of the public would seem to be best conserved by the application of this principle of least total cost. This principle can be applied independent of the question of the equitable distribution of expense between the two companies.

Mr. Fortescue speaks about the effect of the configuration of conductors of the power circuit. The triangular arrangement of wires is in almost every case materially better than either the horizontal or vertical arrangement in limiting the external field of the line. A great deal of information about this is given in the forthcoming book of Technical Reports of the Joint Committee on Inductive Interference in California.

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Address presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 27, 1918.

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ENGINEERS AND THE WAR

BY MAJOR GENERAL WILLIAM M. BLACK
Chief of Engineers, U. S. Army

I FEEL some diffidence in addressing you this evening, partly because, although I might have qualified as an electrical engineer in 1880, I know I could not do it now, and then, I feel that a man who has the privilege of talking to a number of men such as are assembled here at this time has a tremendous responsibility, because the task set before us is one that has been unequalled in the history of the world. We represent a creed, not only in our living but in our Government, which we believe is necessary for the salvation of this world, for bringing to this earth the Kingdom of God.

We are fighting a nation that has a creed directly opposite to ours, a creed that leads them into the belief that Might makes Right, and that anything that is for the supposed good of the State is Right. This attitude has led the people of that nation to break every law that humanity has made for the amelioration of the horrors of war, laws that have gradually grown up through the ages. These laws are being ruthlessly sacrificed by these people in strict accordance with the creed which they hold. That creed and that people must simply be swept from off the face of the earth, and that is the task that we have undertaken.

It is an honor to be privileged to address you on a subject of such importance to our country, the duty of the Engineers in war. Although the part played by Engineers in this war is great and the responsibilities of the profession are correspondingly large, this war, like all wars in the past, is and must be a war carried on in accordance with the principles of the art of war, which are unchanging and which have been recognized and taught ever since organized armies were first created.

Do you realize that almost the only absolutely modern method of warfare now in use is the warfare of the air? The invention of submarines was made during the American Revolution, and

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submarines were used successfully, though to a limited extent, in our own Civil War. Gas and flame fighting are of ancient origin. Trench fighting is hardly better known today than it was to the veterans of Grant and Lee, of Sherman and Johnson. The advances in human knowledge have caused corresponding improvements to be possible in the weapons of warfare. Increased knowledge of chemistry has produced more powerful explosives and improved methods in metallurgy have enabled these explosives to be utilized, by making possible heavier and more powerful guns. Improvements in the means of transportation have enabled larger bodies to be moved more quickly and more readily and to be subsisted and supplied with greater certainty. The telegraph, the telephone and the wireless have afforded a means of prompt communication and have enabled, larger bodies of men to be given co-ordinated action. With such changes, battles are fought on the same principles and won or lost from the same causes as in the time of Alexander the Great. This war has been called a people's war and so it is in the sense that due to modern facilities the entire resources of the people can be utilized to day as they could not have been utilized in the days of old. It has also been called an Engineer's war because in the quickness of movement and in the works necessitated by these modern inventions the services of Engineers become more conspicuous and perhaps more necessary than in the past. But engineering in warfare has always been essential and it is even doubtful whether the science of engineering does not owe its birth to the works of war. An Engineer myself, I would be the last to belittle the work of our profession. It is a matter of pride that the men of our profession, due to the nature of their employment in time of peace, are, of all the civil professions, most prepared to serve the country in war, but to serve the country adequately in war, the Engineer must add to his peace equipment for professional work. The profession of arms is a profession in itself and it is the profession which deals with the very greatest in magnitude of all the endeavors of men. The effective use of an army which is properly constituted exemplifies the best that men can do in organization, in discipline and in the devotion to duty which causes a man to regard his own life as a thing of small moment toward the attainment of the end sought.

There would be a great amount of effort saved if our people recognized more clearly the existence of the technicalities of

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the profession of arms. The Government in Washington is simply deluged with suggestions and so-called inventions for the winning of the war. The records show that about 98 per cent of all of these are without military value and that time and labor have been thrown away by men eager to help, but entirely ignorant of the history and conditions of warfare.

An example with which some of you are familiar is the electrical gun. For years the possibility of such a weapon has been a fascinating line of study to electricians. The principle of the solenoid is the germ. If a series of solenoid coils were to be energized and de-energized in succession sufficiently and rapidly, such a series around a tube can be made to impart a movement of translation and rotation to a projectile. But practical results are today impossible. A six-inch service rifle having a length of 20 feet, fires a projectile weighing 110 lb. with a muzzle velocity of 2600 ft. per sec., or in other words, the projectile leaves the muzzle with a kinetic energy of translation of 115,500 ft-lb. This energy has been stored in the projectile during its travel through the bore of the rifle, or say in 1/65th of a second. The average power expended has therefore been at the rate of 7,607,500 ft-lb. per second or about 14,000 h. p. or 10,500 kilowatts. These figures are simply approximations and neglect entirely the power required for imparting velocity of rotation and for overcoming the friction in the bore. You can easily estimate the weight and dimension of the generating equipment which would be required for even a moderately powerful gun were all the mechanical and electrical problems of its manufacture solved, and making due allowance for the short-load periods. You can understand the impracticability of transporting the electrical plants required for any number of such guns, and the impossibility of distributing this power over shell-swept ground to guns whose position must be constantly shifted, and which must be put in action on a few seconds' notice. I think that you will agree that until new discoveries give a much improved method of storing or generating electricity, smokeless powder will continue to be the most compact and convenient form of stored energy for guns.

And yet there has been a good deal of time and money wasted in trying to perfect such a gun by men whose patriotism is undoubted, and whose ignorance, also, is undoubted. In other words, if a man has an invention or an idea of an invention, by all means let him work on it, but before he goes to Wash-

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ington and takes up the time of men busy trying to devise means to beat the Boches, let him make sure that he knows the conditions of war and what he is trying to do to meet those conditions, and then if he is sure of the means, let him present his ideas and inventions to the people in Washington.

We want all we can get and want the best we can get. We want the inventive power of our country if it can be exercised to do good.

There was a proposition made seriously at Washington recently that the United States should provide a fund, of I do not know how many million dollars, and make a Home for Inventors, where any one who thought he had an invention would be able to go, and work it out at the public expense; and recently, although we had a committee of experts there to pass upon these inventions, the results were so utterly unsatisfactory to the inventors that they came in a perfect horde upon the Secretary of the Navy and the Secretary of War, so much so that they had to make a brand new committee of three men, who could be much better occupied, to go ahead and do this same thing; and I am only waiting for the next drive of inventors to show that this Committee will not suit them one particle better than the old one did.

In addition, without doubt, there are many men in our country of the highest patriotism who are sore-hearted because they are not given something to do directly toward the winning of the war. They do not understand, that some condition peculiar to themselves, possibly age, possibly physical condition, possibly mere ignorance of war and its conditions, compel it that the bit that they must do for their country at this time is to continue in their work in civil life and do their part in keeping up the normal life of the country—in itself a service of importance.

The part which Engineers are now playing in the war is a very great one. The records of the American Institute of Electrical Engineers show that out of a total membership of 9443, there are 973 in the service, or 10.3 per cent of its roster. The American Society of Civil Engineers with 8753 active members has 14⅓ per cent in the service. The American Institute of Mining Engineers 10.4 per cent, and the American Society of Mechanical Engineers 10.1 per cent. But these records are not complete. At the outbreak of the present war there were in the Engineers Corps of the Regular Army about 300 officers and approximately 3500 enlisted men. At the present time,

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there are about 8000 commissioned officers and 200,000 enlisted men, made up of men formerly engaged in works of an engineering character. It is probable that this does not represent much more than one-half of the number of the profession now serving in the Army.

Let us consider the nature of the work of the Engineer, passing from front to rear of the Army.

First in importance is the work of the sappers. They go before and remove obstacles, clearing away obstructions, building bridges and roads, making the trench systems complete, mining, providing light, water, lines for supply (light railways or roads) and military mapping. In this category enter practically all of the branches of the profession. Further to the rear are found the construction and operation of railways; road and bridge construction; the construction of veritable towns for supply depots, with all their accessories, drainage, sewerage, lighting and water supply; construction of quarters and of hospitals; and furthest to the rear, the construction of the ports of debarkation with their wharves storehouses, railway lines, yards and shops, all with their sanitary systems. Separate from these activities, but necessary for their supply, are the Forestry troops who turn the growing timber into lumber of the dimensions required for the various services. Locomotive and car shop troops are performing essential services. Topographic Corps, Sound Ranging Corps and Camouflage Corps are also among the varied activities of the Engineers.

What preparation is required for the fulfillment of these varied duties? For the actual technical work of construction or installation the civil training of the Engineer should prove sufficient when the plans which embody the military features have been prepared, or when the military technique has been learned and assimilated. A fundamental of this military technique is that the time element is to be considered rather than money cost and that the work must be done with whatever materials are available. This requires clearness of conception of the results required, resourcefulness and organization—factors also required for civil work.

As stated earlier, due to the very small numbers of the personnel of the Corps of Engineers of the Regular Army, reliance had to be placed in the members of the profession in civil life. Confidence in their devotion to country and in their ability has not been misplaced. The results already accomplished prove

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this fully. Could more have been done? Undoubtedly, had the profession been better prepared for the call.

Will you permit me to say a few words concerning the general training of our Engineers, based on a professional experience of more than forty years? The conviction has been forced upon me that in educational matters, as in many other affairs of life, we Americans are inclined to go too fast. The basis for any professional career where the highest is to be attained must be a sound general education. Does anyone of you regret the lessons gained in your own experience? Is not the experience of humanity as shown in properly written history of almost equal value? Would Russia now be in the sad condition existing had her people known that the experiments she is trying have always resulted disastrously? Yet is history thus considered in an ordinary technical course? Again, do you not find a knowledge of the general principles of law and of the special rules of the laws of contracts of value? Are these considered essentials? What is the handicap of an Engineer who is unable to express his ideas clearly in spoken and written English? Is this taught thoroughly in our technical courses?

It goes without saying, that the study of pure and applied mathematics is found in all technical courses. But, are these subjects well grasped before their application in special technical courses is studied? Is any faculty of an Engineer of greater value than the ability to form a mental picture of his problems and of its solution? Yet is that study which assists most in this faculty—descriptive geometry—properly apprehended? Is there any branch of the profession which in its application is not based on a knowledge of topographical work, on a knowledge of construction materials and of how these should be used? Is the study of these branches of civil engineering insisted upon sufficiently in the mechanical and electrical courses? In effect would not our professional men be better equipped for their civil work were they not in too great a hurry in their youth to enter directly into life's combat? Does not this war teach that without a long and elaborate preparation down to the last details, an attempted "drive" must fail?

These remarks apply to all Engineers, both military and civil. In the rush of war men cannot always be hand-picked for special jobs and frequently it becomes necessary for an available man to be used for the work immediately necessary, irrespective of his previous training. In this supreme test of

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humanity the best man is he who is prepared to meet any emergency—perhaps not in the most finished way—but to meet it.

There are things that the engineers in this country can do. If they do know enough to give us some ideas for helping along in the killing of Boches, for God's sake let us have them. If they do not, what they can do is to help the supply of men for the winning of the war. We are now short of officers of Engineers, very short, and we are going to be very much shorter. We must have educated engineers for this work, and we must not only have the men for the line work of the army, but we must have mechanicians and artisans and laborers for the special work.

All of you men have spheres of influence—do your best in them, and if you can be used otherwise, and the problem comes up in which we need you, you may be sure you will be called upon. There is this problem now, the supply of men, in which you can help, either by your own personal sacrifice, in going out, or by influencing others.

Now as to soldier work. The movements of drill and the construction methods peculiarly military are easily learned. The knowledge of the art of war which will enable these to be applied promptly and properly is more difficult. But most difficult to acquire is the peculiar mental discipline which makes the soldier. The Army is a huge machine which must work co-ordinately in all of its parts. That competition, which in civil life causes one body to advance further and faster than another, is out of place in an Army. *All* must work together and for one common end. Each man must so subordinate his will and desire to the common good as to work willingly and earnestly in the sphere allotted to him. This does not mean that all initiative is to be suppressed. On the contrary each man must use his initiative to the utmost, but in his own allotted sphere of action. Each must learn to obey and obey from the heart. Through such obedience comes the knowledge of how to command when command becomes a duty. All of this is hard to learn. But each man who is called upon to help in this war must learn it, if he would help effectively.

By all means let us have military training in our schools, but let it be true military training and not tin soldier work.

There is another line of technical military knowledge which must also be studied hard. The machinery for the organization, training, supply and leadership of troops; the methods of ob-

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taining, accounting for and issuing supplies; of keeping returns of the men; and the channels of command must be studied. To civilians in general this is wholly unknown, but if a man is to be of service in the Army, it must be learned until its use becomes automatic.

What have the Engineers done? War was declared April 6, 1917. By the middle of July, nine regiments of Railroad Engineers had been raised and organized and two had actually started for France. In each regiment were two officers of the Corps of Engineers of the Regular Army, the Colonel and the Regimental Adjutant. The remaining officers were all from the Engineer Reserve Corps, some receiving their commissions only when on the point of sailing. Of course, few of the officers had had any previous military training and the tasks of organization were most difficult. Since then, there have been organized:

Five Corps Regiments consisting of Sapper, Searchlight and Sound Ranging troops; 43 Sapper Regiments and trains; 2 Mounted Battalions and trains; 5 Ponton Trains; 4 Inland Waterway Companies; 40 Railway Regiments and Battalions including all classes of Standard Gage and Light Railway troops necessary for the construction, operation and maintenance of railways; 1 Railway Transportation Corps; 1 Highway Regiment; 1 Gas and Flame Regiment; 1 Gas Training Service; 5 Forestry and Auxiliary Forestry Regiments; 1 Surveying and Printing Battalion; 1 Military Mapping Service; 2 Supply and Shop Regiments; 1 Water Supply Regiment; 1 Quarry Regiment; 1 Mining Regiment; 1 Electrical and Mechanical Regiment; 2 Crane operating Companies; 1 Camouflage Battalion; 18 Truck and Auto Companies and 44 Depot Detachments.

The greater part of these organizations is now overseas. Some are serving with the British Army, some with the French, but the majority is with our own troops, in service both at the front and in the rear.

I wish I could go into greater detail as to the work of these Sound Ranging Corps, because it comprises some new electrical work of the highest character, and the apparatus for it has been perfected in this country. We took the best devised at the beginning of the war, and our physicists went to work and have made marked improvements. Perhaps you do not know what sound ranging is. The artillery is stationed in the

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rear of the line. There is almost no direct artillery fire any longer—that is, as a rule, the gun is fired from a point where the target cannot be seen at all. The first thing to be destroyed, invariably, is the enemy's artillery, then the trenches are attacked. The obstructions of wire are torn to pieces, the trenches themselves are practically leveled, and after that is done, in the assault, there is what is termed the barrage fire. That, I suppose you know means a fixed or slowly moving curtain of shells dropped on a certain given line and through which passage is almost impracticable.

On both sides the artillery is carefully camouflaged so it cannot be seen from aeroplanes. To show what care is taken, even the tracks that are made in taking the guns to the front are wiped out, the guns themselves are covered, so that neither from an observation balloon nor an airplane from the enemy's line can the position of the gun be seen, and in order that the flashes of the gun cannot be located, there are dummy guns placed at intervals, and flashes from these guns made by electricity, so that the position of the real guns cannot be known.

In order to determine the position of the real guns, there are delicate instruments which have been devised, which are placed at intervals along the line. These instruments are for the purpose of registering the sound of the gun. There is, first of all the sound of the gun in firing. That is preceded frequently, if the range be great, by the sound of the shell passing through the air, and sometimes by the bursting of the shell itself, before the sound of the gun comes. These are all recorded, and the velocity and the direction of the sound is known. By having these instruments at different points on the line, the position of any one gun can be "spotted," and "spotted" so closely that our own artillery fire can be directed and the gun blotted out. That is one of the improvements of modern warfare rendered possible by the advance in general human knowledge, particularly in electrical knowledge, and these instruments are very exact.

This service of the rear is of great importance and magnitude. Picture to yourselves what is required to transport, house, supply and maintain a million men three thousand miles from home, producing nothing and in their work expending enormous amounts of materials.

Taking the question of storage alone, the provision of space required for an army of 1,000,000 for ninety days aggregates 20,000,000 square feet of floor space of covered storage and

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double that amount of uncovered storage space, with the necessary railway tracks for receipt and shipment and for classification yards, aggregating about 650 miles. Add to this an equal mileage of highways, adequate provision for water supply, sewerage and electric lighting and power and you can realize the work involved in this one item. Add to this the constructions which have been built at the Ports of debarkation (at one of which 375,000 square feet of wharf space had to be provided), the hospitals, barracks, shops, and the lighting, water and sewerage systems required, and some conception of the actual new construction work done, can be formed.

It is estimated that the supply of the army requires the transportation to the front of 25 lb. per man per day. This makes heavy demands on the French railway systems, good as they are. These have had to be supplemented in all but the main line trackage, and a large amount of motive power and of rolling stock has had to be supplied and operated.

Among the special services, the work of the Geologists must be mentioned, and in the line of improved apparatus, it may be stated that new instruments and methods for airplane photography have been devised and introduced. Other new auxiliary aids for fighting have been worked out, some of which have already proved their value on the battle field.

Yes, the Engineers are doing their work well. Be it in constructions in the rear, or under fire, be it in the transportation of ammunition to the firing line, the construction of strong points and obstacles, the construction and destruction of bridges in the face of an enemy, or as in recent instances, under the feet of the enemy, or be it with their rifles in beating back an attack, they are doing and *dying*. All glory to our Comrades in arms in France! There is not a red-blooded American who does not envy them.

But is there not a war duty for us also, for us who are held on this side of the ocean? Yes, undoubtedly. To some is allotted a task in supply, to some a task in manufacture, to some a task in organization. But that is not all. The life of the nation must go on. Her civil machinery must function undisturbed. With so many called away from these civil duties, the onus of the work will be the heavier for those who are left. Let us each then do his bit as and where it presents itself knowing that if each does his best, with love of country and forgetfulness of self as guides, the results are sure.

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Gentlemen, I have been in the front line trenches, and a more abominable place for a man to be in, apart from any enemy, you can hardly imagine. It is raining there much of the time. The mud is up to your ankles, even though the trench be drained and the best attempt made to keep the drainage in good condition. The men at the front are distant from their supports, and they are scattered, maybe a dozen at a place, directly in the face of the enemy, and they have to stay there, and have to be prepared to resist any attack that comes, with a certain knowledge that should the attack come in force, or should a real earnest effort be made to take the line, they in the front line are almost sure to perish, and they are not minding it. They are staying there in the face, not of possible death, but in the face sometimes of almost certain death. They are doing their work, and our engineers have the most dangerous part of that work to do.

You remember in that first British drive to Cambrai, the Boches got in the rear of the firing line. They struck a part of one of our Engineer Railway Regiments, building light railways to the front to bring up ammunition. These men had not their arms with them. All they could do was to scatter and get in shell holes, and as soon as the Boches went by to rally and go back, and take the arms thrown down by the wounded or dead, and then pass over, and they formed their part of the line and did their share of the fighting.

The evening of the great drive, I had the honor of dining with Sir Charles Douglas Haig. He then told me of one of the battalions of the Sixth Engineers, which had been with the British about six weeks, and in that time they had built twenty bridges with spans, ranging from 16 to 60 feet, and he spoke of the wonderful work they had been doing. Two days afterward these men took their place and held about a mile of the British line, and held it so that the Boches could not get through.

The same happened with one of the railway regiments, again caught in the same way, and they formed a part of that miscellaneous army that Gen. Carey got together of the supposedly non-combatant troops, and the line which they formed prevented the capture of Amiens. They held the Germans. There, again, the engineers played their part valiantly.

Just two days ago a cablegram came across from the other side, about one of the companies, Company C, I think, of the First Regiment, which had been ordered to do particularly

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dangerous work in the neighborhood of the Marne. As they started out, two of the officers were killed at once, but the rest went on and did what they were ordered to do.

Then again, you have noted that when the Germans got down to the Marne and were crossing a bridge, some of the engineers waited until the head of the Germans had gotten completely across the bridge, which was filled with the Germans, and they blew up the bridge, and those who got there wished they had not.

Our men are doing their work on the other side wonderfully, and those are our brothers in the profession who have gone over and now wear the uniform.

(General Black then exhibited some moving pictures showing the organization of the Engineers and how they do some of their work, and finally some of the work that they have already accomplished in France, after which he added):

In landing in France, you are struck at once by the number of maimed men you see on the street and by the women in black, and then, as you get to know them, you will see that these people are from their suffering simply the more determined to carry this war through to a successful finish. The same thing is true in England. You are struck by the grim determination to win success at whatever cost. There, due to a difference of temperament, things are taken somewhat differently. I was at a tea in England, and was presented to an English woman of title. She was gowned just as any woman would be for an afternoon reception. In speaking to her I mentioned I had two boys in France. She said, "I did have four, I have only two now," and then seeing me look a little startled, she said, "You know we feel our private griefs must not be allowed to show, that we must not wear mourning, that we must go ahead with our ordinary duties and try to keep up the social life."

We have only just begun to fight. We have not begun to suffer. If you were there, and could see what these people have done, and what they are prepared to do, and then feel as you would feel, how completely they trust us to bring this war to a successful finish, you would feel, too, that it is up to us to equal them in endurance, equal them in sacrifice, and to see that freedom, individual freedom, is preserved for all time in this world. I thank you very much.

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ELECTRIC POWER FOR NITROGEN FIXATION

BY E. KILBURN SCOTT

ABSTRACT OF PAPER

Reference is made to propaganda against processes for making nitrates from air by those interested in keeping the Allies dependant on supplies of nitrate from Chili.

A tabular comparison is made of the operations involved in the *indirect* method and the *direct* method of fixing nitrogen. The indirect method involves the manufacture of carbide of calcium and its combination with nitrogen to form calcium cyanimid, from which ammonia and in turn nitric acid are obtained. The direct method merely consists in combining nitrogen and oxygen of the air in the electric arc.

In the direct method electric energy is the only factor, whereas by the indirect much plant of a very diverse and complicated character is required. Also there are difficulties in connection with the platinum catalyst necessary to convert ammonia into nitric acid. It is claimed that the direct method is better because of the simplicity of plant and of operation, and the possibility of working with off peak power. The suggestion is made that a number of plants for making nitrates by the direct arc process should be erected at existing power houses. Keeping the generating plant more fully employed would improve the load factor and reduce costs.

By making nitrate in a number of centers the transportation of same to the explosive factories would be reduced and the risk of interruption of supplies in case of accident or sabotage would be less than in having a few very large factories.

A diagram is given showing the layout of a battery of by-product coke ovens with an electric power house worked by the surplus gas and a nitrate from air plant to use the electricity. Figures are given showing that the nitric acid made by such a plant is about the right amount to combine with the ammonia to form ammonium nitrate, a compound in great demand at the present time for explosives.

ONE of the most powerful combinations in the world is that connected with the exploitation of Chile Nitrates, and to extend the uses of that material and regulate prices, etc., there is a Chile Nitrate Committee supported by the various interests concerned.

It was created for propaganda work amongst farmers and others, to facilitate the use of nitrate as a fertilizer but since the advent of air nitrates some attention has been given to discrediting the methods of fixing nitrogen from air. This has been done partly by paragraphs in the press throwing doubt on the financial and technical success of such methods, etc. German

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influence, working through political clubs and the press, also assisted the Chile nitrate propaganda while at the same time German scientists were being assisted in every possible way to develop air nitrate processes in their own country.

Years before the war, some of us saw that the question of supplies of Chile nitrate for the manufacture of explosives would be an important factor, and in 1911, at the Portsmouth meeting of the British Association, and later at a meeting of the Society of Arts in London, I sounded a note of warning.

Immediately after the war started, the German government appointed an electrical engineer, head of the Allgemeine Elektricitäts Gesellschaft, to expedite the manufacture of explosives. Air nitrate plants already in operation were greatly extended, and new nitrate plants and the power houses necessary to supply them with electricity, were put in hand. One such power house built early in the war at Bitterfeld, develops 185,000 kv-a. from lignite coal.

Even after three and a half years of war, the Allies still remain practically dependent for explosives on supplies which have to be brought thousands of miles across the sea from a foreign country.

So far as Great Britain is concerned this policy of dependance on supplies from oversea, appears to have been dictated by the Ministers of munitions, two of whom have been lawyer politicians, and one a doctor of medicine. They were probably acting partly on the advice of the head of the department of explosives supplies, also a member of the legal profession.

Considering how much this is an engineer's war, and how expert are the men in charge of the departments in Germany which handle such matters, does it not seem absurd that any of the technical affairs on the Allied side should depend on persons whose particular ability in life is to make speeches that catch the popular vote.

The transportation of Chili nitrate requires much shipping that might be used for other purposes and also occupies attention on the part of the Navy, in order to keep open the sea routes. In 1917 the nitrate imported to United States from Chili amounted to 1,742,540 tons (see figures of Imported Supplies given toward the end of paper) and presumably at least as much more would go direct to Europe from Chili. The average export taken over every day in the year is at least 10,000 tons, and as a round trip is about three months, an easy calculation

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shows that about 1,000,000 tons of shipping valued at $150,000,000. is tied up in this Chili nitrate carrying trade.

Huge sums have also to be paid for the nitrate, and to shippers and brokers for transportation, insurance, etc. On the other hand by being independent of Chili, the Germans have kept their money at consequent financial and economic advantage to the country.

With the right men at the head of affairs, possessing expert first hand knowledge of the engineering and chemical problems involved, the Allies could have become equally independent of Chili and much money and man power which have been absorbed in building obsolete chemical and other plants would have been saved.

To their credit, certain scientists and engineers of this country not only saw the danger, but insisted on the authorities taking action by providing money to establish plants for the manufacture of nitrates.

At the same time, in this country as well as in England, there has been time lost, owing to certain parties maneuvering to obtain the adoption of their own process to the exclusion of others. In so large a field as nitrogen fixation there must necessarily arise numerous improvements in the various processes so that it is not possible today for anyone to gage or forecast their future relative economic values.

This is particularly the case with processes in which electrical energy plays a leading part for it is a sort of ingrained habit of the electrical engineer to simplify and revolutionize existing methods that they eventually become essentially electrical. The whole history of electrical progress, and especially of electrochemistry and metallurgy establishes that fact.

Investigation boards or committees dealing with nitrate problems should be largely made up of engineers who have expert first hand knowledge of electric power conditions and of apparatus, etc. It is not right that chemists should have practically sole power to pass upon processes in a field which electrical engineering is capturing so completely as the production of nitrates.

I feel that the merits of the arc flame process for making nitric acid have not been adequately and sympathetically considered, and this paper is written with the special object of stating them. I wish also to remove the misconception that the arc flame process is dependent on water power and that it can only be in-

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stalled economically on a very large scale. The matter is one of special interest to electrical engineers especially those concerned with the production and supply of electrical energy.

Methods of Making Nitric Acid

Atmospheric nitrogen can be fixed either as nitric acid or as ammonia. The first named forms a basis of all explosives, whilst ammonia is used for one explosive viz. ammonium nitrate and for that purpose it must be combined with nitric acid.

The only way to make nitric acid direct from the air is to combine the nitrogen and oxygen of the air in the electric arc, and then combine the nitrous gases so produced with water to form acid. Owing to its simplicity this is called the *direct* method.

To make ammonia from atmosphere nitrogen the most straightforward method is the synthetic which has been adopted at Nitrate Plant I at Sheffield, Ala. By this method pure nitrogen and pure hydrogen are raised to a high pressure and temperature and in the presence of a catalyst they combine to form ammonia.

Another method called the *indirect* is used at Nitrate Plant II at Sheffield and briefly this is to make carbide of calcium and treat it with nitrogen to form calcium cyanamid, the cyanamid being then acted on by steam to produce ammonia.

Having obtained the ammonia it has now to be made into nitric acid, and this is done by the catalytic process of Prof. Ostwald using platinum as the catalyst. The method adopted at Sheffield, Ala. is to take about half the ammonia produced, and convert it into nitric acid and then combine this acid with the remainder of the ammonia to form ammonium nitrate. Clearly this makes just half the quantity of ammonium nitrate that would be possible, if all the ammonia was to be combined with nitric acid made in some other way, as for example by the direct arc process.

For some considerable time there has been great shortage of ammonia, and therefore, to use any of it as raw material for another nitrogen compound—nitric acid—seems short-sighted policy.

The unit value of nitrogen in ammonia, is just as high as the unit value of nitrogen in nitric acid, so it is economically and financially unsound to at any time make nitric acid from ammonia as a raw material. For this reason it is impossible for the manufacture of nitric acid via cyanamid and ammonia to compete under normal conditions with other methods.

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Those interested in the indirect method have drawn comparisons between it and the direct electric arc process, with the object of showing that the indirect is the better. A tabular statement of all the operations involved in the two processes as given below is the best way to make the comparison.

	Indirect Method Employing Calcium Cyanamid to Make Ammonia and Oxidising the Ammonia to Acid by a Catalyst	Direct Method Employing the Arc Flame Furnace Only
Factories	1. To make calcium carbide. 2. To make cyanamid. 3. To make nitric acid.	1. To make nitric acid.
Operations	1. Burning limestone.	1. Blowing air through electric arc flame to produce nitrous gases.
	2. Grinding lime.	2. Absorption of gases in towers to produce acid.
	3. Grinding coke or anthracite.	
	4. Mixing lime and carbon in correct proportions.	
	5. Making calcium carbide in electric furnaces.	
	6. Grinding carbide to fine powder in neutral atmospheres.	
	7. Making liquid air to produce pure nitrogen.	
	8. Packing calcium carbide into retorts.	
	9. Making calcium cyanamid by adding nitrogen and by heat of electric resistors.	
	10. Emptying cyanamid from retorts.	
	11. Grinding cyanamid to a fine powder.	
	12. Hydrating cyanamid to rid it of unchanged carbide.	
	13. Superheated steam.	
	14. Treatment of cyanamid with steam in autoclaves to produce ammonia.	
	15. Cooling the ammonia and elimination of water.	

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	16. Oxidation of ammonia to produce weak nitrous gases by means of platinum catalyst. 17. Absorption of gases in towers to produce acid.	
	INDIRECT METHOD	DIRECT METHOD
Raw Materials and Renewals	1. Lime. 2. Coke. 3. Carbon electrodes in carbide furnaces. 4. Carbon resistors in cyanamid retorts. 5. Pure nitrogen. 6. Superheated steam. 7. Air. 8. Water.	1. Air. 2. Water. 3. Metal electrodes.
Electric Energy for	1. Carbide furnaces. 2. Grinding carbide. 3. Cyanamid retorts. 4. Grinding cyanamid. 5. Heating catalyst. 6. Motors for power, etc., including several cranes.	1. Arc flame furnaces.
Skilled Labor for	1. Carbide furnaces 2. Cyanamid retorts. 3. Packing cyanamid. 4. Grinding machinery. 5. Making pure nitrogen. 6. Making ammonia. 7. Catalytic process. 8. Absorption plant.	1. Arc flame furnaces. 2. Absorption plant.

It is frequently stated that the amount of electric energy required for a given quantity of nitric acid produced by the indirect process, is less than that required by the direct, and this is put forward as a strong argument in favor of the indirect method. Clearly, however, the only way to compare two methods is to take into account *all* the factors which go to make up the total cost, and appraise them all at their proper values.

If two processes are to be compared as regards one factor only, then it may with equal justice be claimed that the electric energy represented by a few motors and lights required for a plant making acid from sodium nitrate, is less than the electric energy required by all other processes for making acid. Such a statement does not prove anything, and yet it is similar to the one put forward by the advocates of the *indirect* process.

Even when comparisons are made on an energy basis, they are useless, unless *all* forms of energy are included. For example, the steam for the autoclaves of the *indirect* process has to be at 150 lb. per sq. inch superheated to 350 deg. fahr. and this requires

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considerable heat energy in the form of burning coal. Steam is also required for evaporation of water from the products of the end process, and for the distilled water for the acid absorption towers. The heat energy required for these various purposes should therefore be added to the electric energy taken by carbide furnaces, the cyanamid retorts, and for heating the catalysts.

On the other hand, the direct arc process raises its own steam, from heat of the hot gases coming from the furnace. This steam is sufficient for the end process and for distilled water and also under certain circumstances, may be used for generating electric power, so that the process can work regeneratively. At the Norwegian plant at Rjukan II, for example, there are three 4000 kw. steam turbo-generators, generating electricity with steam raised by the furnace gases.

The energy which can be regenerated in this way may be over 15 per cent, and this should of course, be deducted from the total electric energy measured into the furnace, when estimating the energy used for a given yield.

Obviously the cost of plant using the indirect method, will be very much greater than that in the case of the direct, for if we assume that the cost of a carbide furnace and its accessories is about the same as that of an air nitrate furnace with its accessories, then, the indirect process embraces in addition:

1. A complete plant for making cyanamid.
2. A liquid air plant for making pure nitrogen.
3. Powerful machinery for grinding the carbide and the cyanamid.
4. Steam boilers and autoclaves for making ammonia.
5. A complete catalytic plant for oxidizing the ammonia to nitric acid.

In the indirect method it is essential to have all the materials, gases, etc., absolutely pure, for example at the cyanamid works at Odda in Norway it was necessary to carry a pipe up the mountain side so as to ensure a supply of pure air to the liquid air plant.

When carbide is converted into cyanamid some of the former remains unchanged and in order to obviate danger of explosion a special treatment of the mixture is necessary to ensure a total decomposition of the remaining carbide.

To convert ammonia into nitric acid a catalyst is employed and this usually takes the form of a net of platinum wire, the surface of the wire being covered with finely divided platinum

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black. The catalyst is readily poisoned by impurities in the gases etc., when the yield then falls considerably. The process is therefore a very delicate one and requires much skill to operate successfully. To "reactivate" the metal, it is necessary to subject it to an acid treatment and eventually to remelt, in which process it is impossible to avoid loss of this expensive metal.

Russia is almost the sole source of platinum, and whilst our Ally, could be depended upon. Today with Germany practically controlling that country, the position is serious. The various allied Governments have had to commandeer platinum as it is essential for several war purposes. With utmost deliberation and foresight the Germans are working to control the worlds storehouse of platinum in the Ural Mountains and any processes which depend upon this rare metal are going to be very seriously handicapped. I consider that those who have had a hand in starting new processes dependent on platinum are very blameworthy. Non-technical politicians could not be expected to know these things but those who did know should have informed them.

By the direct method the cost of air is nil, and the cost of water is practically that of pumping. On the other hand, the materials required in the indirect method are very expensive and especially difficult to obtain at the present time. Over three fourths of the cost of working the indirect process is represented in materials liable to price fluctuation. These are now much higher than before the war, and will remain at the higher level after the war.

In the direct method less than one-fifth of the total cost is represented in materials dependent on market rates, and the principal item of cost, namely electric power, will, if anything, tend to come down in price.

The direct method is very simple to operate, whilst the indirect requires much skilled and unskilled labor, and some of the operations are dangerous to health. Therefore, the more labor demands increase, the more will the indirect method be handicapped in this respect. There are many separate links involving exact operating, to make the whole run smoothly and the slightest hitch in connection with any one link necessarily holds up the whole system.

The manufacture of cyanamid has to be carried out in retorts of relatively small size involving much labor to set up, etc. This is to enable the nitrogen gas to penetrate to all parts of the con-

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tained carbide. The times of the reaction and the cooling down, etc., are definitely fixed and it is quite impossible to work the process with off-peak power; also should there be an accident or failure of current for a time, the cyanamid retorts and the carbide furnaces may be injured.

All nitrate processes have a military bearing as regards preparedness, in which is involved the question of transportation. The heavy and bulky raw materials necessary for the indirect process places it at a serious disadvantage from this point of view, especially at the present time when the railways are so congested. With the direct process there is no carriage of raw materials.

The indirect process has been strongly advocated in that after the war, cyanamid will be much used as a fertilizer. On the other hand a large number of objections have been voiced against such use, as witness the following extract from a book by Dr. Brion: "Cyanamid cannot be used with a large number of soils such as very sandy or moor soils, or with such soils as tend to become acid. Further it cannot be used for growing tobacco nor for some kinds of fodder. It is useless as a top dressing and can be applied only in dry weather when it must be plowed in at once. Cyanamid attacks the eyes of men handling it."

Whilst some of these objections may have been overcome by making the cyanamid granular and probably also some of them are over-emphasized, it still remains true that cyanamid is by no means as good a fertilizer as nitrate.

The effect of the calcium in calcium cyanamid in the presence of moisture is to cause the reversion of phosphoric acid and therefore it can only be used in limited quantities in a combined fertilizer.

In a legal action in the State of Maine between the Armour Fertilizer works and Ellis Logan, in April, 1916, there was expert testimony that calcium cyanamid destroyed a crop of potatoes and that not more than 60 to 70 lb. of it should be used per ton of fertilizer.

Electric Power

As a basis load for a power house the direct arc process presents the advantage that it can be established anywhere, because the raw materials being only air and water, considerations of transportation do not enter into the situation.

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It is particularly suitable for off-peak or off-season loads, for there is no fused material to solidify, and little to deteriorate in case of stoppage. Some of the furnaces can be switched on and off like an arc lamp, without detriment to brickwork or structural details, or to the process of manufacture.

As there seems to be some doubt as to the possibility of running arc furnaces intermittently on a commercial scale, I would mention that about seven years ago a nitric acid factory was built at Legnano, Italy to utilize 10,000 horse power, especially during the night (see Utilization of Atmospheric Nitrogen by T. H. Norton p. 68). Of course this plant has been considerably extended, especially since the war. I am also credibly informed that in Germany there is a very large arc process plant working with off-peak power. At any rate there is no difficulty in doing it, whereas it is impossible to work intermittently with any other method of fixing atmospheric introgen.

In some ways, it is an advantage to run a plant for 8000 or less hours per year, instead of the full number, because the spare time can be conveniently used for renewals and repairs, Less spare plant is thus required and the plant can be operated by two shifts of men.

Because the plants in Norway are very large and only use hydroelectric power, a mythology has grown up, that the arc flame process can only be worked commercially on a very large scale, and with water power. It is, however, worth while to build plants in units to utilize 10,000 kw.

As a matter of fact hydroelectric power may be a disadvantage because of its distance from industrial centers, for either the factory has to be placed in an out of the way position, or else the power has to be transmitted over a long transmission line. I am of the opinion that electrochemical factories should be placed near the power supply, and the ideal position is alongside the power house especially if off-peak power is used.

In a national emergency it is surely better to bring into immediate use all the surplus equipment that already exists, than to start building new power houses, whether hydraulic or steam, and seeing that the direct-arc flame process is suitable for working with off-peak power, I suggest that a number of nitrate plants be forthwith erected at existing power houses.

By erecting say, ten or more nitrate plants of say 10,000 kw. each at power houses near places where nitrates are required, there would be great saving in transportation and early de-

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liveries of nitrate could be made. Further there would be less risk of temporary interruption of supplies in case of accident or sabotage.

As a matter of fact there are power houses which could easily spare more than 10,000 kw. for over 20 hours a day and through the week end. Also there are power houses fully equipped with steam plant now standing idle which in the present crisis they might just as well be brought into use even if the cost of generation is high.

To give an idea of plant available, the following particulars of power houses will be of interest: From the existing plants in New York City, Mr. F. Hedley has recently estimated that about 115,000 kw. of off peak power might be supplied. In Chicago the existing power houses have a capacity of 470,000 kw. with additional 42,000 kw. now being installed. The maximum peak load of December 26, 1917 was 290,000 kw. for only half an hour, and by shutting off heaters on the surface and elevated cars during this short period, the engineers estimated that at least 90,000 kw. would be available.

There is no doubt that by restricting what may be called the luxurious uses of electricity, a considerable amount of generating plant in various parts of the country can be released for urgent war purposes, and this can be done without much increasing the coal consumption. There are also, the water power plants.

I was in Birmingham, England when for some months the street car service was stopped every day for between meal times, in order to save coal. The people readily adapted themselves to the conditions and the extra walking did the business people good whilst lack of street cars in the afternoon kept the women at home who were not engaged in war work.

In the numerous coal fed power houses of this country, a very large amount of fuel is wasted by having to keep the fires banked in the boilers during the valley periods of the load. Therefore there would be considerable saving by linking such stations to an industrial process which depends essentially on the utilization of electric energy, and the furnaces of which can be started and stopped with the facility of switching on and off an arc lamp.

The direct arc process fills these conditions perfectly, and it is the only chemical or metallurgical process which can be utilized easily with off-peak power. It takes the alternating current exactly as generated, for the furnaces can be built to utilize voltages generally available. There is no fused material to

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become frozen when current is shut off as in the case of carbide of calcium furnaces, steel furnaces, etc.

Some electrolytic processes may be run intermittently but they require direct current, and therefore there is great additional cost for transforming machinery etc.

Hitherto power station managers, have been content to merely supply electric energy and have taken little interest in the purposes for which the energy was used, at any rate from a financial point of view. The time seems to have arrived when this policy should be changed, and every power house, should have its own base load factory to utilize off peak power. I know of no better form of such load, than the manufacture of nitrates from the air by the direct arc process.

By filling in the valleys the load factor is much improved and this would have the immediate effect of reducing costs. There has, however, been too much consideration given to questions of cost. With U-boats on the high seas trying to stop supplies of Chile nitrate, the railways congested with traffic and electrical works engaged in making munitions, it is of little use discussing power costs. The thing to do is to make *full* use of plants already installed.

Recently much has been heard of the suitability of Muscles Shoals, Alabama as a site for the manufacture of nitrates because of the water power which is to be developed there, but it will take at least four years to complete the hydraulic works. In the meantime a large steam power house is being built in order that the cynamid process may be put in to early operation. This includes a 60,000-kw. turbo generator and should anything happen to it, the nitrate plant would be stopped as the various steps of the indirect cyanamid process are so interlocked.

Viewed from this standpoint it would seem to be better in every way to have the manufacture of indispensable materials for explosives manufactured in a number of smaller plants, in widespread centers and by other processes, than the indirect or cyanamid method.

Coke Oven and Nitrate Plants

At the present time ammonium nitrate is required in very large quantities for burster charges for shells, torpedoes, mines grenades, etc. This is made from two components, viz., nitric acid and ammonia, both of which are difficult to transport, the

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first because it is a corrosive acid and the second because in every ton of aqua ammonia there are about 2½ tons of water.

An industrial plant capable of furnishing electric energy as well as a supply of ammonia would be ideal, and it so happens that this is the case with a regenerative coke oven plant. Half the total gas made is available and this can be easily turned into electric energy whilst at the same time the nitrogen contained in the coal provides about the right amount of ammonia necessary to combine with the nitric-acid made from the electric energy by the arc flame process.

In order to show how ideal such a system is for making ammonia nitrate, I have prepared following diagram.

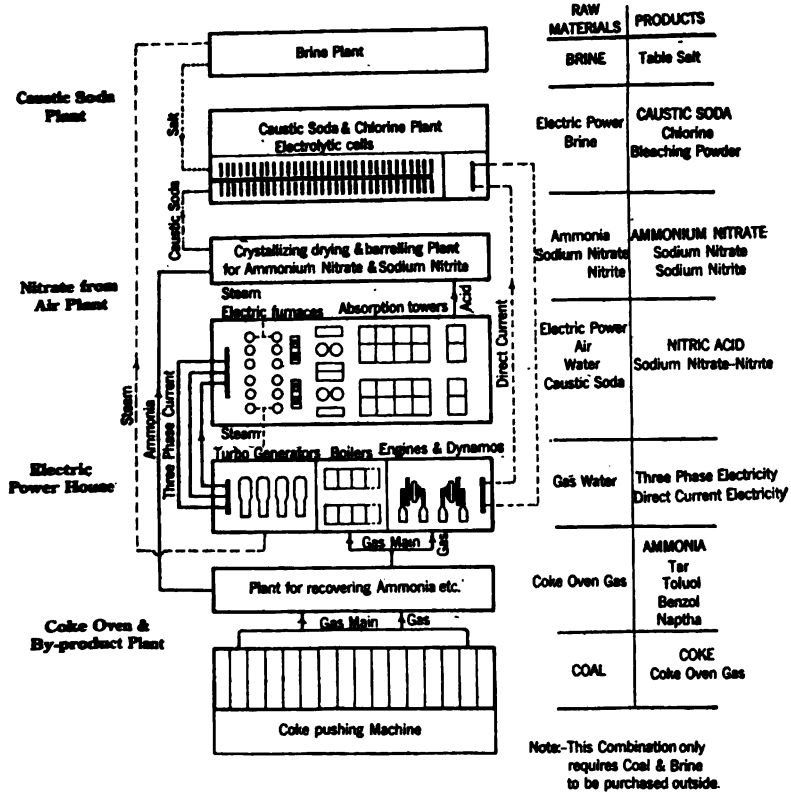

DIAGRAM LAYOUT OF NITRATE FROM AIR PLANT WITH ELECTRIC POWER HOUSE USING COKE OVEN GAS

The scheme provides for a combination of a battery of coke ovens with an ammonia recovery plant together with an electric

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power house in order to utilize the surplus gas. Alongside the power house, there is an electrochemical plant for the manufacture of nitric acid from air by utilizing the three-phase high-tension current. A nitrate house is provided for the purpose of combining the ammonia from the coke ovens with the nitric acid from the elctrochemical plant.

It happens that the by-product of the acid factory is sodium nitrate-nitrite, which is made by combining the gases remaining from the acid towers with caustic soda or soda ash. Electrolytic cells may be laid down as shown in the diagram for purpose of making caustic alkali from brine.

It will thus be seen that the complete project requires only two raw materials viz., coal and brine, and on the other hand, the products which can be made are coke and ammonium nitrate together with toluol, benzol, naptha, tar and sodium nitrate-nitrite.

If electrolytic cells are used there are also the products chlorine and bleaching powder. The chlorine can be combined with the benzol to form chloro-benzol which is an important intermediate in the manufacture of dye-stuffs as well as in the manufacture of picric acid.

Tri-nitrotoluol or T. N. T. is a combination of nitric acid and toluol and it will be noted that the plant supplies both these.

From the point of view of efficient management, and of elimination of transportation charges, the combination is unique, for the ammonia has only to be piped a few yards to the nitrate house and there is no carriage of acid.

As a cheap supply of coal is indispensable for the project, it would be well to locate the plants at industrial centers where this raw material is readily available and which in all probability would be locations where transportation charges are low.

In order to show what can be done with a coke oven plant, the following particulars will be of interest. I take a Koppers type of oven as being the better known, but the figures have general application.

Quality of Coal	Tons per charge	Hours coking time
Low volatile coal	13½	18
Mixture containing 80 per cent high volatile 20 per cent low volatile	12½	16½
High volatile coal	11½	15

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A battery of ovens varies in size but we may as well take a round number of 100 for which the average yields are as follows:

Number of ovens	100
Tons of coal per oven	12½
Hours coking time	16
Total yield of coke	72 per cent
Yield small coal and breeze	5 per cent
Net yield good coke	67 per cent
Ammonium sulphate per ton of coal	25 lb.
Reckoned as ammonia per ton of coal	6½ lb.
Tar per ton of coal	9 gal.
Light oil per ton of coal	3 gal.
Total gas per ton, of coal	11,000 cu. ft.
British thermal units	550 per cu. ft.
Surplus gas	55 per cent
Surplus gas per ton of coal	6,000 cu. ft.

Such a battery of ovens, each of which distils 12½ tons of coal in 16 hours, will deal with

$$\frac{100 \times 12.5 \times 24}{16} = 1{,}900 \text{ tons per day}$$

Assuming 6000 cu. ft. of surplus gas per ton of coal and 550 B.t.u. per cu. ft. the total heat value per hour will be

$$\frac{1900 \times 6000 \times 550}{24} = 260{,}000{,}000 \text{ B.t.u.}$$

If employed in gas engines using 13,000 B.t.u. per h.p-hr. the power will be

$$\frac{260{,}000{,}000}{13{,}000} = 20{,}000 \text{ h. p., or say, } 14{,}000 \text{ kw.}$$

If steam boilers and turbines are used instead of gas engines the power will be less, so to be on the safe side, we will take the round figure of 10,000 kw.

We will also assume that electric furnaces utilizing 10,000 kw. for a whole year, can produce 6,300 tons of 100 per cent acid. Nitric acid capable of furnishing theoretically 8000 tons of ammonium nitrate as indicated below:—

	NH_3 +	HNO_3 =	NH_4NO_3
Molecular weights	17	63	80
In short tons	1700	6300	8000

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Allowing 25 lbs. of sulphate of ammonia or 6½ lb. of ammonia per ton of coal, a total consumption of 1900 tons of coal per day should give.

$$\frac{1900 \times 365 \times 6.5}{20{,}000} = 2250 \text{ tons per annum}$$

It will thus be seen that there is plenty of ammonia to combine with the acid made by the surplus gas, even if a higher yield of acid is allowed per kw-yr. and more power is generated.

I purposely leave out of discussion, questions as to types of nitrogen fixation furnaces and of yields obtained. I may say, however, that it is not right to assume that yields are limited to those usually obtained from certain well known furnaces which must of necessity work with single-phase current.

The amount of ammonium nitrate will be less than the theoretical figure because the efficiency of the reaction is not 100 per cent, also it is usual to convert a certain amount of the gas into sodium nitrate-nitrite. A safe figure would be 6000 tons and at this rate it can be shown that with electric energy at 5 mils per kw-hr. and ammonia at 12 cents a pound, the ammonium nitrate can be made at less than half the price the Government is now paying.

In order to show how large a business the nitrogen industry has become, the following figures (compiled by Dr. Paul J. Fox) give the nitrogen balance sheet for the United States for 1917.

IMPORTED SUPPLIES	Tons of 2,000 lb.	Tons of Nitrogen
Chile saltpetre 95 per cent $NaNO_3$.	1,742,540	272,880
Ordinary saltpetre, potassium, nitrate	4,609	645
Ordinary saltpetre and gunpowder containing 75 per cent KNO_3..	1,500	210
Ammonium sulphate.....	8,135	1,725
Ammonium chloride.....	1,073	280
DOMESTIC SUPPLIES		
Coke oven ammonia—NH_3.....	113,760	93,625
Gas works ammonia—NH_3.....	12,500	10,288
Calcium cyanamid at 20 per cent nitrogen	12,800	10,534
NITROGEN EXPORTED	Tons of 2,000 lbs.	Tons of Nitrogen
Nitric acid, 15 per cent nitrogen...	486	73
Picric acid, 18 per cent nitrogen...	26,610	4,790
Dynamite, 12 per cent nitrogen	8,962	1,255
Gunpowder and smokeless powder, 13 per cent nitrogen.....	223,270	29,025
Ordinary saltpetre.....	875	123

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In addition to the above, there are also about 8800 tons represented nitrogen in the following items which are the figures for 1917.

	Value
Loaded cartridges	$42,000,000
Fuses	34,000,000
Shells and projectiles	74,000,000
All other	202,000,000
Total	253,000,000

It will be noticed that ammonium nitrate is not included in these figures, but I assume it would be about 50,000 tons for 1917.

In Great Britain the consumption of ammoniun nitrate is now probably 400,000 tons a year, and the production here will have to be at least as much. To make this, the theoretical proportion of ammonia required is about 85,000 tons and of nitric acid about 315,000 tons.

It will thus be seen that the coke oven plants in the country could supply all the ammonium nitrate required if they were equipped for the purpose. I do not suggest that existing ovens should be so used, because much of the gas is already allocated for various purposes but for new installations of coke ovens it would be worth while to consider the use of the surplus gas and of the ammonia in the way suggested.

The idea of using coke oven gas for generating electric power for general supply is gaining ground and there are a good many such installations in the North of England where electric energy has been sold at lower rates than appears to be general in the United States. When a company owns the power house as well as the coke ovens it can of course sell the surplus gas to itself at a very low rate or even at zero. A battery of coke ovens is a good paying proposition independent of any money received for surplus gas.

The company operating at Fairmont, West Virginia is completing a new power house at the mouth of a coal mine which will have two 10,000-kw. units, and the company has 12,000 kw. in gas engine units which will be worked with gas from coke ovens, so that by 1919 there will be 15,000 kw. to spare.

Until recently most coke oven ammonia was converted into sulphate, but owing to the war demand for nitrate, more and more of it is being made into aqua-ammonia of about 29 per cent strength. In some cases this is being transported many

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hundreds of miles prior to conversion into ammonium nitrate and since each ton of ammonia necessitates the transportation of about 2½ tons of water, the bearing of this, on the present railway congestion is at once apparent. Tank cars have to be used and they must return empty, so the freight on the actual ammonia carried is extremely high.

There are many coke ovens of the wasteful bee-hive type in operation, which do not recover by-products and the replacement of these by modern coke-ovens would be a great immediate economic gain and meet present war conditions. Coke ovens can be built quicker than large dams for water power and they have the great advantage of giving many products besides coke which are useful for war purposes.

In the present emergency coke ovens are of great value because they give coke for making steel, gas for power purposes, ammonia for nitrate manufacture, and toluol and benzol for explosives.

After the war ammonium nitrate will be in demand for fertilizer as well as for safety explosives and other purposes The high percentage of nitrogen which it contains viz., 35 per cent and the ease with which it can be converted into other compounds makes it especially useful for conveying nitrogen in the fixed form over considerable distances.

It is more profitable to make nitrate than sulphate, because, pound for pound, the nitrate contains nearly twice as much fixed nitrogen and the nitrogen commands a higher price per unit when in the form of ammonium nitrate.

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DISCUSSION ON "ELECTRIC POWER FOR NITROGEN FIXATION" (SCOTT), ATLANTIC CITY, N. J., JULY 27, 1918.

C. P. Steinmetz: I decidedly agree with Mr. Scott in the conviction that the final solution of the nitrate problem is to be found in the arc flame process or direct process. I have read numerous reports during the last fifteen years aiming to prove the contrary, but I have not been convinced, and I have always had the feeling that the chemical engineers who made these reports, while endeavoring to be thoroughly impartial, were so much more familiar with chemical methods and relatively less familiar with electrical phenomena, so that the conclusions derived by them have been influenced unintentionally by their familiarity with the one class of work and unfamiliarity with the other class of work.

I agree that the direct process requires more electric power than the cyanamid process, for instance, but at the same time there are other processes, like the ammonia process, which require no electric power, and the reason why the cyanamid process requires much less power is that of the various operations only a very small part, the first quarter, is carried out electrically. We may put the statement the other way and say that in the direct process nothing but electric power is required, and all the work is done by electric power, while in the other, the indirect processes, electric power plays only a secondary part, or no part at all.

The present situation, as we all realize, is peculiar, that we have to face an emergency. It is not a question whether the one process is better or more economical than the other one, but the question is, first, which process can be brought into operation on a larger scale in the quickest manner. I am not sufficiently familiar with all the data which are available, to judge about this, but I am willing to accept the judgment of those who have gone over the field and decided that the cyanamid process offers the best possibility of getting in operation on a large scale in the quickest possible time, but that in addition thereto extensive experimentation and development should be carried out which may lead to more efficient, and therefore final, methods, but I believe a very great mistake is being made by our nation by not considering in this proposed development, or practically not considering, the arc process.

We must realize that without a powerful nitrate industry we are defenselessly at the mercy of any power which controls the ocean, and that we must have a strong nitrate industry indigenous to this country, not merely for the time of this war, but forever after, and such a nitrate industry can subsist on a large enough scale to be useful in emergencies only if an extensive use is found in peace times, and that use must be in the fertilizer industry.

That means an extremely cheap method of production, and that means as far as power is concerned, it must be capable of

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operation with the cheapest power, that is, secondary power or off-peak power.

Of all the methods of nitrification the only one which is capable of efficient intermittent operation is the arc process, or direct process. The arc is equally as efficient in the second or milli-second after it is started as it is after continuous operation for days. That is not the case, however, with the cyanamid process, where even a very short time interruption means freezing of the carbide furnace and serious loss and interruption.

I saw some time ago a confidential report by a prominent engineer sent by one of our largest utility corporations to Norway to study the Norwegian process, to see whether it could be utilized in the use of off-peak power, and the report in its conclusion said that the Norwegian process,—as I may call the arc process as it was some years ago operated in the Norwegian plants,—if it is operated, not continuously, but half the time, then even if the power cost absolutely nothing, it would not be economical, because the interest on the investment would be greater than the value of the nitrate was at that time. Possibly it is different now, but I have no reason to doubt that the conclusions were correct. I do not know whether the process has been improved at the present time, but that, together with several other features, is a part of the general situation in regard to the Norwegian method.

We hear and have definite information that the Norwegian arc process nitrate plants are commercially successful and profitable, but we also hear that the conditions there are peculiar, such as cannot be found in our country, and the peculiarity is claimed to be the very low cost of power. That is not strictly true in all respects, because off-peak power here can be secured cheaper in many cases than the Norwegian power. The peculiar condition of the Norwegian plant, as it existed some years ago when these reports were made, was a limited power with very cheap capital, so that under those conditions the greatest economy required a development of the highest possible efficiency regardless of the cost of the investment. It is about the same condition which we meet in our largest public utility corporations, like Chicago and Detroit, which, as our President has pointed out, have reached the highest efficiency in electric production, exceeding 20 per cent from the chemical energy of coal to the busbar. An attempt to use apparatus of this high efficiency in an isolated station, supplying a hotel, or in a central station feeding a small city, would be hopelessly uneconomical, for the reason that, because of the size and the load factor the cost of the capital in such a small plant is much higher. We have to balance the efficiency which we can secure against the interest on the investment required to secure the efficiency. This in my opinion, is the characteristic feature of the Norwegian plant, which makes it impossible to transfer it without any change to American conditions, especially to conditions of

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off-peak power, where not only the higher costs of capital under American conditions—especially under the present conditions—but furthermore, the still higher cost of capital due to its intermittent use, makes a plant aiming at the highest efficiency, at the collection of the last traces of the nitrate, makes such a plant uneconomical. This is the feature which, in my opinion, has very largely been killing the arc process in this country, because no competent electrical engineer realizing this situation was fathering it, but it has rather been done by the attempt to transplant the Norwegian method here under circumstances to which it is not adopted. It means redesign and redevelopment of this method, to a very much lower cost of investment, even at the sacrifice of some efficiency. We can better afford to waste 25 per cent of the nitrate if thereby we can get the cost of investment down to a small fraction of what the Norwegian plant costs, and thereby make it economical for off-peak power.

That is the principal point I want to bring to your attention, and furthermore, there is another matter which I can only give from hearsay, that is, that the financial control of the Norwegian nitrate plants were secured some time ago by German industries, and then afterwards were sold, and since that time we have been given to understand, intentionally or unintentionally, that the new German processes are superior, and that there is nothing in the arc process, because they have given up their control of them.

Whether that is really so, or whether they realize that the conditions in Germany are different from ours with regard to the economy of the arc process—the almost complete absence of larger water powers, the great abundance of skilled chemists and skilled chemical labor bringing about conditions which are very much more in favor of chemical processes and against the arc processes. I leave it to you to think over which may have been the reason.

But there is one feature which has seriously mitigated against the arc process, and that is the following: Some time ago a very careful investigation was made by the famous German chemist Nernst, on the temperature equilibrium of oxygen, nitrogen and nitric oxide, and he showed that at the temperature of the carbon arc the equilibrium corresponds to about 4 per cent nitric oxide. That would mean an efficiency of conversion of 13 per cent, if all this 4 per cent could be saved, but we know at these temperatures dissociation is so rapid, that we can expect to save only a part of the nitric oxide from destruction, in dropping the temperature as rapidly as possible. That means the possible efficiency must be much lower than 13 per cent.

It is true that the temperature equilibrium at the carbon arc temperature corresponds to 4 per cent of nitric oxide, and very much lower percentages at lower temperature. It has been proven however that the nitrification in the arc is not a temperature effect, but is an electrical effect, and the proof of that you can find in the TRANSACTIONS of the A. I. E. E. in the discussion

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of a paper by my assistant Mr. J. L. R. Hayden, who made an extensive investigation some years back on nitrification, and has given the results in his discussion. This, however, seems to have been forgotten, since at that time the problem was of lesser importance than now. He showed that in low temperature low-voltage arc, nitric oxide can be produced far in excess of the concentration corresponding to the temperature equilibrium at the temperature of the arc, and therefore the effect cannot be a mere temperature effect. He furthermore showed that the relative efficiency of different temperature arcs does not correspond to the temperature. The carbon arc, which is the hottest, is very far from being the most efficient nitrifying arc. The iron arc and the copper arc each have about the same temperature, but not the same nitrifying efficiency, but rather they are on the two extremes of the series investigated. The copper arc is the least efficient, and the iron arc is the most efficient, showing thereby that the effect of the arc in producing the combination of nitrogen and oxygen is not a temperature effect, but a direct electrical effect.

The reason why I have confidence in the arc process is not only due to the absence of the need of any other raw material but air, which makes it specially suitable for use anywhere where power is available—as pointed out by Mr. Scott, at the coke oven for instance—it is not only the possibility of the use of intermittent power, which holds out the hope of developing a powerful nitrate industry in peace times, but it is also the vast possibility of increasing the efficiency of nitric acid production by the arc process, over that reached in the Norwegian plants. Under favorable conditions, I understand, the present efficiency is from 60 to 80 grams per kilowatt-hour. The theoretical efficiency of nitric oxide production is 2500 grams of NO_3H per kilowatt-hour, so that the present best results are an efficiency of 3 per cent.

You can realize, therefore, the vast possibilities there are in increasing the efficiency, and if at an efficiency of only 3 per cent the process has become commercial under favorable conditions, we can see that there are possibilities in this method which will bear considerable efforts being made in studying the development further, and carrying on the development, which I am sorry to say has practically been neglected since the early pioneer days of Charles Bradley, the electrical engineer, who started a plant at Niagara Falls, but failed, due to the cost of investment in his particular method being out of proportion to the returns.

I want to conclude by repeating that my study of the problem—and I have been closely interested in it, not only theoretically, but I have done considerable experimental work during the past twenty years or more,—has led me to the conclusion that while the cyanamid process may have immediate use by being the quickest available, that the process which I consider

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as the most promising, in the final solution of the problem, is the direct or arc process.

Wm. A. Del Mar (communicated after adjournment): The arc process for the fixation of nitrogen is, in my opinion, the only use to which the off-peak capacity of large power stations can be put, as it is the only electrotechnical process that can thrive under the three characteristic conditions attending the use of such power, namely, absorption of large amounts of power, use of alternating current at the generated voltage and intermittent supply.

The cost of transforming apparatus bars all direct-current electrochemical processes such as electrolytic metal refining, alkali productions, etc. and many thermal processes. Other thermal processes are debarred because they cannot be operated intermittently. Any processes which are exceptional in these respects are too unimportant to absorb the power available from large stations.

Hence if off-peak capacity is ever to be utilized on a large scale, the only promising field for research is the arc-process for nitrogen fixation.

S. Barfoed (communicated after adjournment): As nitric acid is the end product desired it is evident that with the increasing demand upon the nation's man power naturally that system which can produce nitric acid with a minimum of labor is the most logical one to use, other things considered equal. The raw materials needed for the direct method being air and water lend themselves to transport in pipes requiring no attention. Also, the final product, nitric acid, is to be handled in the same manner by means of the new acid proof ferro-alloys. Consequently labor is not needed in this particular part of the cycle of operations.

In the operation of any arc furnaces used by the direct method a row of furnaces may be attended by but two men required for recording and renewing electrodes at intervals.

During the last two and one-half years the writer has been engaged upon the development and operation of a 1000-kv-a. arc furnace using metallic oxide electrodes. The results obtained in this furnace and the simplicity of its operation, also the ease with which it may be kept in repair justifies me in saying that the direct method for making nitric acid as compared with the indirect method has the very advantages which Mr. Scott mentions.

The simple way of getting the end product, nitric acid, with the direct method as the first and only operation must surely be evident to the layman even if he should be a lawyer-politician.

In discussing the proper system to use in nitrogen fixation it must be kept in mind that all the various processes have their proper place of usefulness, all depending upon the raw materials available and the end product it is desired to produce and have the market absorb. The choice of the system is practically

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decided only by the geographical location of the plant. In places where coal has been obtainable at reasonable prices and where men are rather plentiful but where water power is comparatively expensive, the end product may to advantage be cyanamid. In the greater part of the West coal is not obtainable at prices that would make the indirect method of making acid or even the cyanamid process possible under any circumstances.

The scheme outlined for combining a battery of coke ovens with ammonia recovery plant with an arc furnace plant as outlined by Mr. Scott is entirely feasible and is one to which the writer has given considerable thought, but the conditions for its use during peace times must be favorable. Even in certain sections of the West in spite of the existing water powers such a scheme can be worked out successfully for producing ammonium nitrate. Where districts are equally well off as to coal and water the several methods of producing fixed nitrogen can very well exist side by side and not exclude one another, keeping in mind the suitability of each process with a view as to raw material and the end product.

I endorse the formula "Do it Electrically" when nitric acid is to be produced.

I do not believe that at the present time there are many power stations where surplus power may be had. Also, I believe that the load factor of the majority of stations is so high that the time during which off-peak power is available would be too short for making the operation of arc furnaces satisfactory. In the West there is rather a scarcity of power and new developments by existing companies are not likely to be made in time to be of any use during the war, and that is largely because the power it is thought to develop cheaply is to be of great magnitude. There are however, throughout the West numerous small water power sites of 10,000 h. p. or thereabouts which can very cheaply be developed and completed in a short time and which would be entirely suitable for a nitric acid factory. Statements are often made that such water powers cannot be developed under a certain figure. Such statements of course do not mean anything unless they are qualified as to just what is meant to be included in a development. As generally spoken of I believe what is meant is that the development should include step-up transformers, high-tension switch gear, transmission lines, lowering transformers, substation equipment and distributing lines, etc. For such systems the figures as often given may be correct, but it would be a mistake to draw any conclusions from them when it comes to making a hydroelectric development for use with arc furnaces producing nitric acid.

It is often found that the nitric acid factory can be located in close proximity to the power station and it will be at once evident the great saving that can be made by so doing in the cost of the plant. The development would include the hydraulic works and the generating equipment which generates power directly at the voltage required by the furnaces.

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Looking at the matter in this light I have made several investigations where water power of around 10,000 h. p. can be developed for a figure around $60.00 per h. p. and lower. Where this can be done the power cost will be a minor item compared to the fixed charges on the acid factory.

F. W. Sperr, Jr. (communicated after adjournment): A very important phase of the plan which Mr. Kilburn Scott has developed is that it succeeds in reconciling the interests of two great industries—the nitrogen fixation industry and the by-product coke industry. It cannot be denied that, owing largely to the emphasis that has been placed upon fixation by the cyanamid process, there have hitherto been many elements of mutual antagonism in these two industries.

The by-product coke oven must be considered as an essential factor in our national security and military success. It is the greatest and most dependable source of tri-nitro toluol, the most important high explosive, and must be relied upon for our supply of benzol, creosote, naphthalene, anthracene and a host of other products of military value.

Many people still have the old-fashioned idea that beehive coke works better in a blast furnace than by-product coke. As a matter of fact records from a large number of blast furnaces show a saving of over 200 pounds of coke per ton of pig iron due to the substitution of by-product coke for the beehive product. The significance of this in connection with our fuel shortage and the importance of the steel industry is easy to recognize.

In the establishment of by-product coke ovens, we are still sadly behind both economic and military requirements and every agency should be interested in encouraging the construction of new plants as rapidly as possible.

It is to be regretted that the fear of loss through destructive competition with subsidized ammonia produced by the cyanamid process has operated in certain cases as a discouraging factor. In normal times, ammonia is the principal source of revenue, besides coke, to the by-product coke plant. Ammonia produced by the cyanamid process cannot compete with by-product coke oven ammonia except with the aid of a subsidy. Large quantities of subsidized ammonia or a cheap competing fertilizer like cyanamid, thrown upon the agricultural market might well induce hesitation in the building of by-product coke ovens.

On the other hand, Mr. Scott's proposition to combine the manufacture of nitric acid from the atmosphere with the operation of a by-product coke plant, utilizing the surplus gas from the latter to furnish the requisite power, and the ammonia to combine with the nitric acid forming ammonium nitrate, appears to be as economically sound in time of peace as it is desirable in time of war. From the standpoint of the by-product coke oven owner, it looks entirely feasible and encouraging. Under normal conditions, the nitric acid might be sold separately; or it might still be used together with the ammonia as ammonium

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nitrate. In this way the expense of purchasing sulphuric acid to make ammonium sulphate would be avoided and a concentrated fertilizer more valuable than ammonium sulphate would be produced. It is to be hoped that arrangements can be made to build a plant for a large scale trial of this very promising proposition.

E. K. Scott: Members who have to deal with electric powerhouse problems, are naturally interested in improvement of load factor. I know of no load which is more suitable for filling up valleys of a load curve than the arc method of fixing atmospheric nitrogen, for the furnaces can be started and stopped at any time without detriment, as there is no fixed material to freeze when current is off.

A good situation for an air nitrate factory would be on the same plot of land as the power house and the two might well be under the same directorate.

It has often been said that the manufacture of air nitrates is dependent on a very cheap source of power and the implication is made that it must be water power and on a very large scale. I wish to combat this for I consider it feasible to successfully operate moderate sized plants with off peak power from existing power houses.

It is a mistake to unduly emphasize the question of power for there are other equally important factors, *e. g.* transportation of raw materials and finished products. Nitric acid is required in very large quantities for explosives, for dyes, for celluloid and other industries and being difficult to carry, it is best made near to the place where it is to be used.

The nitric acid made in Norway, cannot be exported owing to difficulty of carriage and even when made into nitrate of lime and nitrate of ammonia the cost of carriage to a market abroad is considerable. Take ammonia for example which for many years has been made in Norway with ammonia shipped from England. It has been estimated that the charges for carriage, insurance, transhipment, etc., of the aqua ammonia to Norway and of the finished nitrate of ammonia to England came to more per ton of product than the cost of the electric energy with which the ton was made.

Other things being equal it is often better to be near to a steady market and pay a fair price for power than to go to a considerable distance and get cheaper power. Steam power houses have often an advantage over hydroelectric plants because the latter are usually in inconvenient places.

I fully appreciate the development of hydroelectric power, but think that the building of a coke oven plant with electric power house and nitrate factory is more to the National advantage than if the same money were spent on hydraulic works, etc. The coke oven proposition has certainly wider usefulness.

It may be of some interest to mention that electric energy has been supplied for many years to certain electrochemical

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factories on the North East Coast in England at prices which compare favorably with Niagara. A considerable amount of power developed in that area is from coke ovens and blast furnace gas and by linking the power houses together it has been found possible to bank some boilers in the large coal fed power houses in Newcastle over the week end and obtain supply from the coke oven and waste heat power houses of Durham and North Yorkshire which of course must be kept going at all hours.

Dr. Steinmetz referred to the investment on an air nitrate plant. A large part of this goes in the absorption towers and the building required to house same. Although chemists have always been accustomed to absorb gases in such towers I think a smaller and less costly method can be developed on engineering lines.

I purposely did not enter into questions regarding reactions, etc., that go on in nitrogen fixation furnaces, but I agree with Dr. Steinmetz that the action is not a purely thermal one. The fact that nitric acid may be formed on the windings of extra high-tension machines is an indication.

Suggestions of limitations in electrical methods are usually traceable to those who are interested in old fashioned chemical methods, as for example, that of getting nitric acid from Chili nitrate, but an audience of electrical engineers will appreciate that when electricity once gets a foothold in any industry it holds on.

Some of us remember the days when electric lighting was struggling against firmly entrenched gas interests. The electric street car was a long time before it finally ousted the horse car, especially in England, whilst the introduction of electric power in mines is still opposed by advocates of compressed air and so on.

Electric engineering is now developing rapidly in the largest fields of all, namely, electrochemistry and electrometallurgy. Very large amounts of energy are being called for which means greater power development and wider opportunities for electrical engineers and manufacturers of electrical equipment. Members will no doubt hear from time to time of other methods of making nitrates which do not require electric energy or require less than the arc process. They need have no fear that electrical methods will not outlast all others and I personally believe the arc process is best of all.

The name of Charles S. Bradley has been mentioned. He was a pioneer in this line of work before the Norwegians or Germans, and I think the fact is worthy of commemoration.

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AMERICA'S ENERGY SUPPLY

BY CHARLES P. STEINMETZ

ABSTRACT OF PAPER

The gist of the paper is to demonstrate that the economical utilization of the country's energy supply requires generating electric power wherever hydraulic or fuel energy is available, and *collecting the power electrically, just as we distribute it electrically.*

In the first section a short review of the country's energy supply in fuel and water power is given, and it is shown that the total potential hydraulic energy of the country is about equal to the total utilized fuel energy.

In the second section it is shown that the modern synchronous station is necessary for large hydraulic powers, but the solution of the problem of the economic development of the far more numerous smaller waterpowers is the adoption of the induction generator. However, the simplicity of the induction generator station results from the relegation of all the functions of excitation, regulation and control to the main synchronous station. The economic advantage of the induction generator station is, that its simplicity permits elimination of most of the hydraulic development by using, instead of one large synchronous station, a number of induction generator stations and collecting their power electrically.

The third section considers the characteristics of the induction generator and the induction-generator station, and its method of operation, and discusses the condition of "dropping out of step of the induction generator" and its avoidance.

In the appendix the corresponding problem is pointed out with reference to fuel power, showing that many millions of kilowatts of potential power are wasted by burning fuel and thereby degrading its energy, that could be recovered by interposing simple steam turbine induction generators between the boiler and the steam heating systems, and collecting their power electrically. It is shown that the value of the recovered power would be an appreciable part of that of the fuel, and that organized and controlled by the central stations, this fuel power collection would improve the station load factor, give the advantages of the isolated plant without its disadvantages, and produce a saving of many millions of tons of coal.

I. The Available Sources of Energy

A. COAL

THE only two sources of energy, which are so plentiful as to come into consideration in supplying our modern industrial civilization, are coal, including oil, natural gas, etc., and water power.

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While it would be difficult to estimate the coal consumption directly, it is given fairly closely by the coal production, at least during the last decades, where wood as fuel had become negligible and export and import, besides more or less balancing each other, were small compared with the production. Coal has been mined since 1822, and in Fig. 1 is recorded the coal production of the United States, from the governmental reports. The annual production is marked by circles, the decennial average marked by crosses for every five years. Table I gives the decennial averages, in millions of tons per year.

TABLE I

AVERAGE COAL PRODUCTION OF THE UNITED STATES

(decennial average)

Year	Million tons per year	Per cent increase per year
1825	0.11	....
30	0.32	22.4
35	0.83	19.7
40	1.92	17.0
45	4.00	14.5
50	7.46	10.45
55	10.8	8.35
60	16.6	8.72
65	25.9	9.22
70	40.2	8.58
75	56.8	7.42
80	82.2	7.95
85	122	6.80
90	160	5.40
95	206	5.75
1900	281	6.96
05	404	6.60
10	532	....

In Fig. 1 the logarithms of the coal production in tons are used as ordinates. With this scale, a straight line means a constant proportional increase, that is, the same percentage increase per year, and in the third column of Table I are given the average percentage increase of coal production per year.

This Fig. 1 is extremely interesting by showing the great irregularity of production from year to year, and at the same time a very great regularity over a long period of time. Since 1870 the average production may be represented by a straight line, the values lying irregularly above and below the line, which

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represents an annual increase of 6.35 per cent and thus represents the average coal production[1] C by the equation

$$C = 45.3 \times 10^{0.0267\,(y-1870)} \text{ million tons}$$

or

$$\log C = 0.0267\,(y-1870) + 7.656$$

where y = year.

Before this time, from 1846 to 1884, the coal production could be represented by

$$C = 7.26 \times 10^{.0365\,(y-1850)} \text{ million tons}$$

or

$$\log C = 0.0365\,(y-1850) + 6.861$$

representing an average annual increase of 8.78 per cent.

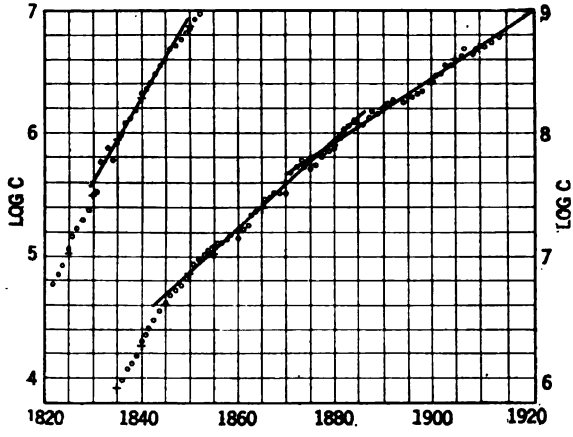

FIG. 1—COAL PRODUCTION OF THE UNITED STATES

It is startling to note how inappreciable, on the rising curve of coal production, is the effect of the most catastrophic political and industrial convulsions, such as the Civil War and the Industrial panic of the early 90's; they are indistinguishable from the constantly recurring annual fluctuations. It means, that the curve is the result of economic laws, which are laws of nature.

Extrapolating from the curve of Fig. 1, which is permissible, due to its regularity, gives 867 million tons as this year's coal consumption. As it is difficult to get a conception of such enormous amounts, I may be allowed to illustrate it. One of the great wonders of the world is the Chinese Wall, running

1. Soft coal and anthracite; and including oil reduced to coal by its fuel value.

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across the country for hundreds of miles, by means of which China unsuccessfully tried to protect its northern frontier against invasion. Using the coal produced in one year as building material, we could with it build a wall like the Chinese Wall, all around the United States, following the Canadian and Mexican frontier, the Atlantic, Gulf and Pacific Coast, and with the chemical energy contained in the next year's coal production, we could lift this entire wall up into space, 200 miles high. Or, with the coal produced in one year used as building material, we could build 400 pyramids, larger than the largest pyramid of Egypt.

It is interesting to note that 100 thousand tons of coal were produced in the United States in 1825; one million tons in 1836; 10 million tons in 1852 and 100 million tons in 1882. The production will reach about 1000 million tons in 1920, and, if it continues to increase at the same rate, it would reach 10,000 million tons in 1958.

Estimating the chemical energy of the average coal as a little above 7000 cal., *the chemical energy of one ton of coal equals approximately the electrical energy of one kilowatt year* (*24 hour service*). That is, one ton of coal is approximately equal in potential energy to one kilowatt-year.

Thus the annual consumption of 867 millions of tons of coal represents, in energy, 867 million kilowatt-years.

However, as the average efficiency of conversion of the chemical energy of fuel into electrical energy is probably about 10 per cent, the coal production, converted into electrical energy, would give about 87 million kilowatts.

Assuming however, that only one half of the coal is used for power, at 10 per cent efficiency, the other half as fuel, for metallurgical work etc., at efficiencies varying from 10 per cent to 80 per cent, with an average efficiency of 40 per cent, then we get 217 million kilowatts (24 hour service) as the total utilized energy of our present annual coal production of 867 million tons.

B. The Potential Water Powers of the United States

Without considering the present limitation in the development of water powers, which permits the use of only the largest and most concentrated powers, we may try to get a conception of the total amount of hydraulic energy which exists in our country, irrespective of whether means have yet been developed

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or ever will be developed for its complete utilization. We therefore proceed to estimate the energy of the total rain fall.

Superimposing the map of rain fall in the United States, upon the map of elevation, we divide the entire territory into sections by rain fall and elevation. This is done in Table II, for the part of our continent between 30 and 50 degrees northern latitude.

TABLE II

TOTAL POTENTIAL WATER POWER OF UNITED STATES

In. rain fall	Ft. elevation	Area $m^2 10^{12} \times$	Avg. elevation m.	Avg. rainfall cm.	Kg-m. per m^2; $10^3 \times$	Kg-m. total $10^{15} \times$
>10	>5000	0.54	2100	12.5	263	142
	1000–5000	0.29	900		112	32.5
10–20	>5000	1.18	2100	37.5	787	930
	1000–5000	1.96	900		338	660
20–30	1000–5000	0.32	900	62.5	563	183
	100–1000	0.97	150		94	91
30–40	1000–5000	0.35	900	87.5	786	275
	100–1000	1.40	150		131	184
40–60	1000–5000	0.27	900	125	1130	305
	100–1000	1.03	150		188	194
						$\Sigma = 2996$
						3000

As obviously only the general magnitude of the energy value is of interest, I have made only few sub-divisions: five of rain fall and four of elevation, as recorded in columns 1 and 2 of Table II[2]. The third column gives the area of each section, in millions of square kilometers, the fourth column the estimated average elevation, in meters, and the fifth column the average rain fall, in centimeters. The sixth column gives the energy, in kilogram-meters per square meter of area, and the last column the total energy of the section, in kilogram-meters, which would be represented by the rain fall, if the total hydraulic energy of every drop of rain were counted, from the elevation where it fell, down to sea level.

As seen from Table II, the total rain fall of the North American Continent between 30 deg. and 50 deg. latitude represents 3000×10^{15} kg-m. This equals 950 million kilowatt years (24 hour service). That is, the total potential water power of the United States, or the hydraulic energy of the total

1. The lowest elevation, < 100 ft., is not included, as having little potential energy.

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rain fall, from the elevation where it fell, down to sea level, gives about 1000 million kilowatts.

However, this is not available, as it would leave no water for agriculture; and even if the entire country were one hydraulic development, there would be losses by seepage and evaporation.

An approximate estimate of the maximum potential power of the rain fall, after a minimum allowance for agriculture and for losses is made in Table III, allowing 12.5 cm. rain fall for wastage, and 37.5 and 25 cm. respectively for agriculture where such is feasible.

TABLE III

AVAILABLE POTENTIAL WATER POWER OF THE UNITED STATES

Avg. rainfall cm.	Avg. elevation m.	Area m^2 10^{12} ×	Wastage cm.	Agriculture, cm.	Available rainfall cm.	Kg. m. per m^2 10^3 ×	Kg-m. total 10^{15} ×
12.5	2100	0.54	12.5	....	....	....	....
....	900	0.29	12.5	....	....	....	....
37.5	2100	0.39	12.5	25	....	....	....
....	2100	0.79	12.5	....	25	525	415
....	900	0.98	12.5	25	....	....	....
....	900	0.98	12.5	....	25	225	220
62.5	900	0.21	12.5	37.5	12.5	112	23
....	900	0.11	12.5	....	50	450	50
....	150	0.97	12.5	37.5	12.5	19	18
87.5	900	0.35	12.5	37.5	37.5	337	118
....	150	1.40	12.5	37.5	37.5	56	78
125	900	0.27	12.5	27.5	75	674	182
....	150	1.03	12.5	37.5	75	112	116
							Σ = 1220

This gives about 1200 × 10^{15} kg-m. as the total available potential energy, which is equal to 380 million kilowatts (24 hour service). Assuming now an efficiency of 60 per cent from the stream to the distribution center, gives 230 million kilowatts (24 hour service) as the maximum possible hydroelectric power, which could be produced, if every river, stream, brook or little creek throughout its entire length, from the Spring to the ocean, and during all seasons, including all the waters of the freshets, were used and could be used. It would mean that there would be no more running water in the country, but stagnant pools connected by pipe lines to turbines exhausting into the next lower pool. Obviously, we could never hope to develop more than a part of this power.

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C. Discussion

It is interesting to note that the maximum possible hydraulic energy of 230 million kilowatts, is little more than the total energy which we now produce from coal, and is about equal to the present total energy consumption of the country, including all forms of energy.

This was rather startling to me. It means that the hope that when coal once begins to fail we may use the water powers of the country as the source of energy, is and must remain a dream, because if all the potential water powers of the country were now developed, and every rain drop used, it would not supply our present energy demand.

Thus hydraulic energy may and should supplement that of coal, but can never entirely replace it as a source of energy. This probably is the strongest argument for efforts to increase the efficiency of our methods of using coal.

A source of energy which is practically unlimited, if it could only be used, is solar radiation. The solar radiation at the earth's surface is estimated at 1.4 cal. per cm.^{2} per min. Assuming 50 per cent cloudiness, this would give an average throughout the year (24 hours per day), of about 0.14 cal. per cm.^{2} horizontal surface per min., and on the total area considered in the preceding table, of 8.3 million square kilometers of North America between 30 and 50 latitude, a total of approximately 800,000 million kilowatts (24 hour service), or a thousand times as much as the total chemical energy of our coal consumption; 800 times as much as the potential energy of the total rainfall.

Considering that the potential energy of the rainfall from surface level to sea level, is a small part of the potential energy spent by solar radiation in raising the rain to the clouds, and that the latter is a small part of the total solar radiation, this is reasonable.

Considering only the 2.7 million square kilometers of Table III, which are assumed as unsuited for agriculture, and assuming that in some future time, and by inventions not yet made, half of the solar radiation could be collected, this would give an energy production of 130,000 million kilowatts.

Thus, even if only one-tenth of this could be realized, or 13,000 million kilowatts, it would be many times larger than all the potential energy of coal and water. Here then would be the great source of energy for the future.

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II. Hydroelectric Station

A. The Modern Synchronous Generator Station

In developing the country's water powers, up to the present time only those of greatest energy concentration have been considered; that is, those where a large volume and a considerable head of water was available within a short distance.

This led to the present type of hydroelectric generating station, as best solving the problem. The equipment of such a station comprises the following apparatus:

Three-phase synchronous direct-connected generators.

Hydraulic turbines of the highest possible efficiency.

Hydraulic turbine speed governing mechanism.

An exciter plant comprising either exciters directly connected to the generators, or several separate exciter machines, connected to separate turbines.

Exciter bus bars.

Voltmeter, and ammeters in exciters and in alternator field circuits.

Field rheostats of the alternators.

Low-tension busbars, either in duplicate, or with transfer or synchronizing bus.

Circuit breakers between generators and busbars, usually non-automatic.

Circuit breakers between transformers and busbars, usually automatic, with time limit.

Voltmeters and potential transformers at the generators.

Synchronoscopes or other synchronizing devices.

Ammeters and current transformers at the generators.

Voltmeter and potential transformer at the busbars.

Ammeters and current transformers at the step-up transformers.

Totaling ammeter for the station output.

Integrating wattmeter.

Relays, interlocking devices etc., etc.

Step-up transformers.

High-tension busbars, possibly in duplicate.

High-tension circuit breakers between transformers and high-tension busbars.

High-tension circuit breakers between high-tension busbars and lines.

Lightning arresters in the transmission lines, with inductances etc.

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Ground detectors, arcing-ground or short-circuit suppressors, voltage indicators etc.

Automatic recording devices (multi-recorder), rarely used though very desirable.

Due to the vast amount of energy controlled by modern stations, the auxiliary and controlling devices in these stations have become so numerous as to make the station a very complex structure, requiring high operating skill and involving high cost of installation. At the same time, not only are all these devices necessary for the safe operation of the station, but we must expect that with the further increase of capacity of our electric systems, additional devices will become necessary for safe and reliable operation. One such device I have already mentioned—automatic recording apparatus, such as the multi-recorder.

With this type of station, it is obviously impossible, in most cases, to develop water powers of small and moderate size. A generating station of a thousand horse power will rarely, and one of a hundred horse power will hardly ever be economical.

On the other hand, a hundred horse power motor installation is a good economical proposition, and the average size of all the motor installations is probably materially below one hundred horse power.

Looking over Tables II and III, especially the latter, in the preceding section, it is startling to see how large a part of the potential water power of the country is represented by comparatively small areas of high elevation, in spite of the relatively low rainfall of these areas. As most of these areas are at considerable distance from the ocean, most of the streams are small in volume. That is, it is the many thousands of small mountain streams and creeks, of relatively small volume of flow, but high gradients, affording fair heads, which apparently make up the bulk of the country's potential water power.

Only a small part of the country's hydraulic energy is found so concentrated locally as to make its development economically feasible with the present type of generating station.

Therefore, some different, and very much simpler type of generating station must be evolved, before we can attempt to develop economically these many thousands of small hydraulic powers, to collect the power of the mountain streams and creeks.

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B. Simplification of Hydroelectric Station

In the following in discussing the simplification of the hydroelectric station to adapt it to the utilization of smaller powers, we limit ourselves to the case where the smaller hydraulic stations feed into a system containing some large hydraulic or steam turbine stations, to which the control of the system may be relegated.

1. We may eliminate the low-tension bus bars, with generator circuit breakers and transformer low-tension circuit breakers and connect each generator directly to its corresponding transformer making one unit of generator and transformer, and do the switching on high-tension busbars, and locating high-tension busbars and circuit breakers outdoors. While it is dangerous to transformers to switch on the high-tension side, due to the possibility of cumulative oscillations, this danger is reduced by the permanent connection of the transformer with the generator circuit, and is less with the smaller units used in small power stations, and thus permissible in this case.

However, the simplification resulting therefrom is not so great, as ammeters, voltmeters and synchronizing devices with their transformers are still retained on the low-tension circuits.

2. As it is not economical to operate at partial load, proper operation of a hydraulic station on a general system is, to operate as many units fully loaded as there is water available, and increase or reduce the number of units (of turbine, generator and transformer, permanently joined together), with the changing amount of available water, thus using all the available energy of the water power.

In this case, the turbine governors, with their more or less complex hydraulic machinery, may be omitted. If then the generators are suddenly shut down by a short circuit which opens the circuit breakers, the turbines will race and run up to their free running speed, until the gates are shut by hand. However, generators, and turbines must stand this, as even with the use of governors, the turbines may momentarily run up to their free speed in case of a sudden opening of the load, before the governors can cut off the water. Where this is not desirable some simple excess speed cut-off may be used.

3. When dropping the governing of the turbines, and running continuously at full load, the question may be raised whether generator ammeters are necessary, as the load is constant, and is all the power the water can give, and it might appear, that am-

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meters with their current transformers, etc. could be omitted. However, with synchronous generators, the current depends not only on the load, but also on the power factor of the load and with excessively low power factor due to wrong excitation, the generators may be overheated by excess current, while the power load is well within their capacity. Thus ammeters are necessary with synchronous generators. As soon, however, as we drop the use of synchronous generators, and adopt induction generators, the ammeters with their current transformers may be omitted, since the current and its power factor is definitely fixed by the load. At the same time, synchronizing devices become unnecessary, together with potential transformers, generator voltmeters, etc. A station voltmeter may be retained for general information, but it not necessary either, as the voltage and frequency of the induction-generator station are fixed by the controlling synchronous main station of the system.

4. With the adoption of the induction generator, the entire exciter plant is eliminated, as the induction generator is excited by lagging currents received from synchronous machines, transmission lines and cables existing in the system. This avoids the use of exciter machines, exciter busses, ammeters, voltmeters, alternator field rheostats, etc., in short, most of the auxiliaries of the present synchronous station become unnecessary.

The solution of the problem of the economic development of smaller water powers is the adoption of the induction generator.

Stripped of all unnecessary equipment, the smaller hydroelectric station thus would comprise:

Hydraulic turbines of simplest form, continuously operating at full load, without governors.

Low-voltage induction generators direct connected to the turbines.

Step-up transformers direct connected to the induction generators.

High-tension circuit breakers connecting the step-up transformers to the transmission line. In smaller stations, even these may be dispensed with and replaced by disconnecting switches and fuses.

Lightning arresters on the transmission line where the climatic or topographical location makes such necessary.

A station voltmeter, a totalling ammeter or integrating wattmeter and a frequency indicator may be added for the informa-

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tion of the station attendant, but are not necessary, as voltage, current, output and frequency are not controlled from the induction generator station, but from the main station, or determined by the available water supply.

It is interesting to compare this induction generator station lay-out with that of the modern synchronous station given above. However, it must not be forgotten that *the simplicity of the induction generator station results from the relegation of all the functions of excitation, regulation and control, to the main synchronous stations* of the system, and the induction generator stations thus are feasible only as adjuncts to at least one large synchronous station, hydraulic or steam turbine, in the system, but can never replace the present synchronous generator stations in their present field of application.

C. Automatic Generating Stations

With the enormous simplification resulting from the use of the induction generator, it appears entirely feasible to make smaller hydroelectric generating stations entirely automatic, operating without attendance beyond occasional—weekly or daily—inspection.

Such an automatic generating station would comprise a turbine with low-voltage induction generator, housed under a shed, and a step-up transformer, outdoors, connecting into the transmission line with time fuses and disconnecting switches.

It is true that in the big synchronous generating stations of thousands of kilowatts, the cost of the auxiliaries, as exciter plant, regulating and controlling devices, etc., is only a small part of the total station cost, and little would therefore be saved by the use of induction generators. No induction generators would, however, be used for such stations. But the cost of auxiliaries and controlling devices, and the cost of the required skilled attendance, decreases far less with decreasing station size than that of the generators—whether synchronous or induction —or in other words, with decreasing size of the station, *per kilowatt output*, the cost of auxiliaries and controlling devices and of attendance increases at a far greater rate than that of the generators, and very soon makes the synchronous station of the present type uneconomical.

It is also true that in the big modern hydraulic power systems, the cost of the generating station usually is a small part of the cost of the hydraulic development. Therefore any saving in

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the cost of the generating station would be of little influence in determining, whether the hydraulic development would be economical. With decreasing size of the water power the cost of the hydraulic development *per kilowatt output* usually increases so rapidly as very soon to make the development of the water power uneconomical, no matter how simple and cheap the station is.

However, the value of the induction generator is not so much in the reduction of the cost of the generating station, as in the reduction of the cost of the hydraulic development, by making it possible to apply to the electric generator the same principle, which has made the electric motor economically so successful: *Collect the power electrically, just as we distribute it electrically.*

We do not, as in the days of the steam engine, convert the electric power into mechanical power at one place, by one big motor, and distribute the power mechanically, by belts and shafts, but we distribute the power electrically, by wires, and convert the electric power to mechanical power, wherever mechanical power is needed, by individual motors throughout mill and factory.

In the same way we must convert the hydraulic, that is, mechanical power into electrical power by individual generators located along the streams or water courses within the territory, wherever power is available, and then collect this power electrically, by medium-voltage collecting lines and high-voltage transmission lines, and so eliminate most of the cost of the hydraulic development, to solve the problem of the economical utilization of the country's water powers. If we attempt to collect the power mechanically, that is, by a hydraulic development gathering the waters of all the streams and creeks of a territory together into one big station, and there convert it into electric power, the cost of the hydraulic development makes it economically hopeless except under unusually favorable conditions, where a very large amount of power is available within a limited territory, or where nature has done the work for us in gathering considerable power at a waterfall, etc.

It is the old problem, and the old solution: If you want to *do it economically, do it electrically.*

Naturally then, we would use induction generators in these small individual stations just as we use induction motors in individual motor installations; but where large power is available, there is the field of the synchronous generator, and the induction generator is undesirable, just as the synchronous motor

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is preferable where large power is required—unless the synchronous motor is excluded by conditions of starting torque, etc.

At first, and for some time to come, we would not consider going to anywhere near as small sizes of induction generators, as we do in induction motors. However, there are undoubtedly many millions of kilowatts available in water powers throughout the country, which can be collected by induction-generator stations from 50 h.p. upwards, and that at fair heads, requiring no abnormal machine design (no very low speed).

Consider an instance—a New England mill river with a descent, in its upper course, of about 1100 ft. (335 m.) within five miles (eight km.), of varying gradient. At three places, where the gradient is steepest, by a few hundred feet of cast iron pipe and a small dam of 20 to 30 ft. (6 to 9 m.) length and a few feet height—just enough to cover the pipe intake—an average head of 150 ft. (45 m.) can be secured, giving an average of 75 h.p. each, or a total of 225 h.p. or 170 kw. This would use somewhat less than half the total potential power. The development of the other half, requiring greater length of pipe line, or involving lower heads, would be left to meet future demands for additional power.

The installation of an electric system of 170 kw. would hardly be worth while, but there are numerous other creeks throughout the territory from which to collect power, and within a few miles passes a high potential transmission line, coming from a big synchronous station, into which the power collecting lines coming from the induction generator stations would be tied and from which they would be controlled.

Thus, the large modern synchronous station has its field, and is about as perfect as we know how to build for large concentrated powers; but beyond this, there is a vast field, and therefore *an economic necessity of the development of a different type of hydraulic generating station to collect the scattered water powers* of the country; and that is the *induction generator station*, to which I wish to draw the attention.

I must caution, however, not to mistake small power and low head power. There are on the lower courses of our streams some hydraulic powers, which are relatively small due to their low heads, and which can not be economically developed by the synchronous generator, due to the low head and correspondingly low speed. The designing characteristics of the induction generator, with regard to low-speed machines, are no better—if

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anything rather worse—than those of the synchronous generator, and the problem of the economical utilization of the low-head water power still requires solution. It is not solved by the induction generator; the latter's characteristic is simplicity of the station, giving the possibility of numerous small automatic generating stations.

III. Induction Generator Station

A. Characteristics of Induction Generator

An induction motor at no load runs at, or rather very close to synchronism. If it is driven above synchronism by mechanical power, current and power again increase, but the electric power is outflowing, and the induction machine consumes mechanical power, and generates electrical power, as an induction generator.

The maximum electrical power, which an induction machine can generate as an induction generator is materially larger than the maximum mechanical power, which the same machine, at the same terminal voltage, can produce as an induction motor.

Resolving the current of the induction machine into an energy component and a wattless or reactive component, the energy current is inflowing, representing consumption of electric power (which is converted to mechanical power) below synchronism. It becomes zero near synchronism, and above synchronism the energy current is in the reverse direction, or outflowing, supplying electric power to the system (which is produced from the mechanical power input into the machine), and the induction machine then is a generator.

The wattless or reactive component is a minimum at synchronism, and increases with the slip from synchronism, and is in the same direction, whether the slip is below synchronism, as a motor, or above synchronism, as a generator. That is, the induction machine always consumes a lagging current (representing the exciting current and the reactance voltages), or, what amounts to the same, produces a leading current. The latter way of putting it is frequently used with induction generators, by saying that the current produced by the induction generator is leading, while the current consumed by the induction motor is lagging. Instead of saying however, that the reactive component of the current generated by the induction generator is leading, we may say, and this makes it often more intelligible, that the induction generator generates an energy current and consumes a lagging reactive current, while the induction motor consumes an energy current and consumes a reactive lagging current.

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As with the increasing voltages and increasing extent of our transmission systems the leading currents taken by transmission lines and underground cables are becoming increasingly larger, the induction generator appears specially advantageous, as tending to offset the effect of line capacity. We may thus say that the induction generator (and induction motor) consumes a lagging reactive current, which is supplied by the synchronous generators, synchronous motors, converters and other synchronous apparatus in the system, and by the capacity of lines and cables. Or we may say that the lagging current consumed by the induction generator neutralizes the leading current consumed by the capacity of lines and cables. Or we may say that the leading current produced by the induction

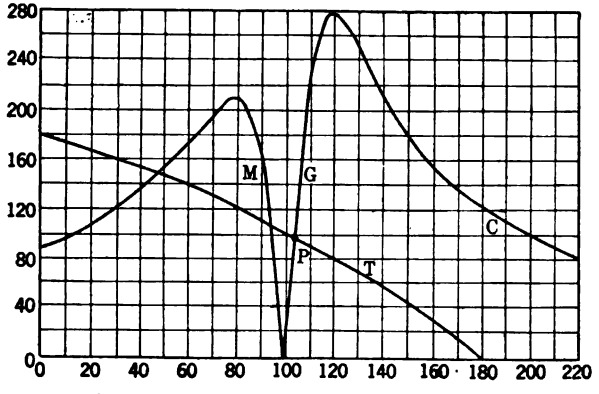

FIG. 2—SMALL HYDROELECTRIC INDUCTION GENERATOR PLANT CONSTANT TERMINAL VOLTAGE

generator supplies the capacity of lines and cables: these are merely three different ways of expressing the same facts.

In Fig. 2 are shown the torque curves, at constant terminal voltage, of a typical moderate size induction machine. *M* is the torque produced as an induction motor below synchronism, and *G* the torque consumed as an induction generator above synchronism, synchronism being chosen as 100 per cent. *T* is an assumed torque curve of a hydraulic turbine.

As seen, the point *P* where *G* and *T* intersect, is 4 per cent above synchronism, and this induction generator thus operates on full load at 4 per cent slip above synchronism or no-load. Assuming now, that the power goes off, by the circuit breakers opening. The turbine then speeds up to 80 per cent above

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synchronism, where the curve T becomes zero. If at this free running turbine speed the circuit is closed and voltage put on the induction generator, the high torque consumed by the induction generator causes the turbine to slow down, and as at all speeds above 104, the torque consumed by the induction generator is very much higher than that given by the turbine, the machine slows down rapidly, to the speed where the induction generator torque has fallen to equality with the turbine torque, at speed 104, and stable condition is restored.

Inversely, if the flow of water should cease, the induction machine slows down to a little below synchronism, and there continues to revolve as induction motor.

In starting, the circuit may be closed before admitting the water, and the turbine started by the induction machine as a motor, on the torque curve M, running up to speed 100, and then, by admitting the water, the machine is speeded up 4 per cent more and thereby made to take the load as generator. Or the turbine may be started by opening the gates, running up to speed 180, and then, by closing the circuit, the induction machine in taking the power slows the speed down to normal.

With larger machines, the most satisfactory way of starting as involving the least disturbance, probably would be, first to open the gates partly while the turbine speeds up, and when it has reached a speed in the neighborhood of synchronism, say between 95 and 105, the circuit is closed and the water gates opened fully.

B. Instability Conditions of Induction Generator

In Fig. 2, the torque consumed by the induction machine, at all turbine speeds above full load P, is much higher than the torque of the turbine. However, the induction generator torque curve has a concave range, marked by C, and if the induction generator should be such as to bring the generator torque curve at C below the turbine torque curve T, the speed, when once increased beyond the range C, would not spontaneously drop back to normal. While in Fig. 2, C is much higher than T, Fig. C represents the theoretical, but not real case of constant terminal voltage at the induction machine. The voltage however is kept constant at the controlling synchronous main station, and thus must vary with the load in the induction generator station. Assuming an extreme case, of 10 per cent resistance and 20 per cent reactance in the line from the induction machine station to

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the next synchronism station, we get the modified torque curve shown in Fig. 3. As seen, at full load P, there is practically no change; about 4 per cent slip above synchronism. The maximum torque of generator G and motor M, and the torque at the concave part of the induction generator curve, C, have greatly

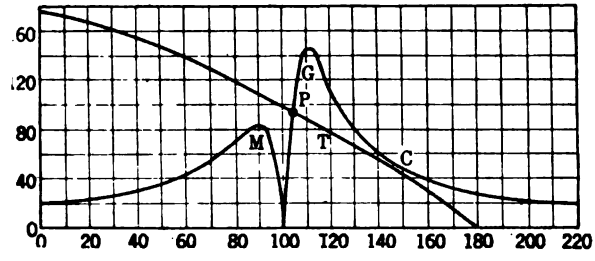

FIG. 3—SMALL HYDROELECTRIC INDUCTION GENERATOR PLANT—CONSTANT VOLTAGE IN SYNCHRONOUS STATION

decreased. However, C is still above T, that is, even under this extreme assumption, the induction generator would pull the turbine down from its racing speed of 180, to the normal full load speed of 104, though the margin has become narrow.

Assuming however an induction machine with much less slip,

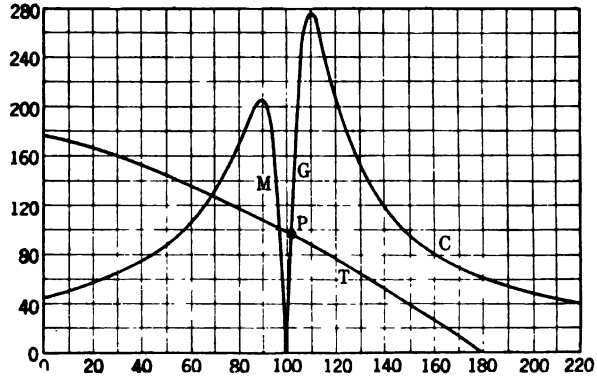

FIG. 4—LARGE HYDROELECTRIC INDUCTION GENERATOR PLANT—CONSTANT TERMINAL VOLTAGE

with only half the rotor resistance of Figs. 2 and 3. At constant terminal voltage, this gives the curves shown in Fig. 4. The full load P is at speed 102, or 2 per cent above synchronism, and while the curve branch C is much lower, the conditions are still perfectly stable. Assuming however, with this type of low

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resistance rotor, a high line impedance, 10 per cent resistance and 20 per cent reactance, as in Fig. 3. We then get the condition shown in Fig. 5. The range C drops below T, and the induction generator torque curve G intersects the turbine torque curve T at three points: P, P_1 and P_2. Of these three theoretical running speeds, $P=102$, $P_1=169$ and $P_2=113.5$, two are stable, P and P_1; while the third one, P_2, is unstable, and from P_2, the speed must either decrease, reaching stability at the normal full load point P, or the machine speed up to P_1.

If with the conditions represented by Fig. 5, the turbine should—by an opening of the circuit for instance—have speeded up to its free running speed 180, closing the circuit does not bring the speed back to normal, P, but the machines slow down only to speed P_1, where stability is reached, at very little output and very large lagging currents in the induction generator. To

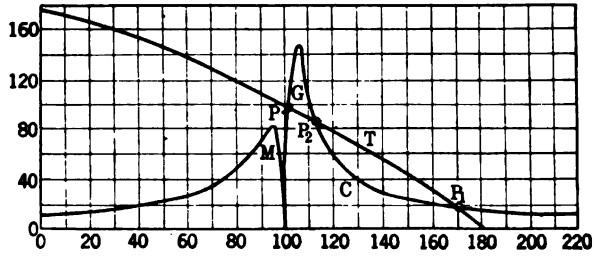

FIG. 5—LARGE HYDROELECTRIC INDUCTION GENERATOR PLANT—CONSTANT VOLTAGE IN SYNCHRONOUS MAIN STATION

restore normal condition then would require shutting off the water, at least sufficiently to drop the turbine torque curve T below C, and then letting the machines slow down to synchronism. They would not go below synchronism, even with the water gates entirely closed, as the induction machine as a motor, on curve M, holds the speed.

A solution in the case Fig. 5 would be the use of a simple excess speed governor, which cuts off the water at 5 to 10 per cent above synchronism.

However, the possibility of difficulty due to the "dropping out of the induction generator" as we may call it in analogy to the dropping out of the induction motor, are rather less real than it appears theoretically. In smaller stations, such as would be operated without attendance, as automatic stations, the torque curve of the induction generator, as a small machine, would be

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of the character of Figs. 2 and 3, and thus not liable to this difficulty. The low resistance type of induction machines, as represented in Figs. 4 and 5, may be expected only with the larger machines, used in larger stations. In those, some attendant would be present to close the water gates in case of the circuit breakers opening, or a simple cheap excess speed cut-off would be installed at the turbines, keeping them within 10 per cent of synchronism, and within this range, no dropping out of the induction generator can occur.

It is desirable however to realize this speed range of possible instability of the induction generator, so as to avoid it in the design of induction generators and stations.

APPENDIX

Collection of Fuel Power by Steam Turbine Induction Generator

A. The Automatic Steam Turbine Induction Generator Station

The same reason which in the preceding led to the conclusion that in the (automatic) induction generator station is to be found the solution of the problem of collecting the numerous small amounts of hydraulic energy, which are scattered throughout our country along creeks and mountain streams, also applies, and to the same extent, to the problem of collecting the innumerable small quantities of mechanical or electrical energy, which are, or can be made available wherever fuel is consumed for heating purposes. Of the hundred millions tons of coal, which are annually consumed for heating purposes, most is used as steam heat. Suppose then, we generate the steam at high pressure—as is done now in many cases for reasons of heating economy—and interpose between steam boiler and heating system some simple form of high pressure steam turbine, directly connected to an induction generator, and tie the latter into the general electrical power distribution system. Whenever the heating system is in operation, electric power is generated, as we may say as "by-product" of the heating plant, and fed into the electric system.

The power would not be generated continuously, but mainly in winter, and largely during the day and especially the evening. That is, the maximum power generation by such fuel power collecting plant essentially coincides with the lighting peak of the central station, thus occurs at the time of the day, and the season when power is most valuable. The effect of such fuel

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power collection on the central station should result in a material improvement of the station load factor, by cuttnig off the lighting peaks.

The only difference between such steam turbine induction generator stations, collecting the available fuel power scattered throughout the cities and towns, and the hydraulic induction generator stations collecting the powers of the streams throughout the country, is that in the steam turbine plant an excess speed cut-off must be provided, as the free running steam turbine speed is usually not limited to less than double speed, as is the case with the hydraulic turbine. Otherwise however, no speed governing is required. A further difference is, that the greater simplicity and therefore lower investment of the steam turbine plant would permit going down to smaller powers, a few kilowatts perhaps.

It is interesting to note, that even with a very inefficient steam turbine, the electric generation of such fuel power collecting plant interposed between boiler and heating system, takes place with practically 100 per cent efficiency, because whatever energy is wasted by the inefficiency of the steam turbine plant, remains as heat in the steam, and the only loss is the radiation from turbine and generator, and even this in most cases is useful in heating the place where the plant is located. The only advantage of a highly efficient turbine, is that larger amounts of electric power can be recovered from the fuel, and the question thus is that between the investment in the plant, and the value of the recovered power.

If then the total efficiency, from the chemical energy of the fuel to the electric power, were only 3 per cent, it would mean that 3 per cent more coal would have to be burned, to feed the same heat units into the heating system. At an average energy value of 30,000 kj. per kg. of coal, this would give per ton of coal, 900,000 kj. or 250 kw-hr. At a bulk value of ½ cent per kw-hr. it would represent a power recovery value of $1.25 per ton of coal. This is quite considerable, more than sufficient to pay the interest on the investment in the very simple plant required.

At first, the steam turbine induction generator plant, proposed for the collection of fuel power, would appear similar to the isolated plant which, though often proved uneconomical, still has successfully maintained its hold in our northern latitudes, where heating is necessary through a considerable part of the year. However, the difference between the steam turbine in-

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duction generator plant and the isolated steam electric plants in our cities, is the same as that between the automatic hydroelectric induction generator station, and the present standard synchronous generator station: by getting rid of all the complexity and complication of the latter, the induction generator station becomes economically feasible in small sizes; but it does so only by ceasing to be an independent station, by turning over the functions of regulation and control to the central main station and so becoming an adjunct to the latter. But by this very feature, the turbo induction generator plant might afford to the central station, the public utility corporation, a very effective means of combatting the installation of isolated plants, by relieving the prospective owner of the isolated plant of all trouble, care and expense and incidental unreliability thereof, supplying central station power for lighting, but at the same time utilizing the potential power of the fuel burned for heating purposes. The simplest arrangement probably would be, that the fuel power collecting plants scattered throughout the city would, as automatic stations, be taken care of by the public utility corporation, their power paid at its proper rates, those of uncontrolled bulk power, while the power used for lighting is bought from the central station at the proper lighting rates.

As this however means a new adjustment of the relation between customer and central station, and is not merely an engineering matter like the hydroelectric power collection, I have placed it in an appendix.

B. Discussion

We realize that our present method of using our coal resources is terribly inefficient. We know that in the conversion of the chemical energy of coal into mechanical or electrical energy, we have to pass through heat energy and thereby submit to the excessively low efficiency of transformation from the low grade heat energy to the high grade electrical energy. We get at best 10 to 20 per cent of the chemical energy of the coal as electrical energy; the remaining 80 to 90 per cent we throw away as heat in the condensing water, or worse still, have to pay for getting rid of it. At the same time we burn many millions of tons of coal to produce heat energy, and by degrading the chemical energy into heat, waste the potential high grade energy which those millions of tons of coal could supply us.

It is an economic crime to burn coal for mere heating without

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first taking out as much high grade energy, mechanical or electrical, as is economically feasible. It is this feature, of using the available high grade energy of the coal, before using it for heating, which makes the isolated station successful, though it has every other feature against it. To a limited extent, combined electric and central steam heating plants have been installed, but their limitation is in the attempt to distribute heat energy, after producing it in bulk, from a central station. Here again we have the same rule; to do it efficiently, do it electrically. In the efficiency of distribution or its reverse, collection, no other form of energy can compete with electric energy, and the economic solution appears to be to burn the fuel wherever heating is required, but first take out its available high grade energy, and collect it electrically.

Assume we use 200 million tons of coal per year for power, at an average total efficiency of 12 per cent, giving us 24 million kw. (referred to 24-hr. service) and use 200 million tons of coal for heating purposes, wasting its potential power.

If then we could utilize the waste heat of the coal used for power generation, even if thereby the average total efficiency were reduced to 10 per cent, we would require only 240 million tons of coal, for producing the power, and would have left a heating equivalent of 216 million tons of coal, or more than required for heating. That is, the coal consumption would be reduced from 400 million to 240 million of tons, a saving of 160 million tons of coal annually.

Or, if from the 200 million tons of coal, which we degrade by burning it for fuel, we could first abstract the available high grade power, assuming even only 5 per cent efficiency, this would give us 10 million kw. (24-hr. rate), at an additional coal consumption of 10 million tons, while the production of the 10 million kw. now requires 100 million tons of coal, more or less, thus getting a saving of 90 million tons of coal; or putting it the other way, a gain of 9 million kw.—12 million horse power—24-hr. service, or 36 million horse power for an 8-hr. working day.

It is obvious that we never could completely accomplish this; but even if we recover only one-quarter, or even only one-tenth of this waste, it would be a vast increase in our national efficiency.

Thus the solution of the coal problem, that is, the more economic use of fuel energy, is not only the increase of the thermodynamic efficiency of the heat engine, in which a radical advance

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Per ton of coal: Chemical energy, 30 × 10^6 kj: Heat energy of steam from boiler, at 75 per cent. boiler efficiency, 22.5 × 10^6 kj.

	Boiler press		Dist. press		Carnot-efficiency per cent.	Output at 50 per cent. efficiency		Value of power at ½c. per kw-hr. $..	Avg. kw., assuming 25% time of use	Tons of coal per kw.	Size of induction generator, per 100 tons coal annually
	atm:	lb.	atm:	lb.		kj. × 1000	kw-hr.				
Steam Heating	6	90	1.25	19	12.3	1380	385	1.92	0.176	5.7	25 kw.
	15	220	1.25	19	19.8	2220	620	3.10	0.283	3.5	45 kw.
Vacuum Heating	6	90	0.48	7	18.1	2030	565	2.82	0.258	3.9	40 kw.
	15	220	0.48	7	25.0	2820	810	4.05	0.37	2.7	55 kw.

is limited by formidable difficulties; but is the recovery of the potential energy of all the fuel, by electric collection.

C. Turbo Induction Generator

Assume then that wherever fuel is burned to produce steam for heating purposes, instead of a low-pressure boiler giving a few pounds over-pressure only, we generate the steam at high pressure, at six atmospheres (90 lbs.) or, in larger plants, even at 15 atmospheres (220 lb.) passing the steam through a high pressure turbine wheel directly connected to an induction generator tied into the electric supply system, and then exhaust the steam at 1.25 atmospheres (19 lb.) into the steam heating system, or at 0.48 atmospheres (7 lb.) into a vacuum heating system.

At a fuel value of the coal of 30,000 kj. per kg. we have (see table)

From this it would follow that the average magnitude of the steam turbine induction generator plant for power collection from fuel in heating plants, would be about one-quarter to one-half kw. per ton of coal burned annually, under the assumption, that the use of the heating plant is equivalent to full capacity during one quarter of the time, and the turbine induction generator plant 50 per cent larger, to take care of maximum loads.

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As seen, the value of the recovered power would be a substantial percentage of the fuel cost.

With 100 million tons of coal used for heating purposes annually, assuming an average recovery of 600 kw-hr. per ton, this gives a total of 60,000 million kw-hr. per year. One-quarter of this is more electric power than is now produced at Niagara, Chicago, New York and a few other of the biggest electric systems together.

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DISCUSSION ON "AMERICA'S ENERGY SUPPLY" (STEINMETZ), ATLANTIC CITY, N. J., JUNE 27, 1918.

A. M. Schoen: In connection with the discussion of the question of our fuel supply and the need of additional power, it occurs to me to mention that about the year 1911, I think it was, I had the honor of representing the Institute at a hearing before the Committee on Agriculture from Congress on the subject of forest conservation. At that time the interests of the Institute lay in the conservation of the waterpower by protecting forests. There was a paper presented at that meeting by Mr. M. O. Leighton, engineer for the U. S. Geological Survey, and in that paper he went comprehensively into a survey he had made of the water powers of the Southern Appalachians. He called attention to the fact that by throwing dams across the ravines at the heads of a number of these streams the water could be impounded, and the low water stage of certain rivers could be materially raised.

The rainfall in the Southern Appalachians is very heavy, I think next to the Puget Sound region the heaviest we have in this country. Mr. Leighton's work was done with a view to improving rivers at that time navigable and rendering navigable others suffering from extreme low water. The same data, however, would indicate that through this method of protecting against freshet and drought, the low water stages could be so improved as to furnish sufficient water to change a considerable part of the power developed at some of the hydroelectric plants from secondary to primary, thus utilizing machinery at present idle during a considerable part of the year. With the present appeal from the Government to help save the coal, now would seem to be the accepted time to make use of this additional power. Mr. Leighton's paper was published in 1908 and will doubtless be found of interest in this connection.

B. A. Behrend: Dr. Steinmetz's interesting suggestion meets with one or the other of the horns of a dilemma which has confronted engineers in regard to the distribution of heat. If Mr. Steinmetz's idea is carried out literally as he wishes it to be, it is necessary to install in each house a boiler, a steam turbine, and an induction generator, so that the latent heat of the steam can be utilized for heating purposes. If this is not done, it is necessary to group together a number of houses in which the old difficulty of distributing heat has to be faced. Mr. Steinmetz does not wish us to consider the second alternative which has been carried out in several plants, particularly in Milwaukee in a station installed there some twelve years ago by Mr. John I. Beggs, for which the speaker designed the electric generating units.

Considering, therefore, the scheme as advocated by Dr. Steinmetz, we have to familiarize ourselves with the idea that dwellings will have to be equipped as indicated above, and

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whether this is feasible, or whether it is possible to operate such plants without attendants of any kind is a question the consideration of which must be left to the individual judgment of those who have heard Mr. Steinmetz's paper.

In this connection, allow me to point out the suggestion made many years ago by Lord Kelvin to reverse the Carnot cycle and to return the heat thus gained into the building to be warmed. In making this proposal, Lord Kelvin stated that it would not be resorted to until a shortage of fuel was experienced. (See Mathematical and Physical Papers by Lord Kelvin, Vol. I, p. 516.)

S. Barfoed (communicated after adjournment): In large sections of the Pacific Coast one mentioned source of energy supply, namely oil, is being used in an ever increasing amount, in spite of the fact that water power is plentiful in the undeveloped stage.

This oil, which is being burnt for its heat value only is the country's legitimate lubricating stock. It is needed in an ever increasing amount for machinery of all descriptions and should not be used as a source of energy supply merely because it will burn.

The various factors contributing to this misuse are well known to engineers and need no longer be discussed by them.

How the energy sources may be utilized and coordinated to best advantage and with a minimum of waste from an engineering point of view seems almost paltry compared to the vexatious work necessary to bring about an understanding of the problem on the part of the general public and its chosen representatives on the various legislative and executive bodies.

The trained engineer thinks generally not along the lines of least resistance, but he will have all he can do to carry out but a small fraction of Dr. Steinmetz' simple and beautiful dream while our present generation is alive.

Ralph Bennett (communicated after adjournment): In the first section of his article Mr. Steinmetz discusses in a general way the total possible energy to be derived from the use of falling water and of the sun's rays.

The conclusion drawn is that, even with a workable automatic generating station the energy of the water must be supplemented by other sources.

But if it be granted that such stations are feasible and that the electrical network extends for distances so great that local climatical changes may be taken as more or less equalized then other sources not stated become available.

The total energy of the winds is almost incalculable. Efforts to utilize them have been successful only in a small way because the need is for local power at definite times while the wind is a variable source, so far as any small areas are concerned, and cannot be controlled except to reduce the energy delivered by any single power unit.

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Two methods of overcoming this difficulty have been generally suggested. Both assume local isolated plants but one stores energy in electrical storage batteries while in the other scheme the energy is stored in water lifted into reservoirs. In both cases the cost of equipment and maintenance is so great that other sources of energy are more reliable and economical.

As simple units feeding into a far extending network wind engines might have a much greater application.

The average wind velocity over the United States is in the vicinity of 10 m. (16 km.) per hr. and 50-ft. (15.2 m.) mills will produce about 5 h. p. at this velocity.

The number of mills which can be used is limited only by the reduction in current velocity which would result from over close spacing. If only one per acre were permissible on the 1,920,000,000 acres of continental United States there would be developed a continuous output averaged over the country of over 8,700,000,000 h. p.

The actual practicable production from this source may really be very considerable in the not distant future for there are known zones of highly sustained wind velocities and commercial mills are already well developed in small sizes. Only a power market capable of paying the high interest charges on the investment is needed to make such a plant possible.

Two other intermittent sources of large amounts of energy are the waves and the tides. Both have been the source of endless experimentation by inventors. But few successful motors have developed. The main difficulty appears to be the extremely rugged construction required as compared with the average energy developed. But connected to networks fed from many and scattered sources the output here might, as in the case of windmills, vary with the power available and not with the local demand.

The coasts of the United States extend for over 21,300 mi. (34,279 km.). Along most of this length there is a tidal range of 5 ft. or better and a constant wave action.

Possibly there may be available over 5000 h. p. per mile of coast from the waves alone and neglecting any energy which might be developed off shore.

This would amount to 106,500,000 h. p. for the entire shore line.

Doubtless successful motors could be built if the market would warrant the expenditure.

Tidal motors are a more difficult problem since the only method considered at all possible is the use of tidal basins and only in rare instances can these be had.

Other sources of stored energy besides coal are in extensive use. At the present time the oil production of the United States equals the equivalent of one-eighth of the coal used.

Vast oil fields are as yet untouched. No complete exploration for new fields has been made.

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Oil shales and bitumen charged rocks exist in enormous beds and from them as well as the exhausted sands of old oil fields further supplies of fuel can be drawn as the demand requires.

One half of all the timber cut is wasted, yet the fuel and by-product value of this "refuse" is greater than that of the lumber commercially used.

With the extension of the networks and a policy of free acceptance of offered power the use of these and numerous other sources of smaller or larger supplies of energy will become possible and the necessary use of the coal and high-grade oil reserves can be materially restricted without hardship.

C. P. Steinmetz: In discussing fuel consumption, I have included oil and natural gas, reduced to coal by their approximate fuel value. This is the reason why the tonnage of coal given in my paper is larger than that actually mined. It includes the coal equivalent of the other fuels.

Very interesting and of some importance, is the subject regarding the use of wind power, made by Mr. R. Bennett. At present, due to the great irregularity of the wind, the windmill is of use mainly for such power demands, where great irregularity is permissible, that is, pumping for drainage, etc. Storage of the energy by storage battery is not economical—unless in very exceptional cases—due to the high investment, low efficiency and rapid depreciation of the battery. The only way of utilizing wind power to a greater extent, thus would be by electric collection, feeding whatever power the wind gives into a general system together with hydraulic power, steam power, etc. The only economical way this could be done, appears to me, by the use of an induction generator attached to the wind-wheel. Such low-voltage induction generator would thus be located at the top of the tower and there would also be located the step-up transformer raising to the voltage of the collecting lines. This will probably be about 10,000 volts, that is, low enough not to increase the cost of the transformer and sufficiently high to carry the power for some distance. Even the collecting lines would not come down from the wind-mill tower but jump directly to the next tower, that is, the wind-wheel tower would be used as transmission line tower.

Such a system of collecting wind-power then would comprise a high-voltage trunk line—100,000 volts—main collecting lines issuing at right angles from the trunk line every few miles, at about 10,000 volts, and tied to the trunk line by step-up transformers, and branch collecting lines issuing from the main collecting lines every few hundred feet at right angles, thus about parallel to the trunk line. All these collecting lines would be tower lines and each tower contains a wind-wheel with low-voltage induction generator and a step-up transformer, connecting into the collecting line. Possibly some simple cutout device would also be required, connecting the wind-

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wheel-induction generator-transformer into the collecting line by centrifugal force and disconnecting it by reverse-power relay.

I believe such an arrangement would be economically feasible, that is, the value of the collected wind-power would pay the interest on the investment and maintenance and a fair profit, even to-day in some localities, where fairly sustained wind velocities exist, power is valuable, and the cost of fuel high.

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PRE-CHARGED CONDENSERS IN SERIES AND IN PARALLEL

BY V. KARAPETOFF

ABSTRACT OF PAPER

A condenser is charged from a source of direct voltage, and then is used as a booster in series with this source to charge another condenser. By repeating this process a large number of times the second condenser is finally subjected to twice the voltage of the source. This is the principle of the Delon apparatus for testing cables, and is explained in a numerical example. Then the more general case of two or more "pre-charged" condensers in series is considered, when these condensers are connected to some source of direct voltage; it is shown how to determine the final distribution of voltages among them. A similar problem is solved for pre-charged condensers in parallel. Finally a general network of pre-charged condensers is considered, and equations are derived similar to Kirchoff's laws, from which the final distribution of voltages and charges may be computed knowing the initial distribution.

IN A paper presented before the Institution of Electrical Engineers (British) in February 1916, Mr. O. L. Record described Delon's apparatus for testing the insulation of a high-tension cable.[1] While endeavoring to make clear to himself the theory of this ingenious device the present writer has investigated in general the action of a pre-charged condenser used as a booster in series with some source of e.m.f. for charging another condenser. This led him further to deduce the equations of a general network of such condensers. Before giving these more general relationships it is of interest to explain the principle of Delon's apparatus in a simple numerical example.

The diagram of connections is shown in Fig. 1, in which C is a two-conductor cable under test, T is the testing transformer, R is a synchronous revolving rectifier arm, and C_1, C_2 are two "booster" condensers. The theory of the apparatus, confirmed by actual experience, shows that the cable is subjected to a continuous voltage which is numerically equal to twice the ampli-

1. Journal Inst. Elec. Engrs. Vol. 54 (1916), p. 610; abstracted in the *Electrical World*, Vol. 67 (1916), p. 665.

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tude of the transformer voltage. Thus a cable can be readily tested at quite a high voltage, without having a source of considerable reactive kilovolt-amperes, and without subjecting the cable to dielectric hysteresis or dangerous oscillations.

Theory of Delon's Apparatus. Let the capacities of condensers C_1 and C_2 each be equal to one quarter of that of cable C, and let the rectifying arm occupy the position shown in Fig. 1 at the instant when the lower terminal of the transformer is positive, and the voltage is at a maximum. Let the instantaneous value of this maximum voltage be 100 kilovolts and let the two condensers and the cable not be previously charged at all. Then at the instant under consideration condenser C_1 is subjected to the full potential difference of 100 kv. between points A_1 and B, and is being "pre-charged" for the next half of the cycle. For the sake of simplicity we shall assume that the duration of the contact is sufficient to charge C_1 to 100 kv.

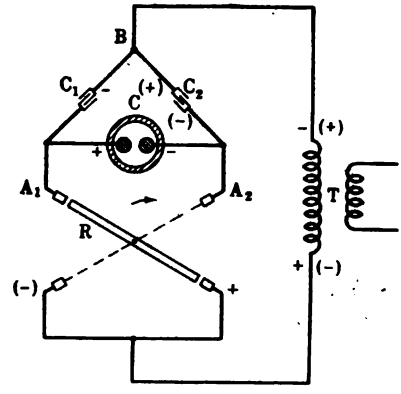

FIG. 1—DIAGRAM OF CONNECTIONS FOR TESTING TWO-CONDUCTOR CABLE

At the same instant the cable C is in series with condenser C_2 and the two together are subjected to 100 kv. Since the capacities C and C_2 are in the ratio of 4 to 1, the voltage across C is 20 kv., that across C_2 is 80 kv. It is important to note that the polarities marked at condensers C_1 and C_2 are those during the process of pre-charging. The polarity of C_2 during the first charge just described is opposite of that shown in the figure.

Half a cycle later the synchronous arm touches contact A_2 and the lower transformer terminal is negative. The corresponding polarities are shown in parentheses. The pre-charged condenser C_1 now serves as a booster in series with the transformer and helps to raise the charge on the cable, while condenser C_2 is being "pre-charged" between A_2 and B for the next impulse. We shall now determine the voltage to which the cable is charged after the second impulse.

Since the capacities C and C_1 are in the ratio of 4 to 1, any gain in voltage X across C corresponds to a loss of $4X$ across

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C_1. In other words, if the new voltage across C is $(20 + X)$, that across C_1 is $(100 - 4X)$. But the total instantaneous voltage between A_2 and B is 100 kv., so that

$$(20 + X) - (100 - 4X) = 100,$$

from which $X = 36$ kv. Thus, after the second contact the cable is charged to 56 kv., while the original voltage across C_1 has dropped to -44 kv.

When the synchronous arm comes again in position A_1, the cable is connected in series with condenser C_2 which in the meanwhile has been charged to 100 kv. We thus have the equation

$$(56 + X) - (100 - 4X) = 100$$

from which $X = 28.8$ kv., so that the cable is now charged to 84.8 kv.

Carrying this reasoning further one readily finds that the voltage across the cable indefinitely approaches the value 200 kv. which is the double of the amplitude of the transformer voltage. Only after this value has been reached do the booster condensers and the transformer cease to charge the cable further. This is because the total voltage of the transformer plus that of a booster condenser is equal to 200 kv., and a permanent equilibrium is established. Let the cable voltage after the $n-$ th charge be E_n. Then, by analogy with the foregoing expressions, we have for the $(n + 1)$ th application of voltage

$$(E_n + X) - (100 - 4X) = 100$$

or

$$X = (200 - E_n)/5 \qquad (1)$$

so that

$$E_{n+1} = E_n + X = 40 + 0.8\,E_n \qquad (2)$$

It will thus again be seen that the charging will stop or X will become zero when $E_n = 200$ kv.

Applying formula (2) to the first few charges (except the initial one) we get the following values:

20; 56; 84.8; 107.8; 126.3; 141; 152.8 kv.

Here the first value, 20 kv., does not satisfy the left-hand side of equation (2) when on the right-hand side one puts $E_n = 0$. However, it must be remembered that when the voltage is applied for the first time the booster condenser is not pre-charged so that equation (1) does not hold true either. Both equations hold true for every impulse except the first one.

One could imagine n condensers in place of each booster condenser, and with a proper commutating arrangement these

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condensers could be pre-charged in parallel and discharged in series. In this case the cable could be subjected to a voltage equal to $n + 1$ times that of the amplitude of the transformer voltage. The idea of charging condensers in parallel and discharging them in series is not new and has been used for other purposes.

Two Condensers in Series. Let us now consider a more general case (Fig. 2) of a condenser of capacity C pre-charged to a voltage E_0 and of a booster condenser C' pre-charged to a voltage E_0'. Let the corresponding charges or displacements of electricity in the condensers be Q_0 and Q_0'. The operations will consist in charging C to a higher voltage by means of pre-charged C' in series with a battery e, then pre-charging C' again, raising the voltage of C, and so forth. The problem is to determine the consecutive values of the voltage across C. These values will be denoted by E_0, E_1, E_2, $\ldots E_n$, while the corresponding pre-charge voltages of the booster condenser will be called E_0', E_1', $E_2' \ldots E_n'$.

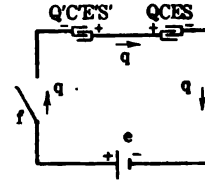

FIG. 2—TWO CONDENSERS IN SERIES

We have the fundamental relations

$$Q_0 = C E_0 \tag{3}$$

$$Q_0' = C' E_0' \tag{4}$$

Since the two condensers are connected in series, it is more convenient to use the reciprocal of capacity namely the so-called *elastance* of a condenser.[2] Introducing the elastances

$$\left.\begin{aligned} S &= 1/C \\ S' &= 1/C \end{aligned}\right\} \tag{5}$$

we get

$$\left.\begin{aligned} E_0 &= Q_0 S \\ E_0' &= Q_0' S' \end{aligned}\right\} \tag{6}$$

Let now condenser C' be connected in series with a constant-voltage battery e, and the switch f closed. With the polarities shown in the sketch the condenser C' assists the battery in further charging condenser C, and a quantity of electricity q

2. When a capacity is expressed in microfarads, the corresponding elastance is measured in mega-darafs. See the author's "Electric Circuit" (McGraw-Hill, 1912), p. 148; also his paper "Sur quelques calculs pratiques des champs électrostatiques," *Trans. Congresso Internazionale delle Applicazioni Elettriche*, Torino, 1911.

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is displaced through the circuit. The charge on C is increased to $(Q + q)$, that on C' is reduced to $(Q' - q)$. The corresponding voltages now are

$$E_1 = (Q_0 + q)\, S \tag{7}$$

$$E_1' = (Q_0' - q)\, S' \tag{8}$$

An equilibrium is established when

$$e + E_1' = E_1 \tag{9}$$

Substituting in this equation the values from equations (6), (7) and (8) we get

$$e + E_0' - q\, S' = E_0 + q\, S,$$

or

$$e + E_0' - E_0 = q\,(S + S') \tag{10}$$

so that

$$q = (e + E_0' - E_0)/(S + S') \tag{11}$$

All the quantities on the right hand side of this expression are known and q may be computed. Substituting this value of q in equation (7) we get

$$E_1 = E_0 + (e + E_0' - E_0)[S/(S + S')] \tag{12}$$

This expression permits one to determine the new voltage of condenser C knowing the initial voltages and the elastances of the condensers.

The larger the booster condenser the more it helps in charging the other condenser, but on the other hand the more expensive it is. Since the result depends only upon the ratio of the elastances we shall denote

$$S'/(S + S') = \alpha \tag{13}$$

$$S/(S + S') = 1 - \alpha \tag{14}$$

In the most efficient and most expensive case of an infinitely large auxiliary condenser ($S' = 0$) we have $\alpha = 0$. In the other extreme, when it is impossible to charge C because $C' = 0$ or $S' = \infty$, the ratio is $\alpha = 1$. In all cases α is a positive regular fraction, and may be called "slack factor." Equation (12) becomes

$$E_1 = (1 - \alpha)\,(e + E_0') + \alpha\, E_0 \tag{15}$$

or

$$E_1 = (1 - \alpha)\, V_1 + \alpha\, E_0 \tag{16}$$

Here

$$V_1 = e + E_0' \tag{17}$$

is the total charging voltage applied to condenser C at the instant of closing switch f.

Let now the switch be opened, the condenser C' pre-charged

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to some other voltage E_1' and the switch closed again. By analogy with equation (**16**), the new voltage to which condenser C will be charged is

$$E_2 = (1 - \alpha)\ V_2 + \alpha E_1,$$

where $V_2 = e + E_1'$ is the new total charging voltage which in a general case may be different from V_1. If this charging process be repeated n times we get

$$\left.\begin{aligned}
E_1 &= (1 - \alpha)\ V_1 + \alpha E_0 \\
E_2 &= (1 - \alpha)\ V_2 + \alpha E_1 = (1 - \alpha)\ (V_2 + \alpha V_1) + \alpha^2 E_0 \\
E_3 &= (1 - \alpha)\ V_3 + \alpha E_2 = (1 - \alpha)\ (V_3 + \alpha V_2 + \alpha^2 V_1) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \alpha^3 E_0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
E_n &= (1 - \alpha)\ (V_n + \alpha V_{n-1} + \alpha^2 V_{n-2} + \ldots + \alpha^{n-1} V_1) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \alpha^n E_0
\end{aligned}\right\} \quad (18)$$

No matter how many times the charging process be repeated the voltage across C remains finite because it cannot possibly exceed the highest charging voltage V applied to it.

In practise, the condenser C' would usually be pre-charged every time to the same constant value, say E', so that

$$V_1 = V_2 = \ldots = V_n = V = e + E' \qquad (19)$$

The preceding expression is then simplified to

$$E_n = (1 - \alpha)\ (e + E')\ (1 + \alpha + \alpha^2 + \ldots + \alpha^{n-1}) + \alpha^n E_0$$

or

$$E_n = (1 - \alpha^n)\ V + \alpha^n E_0 \qquad (20)$$

Compare this expression with equation (**16**). After an infinite number of applications of voltage, that is $n = \infty$,

$$\alpha^n = 0, \text{ and}$$

$$E_\infty = e + E' \qquad (21)$$

independent of α or E_0. However, the values of α and E_0 determine the law according to which the voltage E approaches its ultimate value. With a small slack factor, α, that is with an expensive auxiliary condenser, the final value is practically reached after a few charges, while with an α near unity it may require many thousands of applications.

In the simplest case the same battery e would be used for pre-charging condenser C', so that $E' = e$. Equations (**20**) and (**21**) become

$$E_n = 2\,e\,(1 - \alpha^n) + \alpha^n E_0 \qquad (22)$$

$$E_\infty = 2\,e \qquad (23)$$

These two relationships apply in Delon apparatus.

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Several Condensers in Series. We shall now consider a still more general case of several pre-charged condensers in series. Let the initial voltage, charge, and elastance of the $k-$th condenser be E_{k0}, Q_k and S_k respectively, so that originally

$$E_{k0} = Q_k S_k \tag{24}$$

Let all these condensers be connected in series and also in series with a source of constant voltage e, say a battery, and the switch closed. It is required to find the new voltages across the individual condensers.

We shall consider as positive those voltages E_{k0} which "buck" or oppose the battery voltage, while the voltages which help the battery will be considered negative. Let q be the displacement of electricity or the charge which passes through the circuit when the switch is closed. The new charge in the $k-$th condenser is $Q_k + q$, and the new voltage

$$E_{k1} = (Q_k + q) S_k = E_{k0} + q S_k \tag{25}$$

But when the equilibrium is established

$$\Sigma E_{k1} = e \tag{26}$$

or

$$\Sigma E_{k0} + q \Sigma S_k = e \tag{27}$$

Hence, the additional charge

$$q = (e - \Sigma E_{k0})/\Sigma S_k \tag{28}$$

But $e - \Sigma E_{k0}$ is the *net* voltage before the switch is closed, and ΣS_k is the *equivalent* elastance of the whole circuit. We denote

$$e - \Sigma E_{k0} = e_n \tag{29}$$

$$\Sigma S_k = S_{eq} \tag{30}$$

The preceding expression for q becomes

$$q = e_n/S_{eq} \tag{31}$$

and the new voltage across the $k-$th condenser

$$E_{k1} = E_{k0} + e_n(S_k/S_{eq}.) \tag{32}$$

Condensers in Parallel. The case of several pre-charged condensers in parallel allows of a very simple solution. Let condensers of capacity $C_1, C_2 \ldots . C_k \ldots . C_n$ be pre-charged to voltages $E_1, E_2 \ldots . E_k \ldots . E_n$ respectively and then put in parallel. Let it be required to find the common voltage E after the equalization of the charges. The $k-$th condenser has lost voltage $E_k - E$, which means that it has lost a charge equal to $(E_k - E)$ C_k. But in the absence of a source of e.m.f. the sum of the

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charges after the equalization is the same as before, so that the sum of the changes of charge must be zero. We thus have the equation

$$\Sigma\ (E_k - E)\ C_k = 0 \tag{33}$$

or

$$\Sigma\ E_k\ C_k - E\ \Sigma\ C_k = 0$$

from which

$$E = \Sigma\ E_k\ C_k / \Sigma\ C_k \tag{34}$$

When using this equation one has to be careful regarding the sign of each individual E_k. If all the positive terminals are connected together and all the negative ones together, then all the E_k's should enter in the equation with the same sign, say plus. But if some positive and other negative terminals are connected together, one should select a positive direction of e.m.f. and enter the individual E_k's in equation (**34**) with the sign plus or minus accordingly. As a practical application of pre-charged condensers in parallel the "method of mixtures" may be mentioned for measuring small capacities.[3]

Network of Condensers. As a most general case treated in this article, let a number of pre-charged condensers be connected in any arbitrary manner so as to form a network similar to a network of conductors. Let the initial voltage of the $k-$ th condenser be E_{k0}, the corresponding charge Q_{k0} and the elastance S_k, so that

$$E_{k0} = Q_{k0}\ S_k \tag{35}$$

Let various sources of d-c. voltage, e_1, e_2, e_3, etc. be distributed throughout the network. In the beginning a sufficient number of switches are supposed to be opened to prevent an equalization of charges and voltages. Then all these switches are closed, and it is required to find the new voltages and charges on the condensers.

The problem is solved in a manner similar to that in which currents are found in a complicated network of conductors, namely by applying equations analogous to Kirchoff's laws. As soon as the switches have been closed, the distribution of charges on the condensers changes, the $k-$ th condenser receiving an additional charge say q_k. Let q_k be considered positive when a positive displacement moves away from a junction point of three or more condensers. Since electricity behaves like an

3. See for example V. Karapetoff, Experimental Electrical Engineering, (Wiley) Vol. II, p. 9.

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incompressible fluid, we have a relationship similar to the first Kirchoff law, namely

$$\Sigma\, q_k = 0 \tag{36}$$

An independent equation of this kind may be written for each junction point of the network but one.

For each dielectric circuit that can be traced within the network the second Kirchoff law may be applied which in this case simply becomes

$$\Sigma\, e = \Sigma\, E_{k1} = \Sigma\, (Q_{k0} + q_k)\, S_k.$$

But originally we had $Q_{k0}\, S_k = E_{k0}$, so that the preceding expression becomes

$$\Sigma\, e = \Sigma\, E_{k0} + \Sigma\, q_k\, S_k \tag{37}$$

A sufficient number of equations of this form can be written down, together with equations of the kind (36), to enable one to solve them as simultaneous equations for the unknown quantities q_k. Knowing a q_k, the corresponding new voltage across the condenser may be computed from the relationship

$$E_{k1} = (Q_{k0} + q_k)\, S_k = E_{k0} + q_k\, S_k \tag{38}$$

In the beginning the directions of q_k may be assumed arbitrarily; then those which actually take place in the opposite direction will come out negative, from the solution of equations (36) and (37).

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DISCUSSION ON "PRE-CHARGED CONDENSERS IN SERIES AND IN PARALLEL" (KARAPETOFF), ATLANTIC CITY, N. J., JUNE 28, 1918.

C. O. Mailloux: I wish to say I saw the apparatus perform at the Electrical Exposition at Versailles in 1908, and, of course, it was at that time one of the most interesting features of the exposition.

L. W. Chubb: Prof. Karapetoff's problem becomes more interesting when resistance load instead of static charges are considered. It is perfectly feasible, instead of using condensers in this apparatus, to use batteries or direct-current generators, in which case the fluctuation of energy can be made very great, and power can actually be utilized at a potential higher than the potential of the source. We have used in certain cases a limited power from transformers giving approximately half of the voltage required, and in the case of one or two hundred kv. it is a great advantage to have a transformer, say, at 100 kv. for a load at 175 kv. When these loads are applied the equations for the apparatus become very much more complicated, because while the connections with the pre-charged condensers are being made, there is, of course, a leakage dependent upon the nature of the load and the time that it is on.

W. V. Lyon (Read by B. A. Behrend): The solution for the distribution of electricity among a network of condensers is, as Prof. Karapetoff points out, readily obtained by applying principles similar to those commonly known as "Kirchoff's Laws". The writer has long used problems involving such networks to emphasize the generality of these principles.*

Another point of view should be both interesting and instructive. The current supplied to a single conductor or to any network of resistances, in which there are no electromotive forces acting, will so distribute itself that the total joule heating loss is a minimum. Mathematically this reduces to the statement that the sum of the resistance drops around a closed circuit is zero.

When a number of condensers that have been pre-charged are reconnected in any form of network and allowed to come to an unrestrained equilibrium†, the charges will so redistribute themselves that the total stored energy is a minimum. Mathematically this is also equivalent to the statement that the voltage drop around any closed circuit is zero. If this principle is combined with a second, namely, the algebraic sum of the charges at any insulated junction is constant, a solution for the redistributed charge is at once obtained. If an electromotive force is *inserted in* any branch of this stabilized network there will be produced an *additional charge* on each condenser equal to that which would have been produced had the con-

*Problems in Electrical Engineering (pp, 96, 97, 98) McGraw-Hill Book Co., 1908.

†No electromotive forces acting in the network.

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densers originally been discharged. The resultant effect of more than one such electromotive force inserted in the network is determined by calculating the effect of each electromotive force as if it acted alone, all of the others being short-circuited. The net additional charge on any condenser is equal to the algebraic sum of the charges due to the component electromotive forces. The *total* charge on any condenser is the algebraic sum of the stabilized charge and the net additional charge. The stabilized charge should always be calculated for the condition that all of the electromotive forces are short-circuited.

As a means of solution there is little to commend this method. As a method of analysis, however, it emphasizes two important principles. These are the principles concerning the dissipation of energy in a system, and the combination of component effects in producing a resultant condition.

V. Karapetoff: Perhaps one of the reasons why Delon apparatus has not been used in this country is that we have now the possibility of obtaining high voltages of a unidirectional character by means of hot cathode valves, and I hope that a method will be developed, if it has not been already developed, using these hot cathode valves, for producing high unidirectional voltage for testing cables.

Regarding my nomenclature, I do not like the word daraf, but I wanted to create a disturbance among our rigorists in order to induce the Committee on Nomenclature to take some action. To make the situation particularly objectionable, I also spelled the word henry backward, making it yrneh, which word is used in my "Magnetic Circuit". It is not for an individual to coin a new word, and I felt that I was more consistent in following the established practise (ohm, mho) than in inventing a new word. I will be the first one to adopt a new word for reluctance, the reciprocal of henry, and for the daraf, the reciprocal for farad, as soon as an international action has been taken.

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Presented at the 34th Annual Convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 28, 1918.

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METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS

BY C. L. FORTESCUE

ABSTRACT OF PAPER

In the introduction a general discussion of unsymmetrical systems of co-planar vectors leads to the conclusion that they may be represented by symmetrical systems of the same number of vectors, the number of symmetrical systems required to define the given system being equal to its degrees of freedom. A few trigonometrical theorems which are to be used in the paper are called to mind. The paper is subdivided into three parts, an abstract of which follows. It is recommended that only that part of Part I up to formula (33) and the portion dealing with star-delta transformations be read before proceeding with Part II.

Part I deals with the resolution of unsymmetrical groups of numbers into symmetrical groups. These numbers may represent rotating vectors of systems of operators. A new operator termed the sequence operator is introduced which simplifies the manipulation. Formulas are derived for three-phase circuits. Star-delta transformations for symmetrical co-ordinates are given and expressions for power deduced. A short discussion of harmonics in three-phase systems is given.

Part II deals with the practical application of this method to symmetrical rotating machines operating on unsymmetrical circuits. General formulas are derived and such special cases, as the single-phase induction motor, synchronous motor-generator, phase converters of various types, are discussed.

INTRODUCTION

IN THE latter part of 1913 the writer had occasion to investigate mathematically the operation of induction motors under unbalanced conditions. The work was first carried out, having particularly in mind the determination of the operating characteristics of phase converters which may be considered as a particular case of unbalanced motor operation, but the scope of the subject broadened out very quickly and the writer undertook this paper in the belief that the subject would be of interest to many.

The most striking thing about the results obtained was their symmetry; the solution always reduced to the sum of two or more symmetrical solutions. The writer was then led to inquire if there were no general principles by which the solution of unbalanced polyphase systems could be reduced to the solu-

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tion of two or more balanced cases. The present paper is an endeavor to present a general method of solving polyphase network which has peculiar advantages when applied to the type of polyphase networks which include rotating machines.

In physical investigations success depends often on a happy choice of co-ordinates. An electrical network being a dynamic system should also be aided by the selection of a suitable system of co-ordinates. The co-ordinates of a system are quantities which when given, completely define the system. Thus a system of three co-planar concurrent vectors are defined when their magnitude and their angular position with respect to some fixed direction are given. Such a system may be said to have six degrees of freedom, for each vector may vary in magnitude and phase position without regard to the others. If, however, we impose the condition that the vector sum of these vectors shall be zero, we find that with the direction of one vector given, the other two vectors are completely defined when their magnitude alone is given, the system has therefore lost two degrees of freedom by imposing the above condition which in dynamical theory is termed a "constraint". If we impose a further condition that the vectors be symmetrically disposed about their common origin this system will now have but two degrees of freedom.

It is evident from the above definition that a system of n coplanar concurrent vectors may have $2n$ degrees of freedom and that a system of n symmetrically spaced vectors of equal magnitude has but two degrees of freedom. It should be possible then by a simple transformation to define the system of n arbitrary congruent vectors by n other systems of concurrent vectors which are symmetrical and have a common point. The n symmetrical systems so obtained are the symmetrical co-ordinates of the given system of vectors and completely define it.

This method of representing polyphase systems has been employed in the past to a limited extent, but up to the present time there has been as far as the author is aware no systematic presentation of the method. The writer hopes by this paper to interest others in the application of the method, which will be found to be a valuable instrument for the solution of certain classes of polyphase networks.

In dealing with alternating currents in this paper, use is made of the complex variable which in its most general form

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may be represented as a vector of variable length rotating about a given point at variable angular velocity or better as the resultant of a number of vectors each of constant length rotating at different angular velocities in the same direction about a given point. This vector is represented in the text by $\check{I}$, $\check{E}$, etc., and the conjugate vector which rotates at the same speed in the opposite direction is represented by $\hat{I}$, $\hat{E}$, etc. The effective value of the vector is represented by the symbol without the distinguishing mark as I, E, etc. The impedances Z_a, Z_b, Z_{ab}, etc., are generalized expressions for the self and mutual impedances. For a circuit A the self-impedance operator will be denoted by Z_{aa} or Z_a. In the case of two circuits A and B the self impedance operators would be Z_{aa} Z_{bb} and the mutual impedance operator Z_{ab}. The subletters denote the circuits to which the operators apply. These operators are generally functions of the operator, $D = \frac{d}{dt}$ and the characteristics of the circuit; these characteristics are constants only when there is no physical motion. It will therefore be necessary to carefully distinguish between $Z_a \check{I}_a$ and $\check{I}_a Z_a$ when Z_a has the form of a differential operator. In the first case a differential operation is carried out on the time variable $\check{I}_a$ in the second case the differential operator is merely multiplied by $\check{I}_a$.

The most general expression for a simple harmonic quantity e is

$$e = A \cos pt - B \sin pt$$

in exponential form this becomes

$$e = \frac{A + jB}{2} \epsilon^{jpt} + \frac{A - jB}{2} \epsilon^{-jpt}$$

$(A + jB)\,\epsilon^{jpt}$ represents a vector of length $\sqrt{A^2 + B^2}$ rotating in the positive direction with angular velocity p while $(A - jB)\,\epsilon^{-jpt}$ is the conjugate vector rotating at the same angular velocity in the opposite direction. Since ϵ^{jpt} is equal to $\cos pt + j \sin pt$, the positively rotating vector $\check{E} = (A + jB)\,\epsilon^{jpt}$ will be

$$\check{E} = A \cos pt - B \sin pt + j\,(A \sin pt + B \cos pt)$$

or the real part of $\check{E}$ which is its projection on a given axis is equal to e and therefore $\check{E}$ may be taken to represent e in phase and magnitude. It should be noted that the conjugate vector $\hat{E}$ is equally available, but it is not so convenient since the

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operation $\frac{d}{dt}\,\epsilon^{-jpt}$ gives $-jp\,\epsilon^{-jpt}$ and the imaginary part of the impedance operator will have a negative sign.

The complex roots of unity will be referred to from time to time in the paper. Thus the complete solution of the equation $x^n - 1 = 0$ requires n different values of x, only one of which is real when n is an odd integer. To obtain the other roots we have the relation

$$\begin{aligned} 1 &= \cos 2\pi r + j \sin 2\pi r \\ &= \epsilon^{j2\pi r} \end{aligned}$$

Where r is any integer. We have therefore

$$1^{\frac{1}{n}} = \epsilon^{j\frac{2\pi r}{n}}$$

and by giving successive integral values to r from 1 to n, all the n roots of $x^n - 1 = 0$ are obtained namely,

$$a_1 = \epsilon^{j\frac{2\pi}{n}} = \cos\frac{2\pi}{n} + j\sin\frac{2\pi}{n}$$

$$a_2 = \epsilon^{j\frac{4\pi}{n}} = \cos\frac{4\pi}{n} + j\sin\frac{4\pi}{n}$$

$$a_3 = \epsilon^{j\frac{6\pi}{n}} = \cos\frac{6\pi}{n} + j\sin\frac{6\pi}{n}$$

$$a_n = \epsilon^{j2\pi} = 1$$

It will be observed that $a_2\, a_3 \ldots . a_n$ are respectively equal to $a_1^2\, a_1^3 \ldots . a_1^{(n-1)}$.

When there is relative motion between the different parts of a circuit as for example in rotating machinery, the mutual inductances enter into the equation as time variables and when the motion is angular the quantities ϵ^{jwt} and ϵ^{-jwt} will appear in the operators. In this case we do not reject the portion of the operator having ϵ^{-jwt} as a factor, because the equations require that each vector shall be operated on by the operator as a whole which when it takes the form of a harmonic time function will contain terms with ϵ^{jwt} and ϵ^{-jwt} in conjugate relation. In some cases as a result of this, solutions will appear with indices of ϵ which are negative time variables; in such cases in the final statement the vectors with negative index should be replaced by their conjugates which rotate in the positive direction.

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This paper is subdivided as follows:

Part I.—"The Method of Symmetrical Co-ordinates." Deals with the theory of the method, and its application to simple polyphase circuits.

Part II.—Application to Symmetrical Machines on Unbalanced Polyphase Circuits. Takes up Induction Motors, Generator and Synchronous Motor, Phase Balancers and Phase Convertors.

Part III. Application to Machines having Unsymmetrical Windings.

In the Appendix the mathematical representation of field forms and the derivation of the constants of different forms of networks is taken up.

The portions of Part I dealing with unsymmetrical windings are not required for the applications taken up in Part II and may be deferred to a later reading. The greater part of Part I is taken up in deriving formulas for special cases from the general formulas (**30**) and (**33**), and the reading of the text following these equations may be confined to the special cases of immediate interest.

I wish to express my appreciation of the valuable help and suggestions that have been given me in the preparation of this paper by Prof. Karapetoff who suggested that the subject be presented in a mathematical paper and by Dr. J. Slepian to whom I am indebted for the idea of sequence operators and by others who have been interested in the paper.

PART I

Method of Symmetrical Generalized Co-ordinates

Resolution of Unbalanced Systems of Vectors and Operators

The complex time function $\check{E}$ may be used instead of the harmonic time function e in any equation algebraic or differential in which it appears linearly. The reason of this is because if any linear operation is performed on $\check{E}$ the same operation performed on its conjugate $\hat{E}$ will give a result which is conjugate to that obtained from $\check{E}$, and the sum of the two results obtained is a solution of the same operation performed on $\check{E} + \hat{E}$, or $2\,e$.

It is customary to interpret $\check{E}$ and $\hat{E}$ as coplanar vectors, rotating about a common point and e as the projection of either vector on a given line, $\check{E}$ being a positively rotating vector and

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$\hat{E}$ being a negatively rotating vector, and their projection on the given line being

$$e = \frac{\check{E} + \hat{E}}{2} \tag{1}$$

Obviously if this interpretation is accepted one of the two vectors becomes superfluous and the positively rotating vector $\check{E}$ may be taken to represent the variable "e" and we may define "e" by saying that "e" is the projection of the vector $\check{E}$ on a given line or else by saying that "e" is the real part of the complex variable $\check{E}$.

If (1), $a, a^2 \ldots a^{n-1}$ are the n roots of the equation $x^n - 1 = 0$ a symmetrical polyphase system of n phases may be represented by

$$\left.\begin{aligned} \check{E}_{11} &= \check{E}_{11} \\ \check{E}_{21} &= a\,\check{E}_{11} \\ \check{E}_{31} &= a^2\,\check{E}_{11} \\ &\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots \\ \check{E}_{n1} &= a^{n-1}\,\check{E}_{11} \end{aligned}\right\} \tag{2}$$

Another n phase system may be obtained by taking

$$\left.\begin{aligned} \check{E}_{12} &= \check{E}_{12} \\ \check{E}_{22} &= a^2\,\check{E}_{12} \\ \check{E}_{32} &= a^4\,\check{E}_{12} \\ &\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots \\ \check{E}_{n2} &= a^{2(n-1)}\check{E}_{12} \end{aligned}\right\} \tag{3}$$

and this also is symmetrical, although it is entirely different from (2).

Since $1 + a + a^2 + a^{n-1} = 0$, the sum of all the vectors of a symmetrical polyphase system is zero.

If $\check{E}_1\ \check{E}_2\ \check{E}_3 \ldots \check{E}_n$ be a system of n vectors, the following identities may be proved by inspection:

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$$
\left.
\begin{aligned}
\check{E}_1 \equiv & \frac{\check{E}_1 + \check{E}_2 + \check{E}_3 + \ldots . \check{E}_n}{n} \\
& + \frac{\check{E}_1 + a\,\check{E}_2 + a^2\,\check{E}_3 + \ldots . a^{n-1}\,\check{E}_n}{n} \\
& + \frac{\check{E}_1 + a^2\,\check{E}_2 + a^4\check{E}_3 + \ldots . a^{2(n-1)}\,\check{E}_n}{n} \\
& + \frac{\check{E}_1 + a^{r-1}\,\check{E}_2 + a^{2(r-1)}\,\check{E}_3 + \ldots . a^{(n-1)(r-1)}\,\check{E}_n}{n} \\
& + \ldots . \frac{\check{E}_1 + a^{-1}\,\check{E}_2 + a^{-2}\,\check{E}_3 + \ldots . a^{-(n-1)}\,\check{E}_n}{n} \\
\check{E}_2 \equiv & \frac{\check{E}_1 + \check{E}_2 + \check{E}_3 + \ldots . \check{E}_n}{n} \\
& + a^{-1}\,\frac{\check{E}_1 + a\,\check{E}_2 + a^2\,\check{E}_3 + \ldots . a^{n-1}\,\check{E}_n}{n} \\
& + a^{-2}\,\frac{\check{E}_1 + a^2\,\check{E}_2 + a^4\,\check{E}_3 + \ldots . a^{2(n-1)}\,\check{E}_n}{n} \\
& + a^{-(r-1)}\,\frac{\check{E}_1 + a^{r-1}\,\check{E}_2 + a^{2(r-1)}\,\check{E}_3 + a^{(n-1)(r-1)}\,\check{E}_n}{n} \\
& + a^{-(n-1)}\,\frac{\check{E}_1 + a^{-1}\,\check{E}_2 + a^{-2}\,\check{E}_3 + \ldots . a^{-(n-1)}\,\check{E}_n}{n} \\
& \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
& \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\check{E}_n \equiv & \frac{\check{E}_1 + \check{E}_2 + \check{E}_3 + \ldots . \check{E}_n}{n} \\
& + a^{-(n-1)}\,\frac{\check{E}_1 + a\,\check{E}_2 + a^2\,\check{E}_3 + \ldots . a^{n-1}\,\check{E}_n}{n} \\
& + a^{-2(n-1)}\,\frac{\check{E}_1 + a^2\,\check{E}_2 + a^4\,\check{E}_4 + \ldots . a^{2(n-1)}\,\check{E}_n}{n} \\
& + a^{-(n-1)(r-1)}\,\frac{\check{E}_1 + a^{r-1}\,\check{E}_2 + \ldots . a^{(n-1)(r-1)}\,\check{E}_n}{n} \\
& + a^{-1}\,\frac{\check{E}_1 + a^{-1}\,\check{E}_2 + a^{-2}\,\check{E}_3 + \ldots . a^{-(n-1)}\,\check{E}_n}{n}
\end{aligned}
\right\} \quad (4)
$$

It will be noted that in the expression for $\check{E}_1$ in the above formulas if the first term of each component is taken the result is $n\,\frac{\check{E}_1}{n}$ or $\check{E}_1$. If the succeeding terms of each component involving

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$\check{E}_2\,\check{E}_3 \ldots \check{E}_n$ respectively, are taken separately they add up to expressions of the form $\frac{\check{E}_r}{n}\,(1+a+a^2+\ldots a^{n-1})$ which are all equal to zero since $(1+a+a^2+\ldots a^{n-1})$ is equal to zero. In like manner in the expression for $\check{E}_2\,\check{E}_3 \ldots \check{E}_n$ respectively, all the terms of the components involving each of the quantities $\check{E}_1\,\check{E}_2\,\check{E}_3 \ldots$ etc. excepting the terms involving that one of which the components are to be determined add up to expressions of the form $\frac{\check{E}_r}{n}$ $(1+a+a^2+\ldots a^{n-1})$ all of which are equal to zero, the remaining terms add up to $\check{E}_2\,\check{E}_3 \ldots E_n$ respectively. It will now be apparent that (4), is true whatever may be the nature of $\check{E}_1\,\check{E}_2$ etc., and therefore it is true of all numbers, real complex or imaginary, whatever they may represent and therefore similar relations may be obtained for current vectors and they may be extended to include not only vectors but also the operators.

In order to simplify the expressions which become unwieldy when applied to the general n-phase system, let us consider a three-phase system of vectors $\check{E}_a\,\check{E}_b\,\check{E}_c$. Then we have the following identities:

$$\left.\begin{aligned}
\check{E}_a &\equiv \frac{\check{E}_a+\check{E}_b+\check{E}_c}{3} + \frac{E_a+a\,\check{E}_b+a^2\,E_c}{3} + \frac{\check{E}_a+a^2\,\check{E}_b+a\,\check{E}_c}{3} \\
\check{E}_b &\equiv \frac{\check{E}_a+\check{E}_b+\check{E}_c}{3} + a^2\,\frac{\check{E}_a+a\,\check{E}_b+a^2\,E_c}{3} + a\,\frac{\check{E}_a+a^2\,\check{E}_b+a\,\check{E}_c}{3} \\
\check{E}_c &\equiv \frac{\check{E}_a+\check{E}_b+\check{E}_c}{3} + a\,\frac{\check{E}_a+a\,\check{E}_b+a^2\,\check{E}_c}{3} + a^2\,\frac{\check{E}_a+a^2\,\check{E}_b+a\,\check{E}_c}{3}
\end{aligned}\right\} \quad (5)$$

(4) states the law that a system of n vectors or quantities may be resolved when n is prime into n different symmetrical groups or systems, one of which consists of n equal vectors and the remaining $(n-1)$ systems consist of n equi-spaced vectors which with the first mentioned groups of equal vectors forms

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an equal number of symmetrical n-phase systems. When n is not prime some of the n-phase systems degenerate into repetitions of systems having numbers of phases corresponding to the factors of n.

Equation (5) states that any three vectors $\check{E}_a$ $\check{E}_b$ $\check{E}_c$ may be resolved into a system of three equal vectors $\check{E}_{a0}$ $\check{E}_{a0}$ $\check{E}_{a0}$ and two symmetrical three-phase systems $\check{E}_{a1}$, $a^2\,\check{E}_{a1}$, $a\,\check{E}_{a1}$, and $\check{E}_{a2}$,

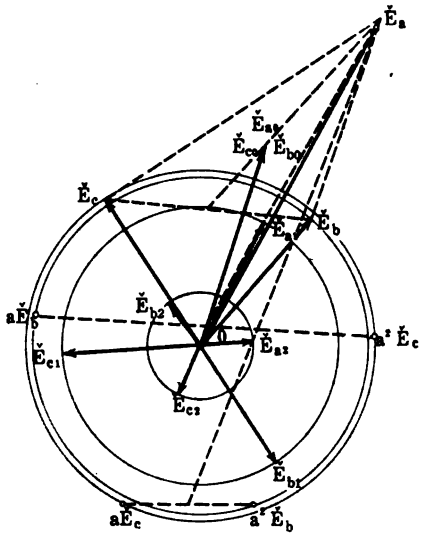

FIG. 1—GRAPHICAL REPRESENTATION OF EQUATION 5.

$a\,\check{E}_{a2}$, $a^2\,\check{E}_{a2}$, the first of positive phase sequence and the second of negative phase sequence, or

$$\left.\begin{aligned} \check{E}_a &= \check{E}_{a0} + \check{E}_{a1} + \check{E}_{a2} \\ \check{E}_b &= \check{E}_{a0} + a^2\,\check{E}_{a1} + a\,\check{E}_{a2} \\ \check{E}_c &= \check{E}_{a0} + a\,\check{E}_{a1} + a^2\,\check{E}_{a2} \end{aligned}\right\} \qquad (6)$$

Similarly

$$\left.\begin{aligned} \check{I}_a &= \check{I}_{a0} + \check{I}_{a1} + \check{I}_{a2} \\ \check{I}_b &= \check{I}_{a0} + a_2\,\check{I}_{a1} + a\,\check{I}_{a2} \\ \check{I}_c &= \check{I}_{a0} + a\,\check{I}_{a1} + a^2\,\check{I}_{a2} \end{aligned}\right\} \qquad (7)$$

Figs. (1) and (2) show a graphical method of resolving three vectors $\check{E}_a$ $\check{E}_b$ and $\check{E}_c$ into their symmetrical three-phase components corresponding to equations (5). The construction is as follows:—$\check{E}_{a0}$ $\check{E}_{b0}$, $\check{E}_{c0}$ are obtained by drawing a line from 0 to

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the centroid of the triangle $\check{E}_a\ \check{E}_b\ \check{E}_c$. $\check{E}_{a1}\ \check{E}_{b1}$, $\check{E}_{c1}$ are obtained by rotating $\check{E}_b$ positively through an angle $\frac{2\pi}{3}$, and $\check{E}_c$ negatively through the same angle giving the points $a\ \check{E}_b$ and $a^2\ \check{E}_c$ respectively. $\check{E}_{a1}$ is the vector obtained by a line drawn from 0 to the centroid of the triangle $\check{E}_a$, $a\ \check{E}_b$, $a^2\ \check{E}_c$; and $\check{E}_{b1}$ and $\check{E}_{c1}$ *lag* this vector by $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ respectively. To obtain $\check{E}_{a2}\ \check{E}_{b2}$ $\check{E}_{c2}$, $\check{E}_b$ is rotated negatively and $\check{E}_c$ positively through the angle $\frac{2\pi}{3}$ giving the points $a_2\ \check{E}_b$ and $a\ \check{E}_c$ respectively; the line drawn from 0 to the centroid of the triangle $\check{E}_a$, $a^2\ \check{E}_b$, $a\ \check{E}_c$ is the

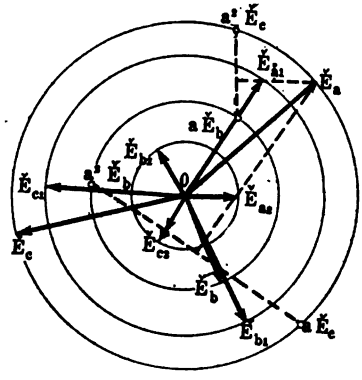

FIG. 2—GRAPHICAL REPRESENTATION OF EQUATION 5.

vector $\check{E}_{a2}$, $\check{E}_{b2}$ and $\check{E}_{c2}$ *lead* this vector by the angles $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ respectively.

The system of operators $Z_{aa}\ Z_{bb}\ Z_{cc}\ Z_{ab}\ Z_{bc}\ Z_{ca}$ may be resolved in a similar manner into symmetrical groups,

$$\left.\begin{aligned} Z_{aa} &= Z_{aa0} + Z_{aa1} + Z_{aa2} \\ Z_{bb} &= Z_{aa0} + a^2\ Z_{aa1} + a\ Z_{aa2} \\ Z_{cc} &= Z_{aa0} + a\ Z_{aa1} + a^2\ Z_{aa2} \end{aligned}\right\} \qquad (8)$$

$$\left.\begin{aligned} Z_{ab} &= Z_{ab0} + Z_{ab1} + Z_{ab2} \\ Z_{bc} &= Z_{ab0} + a^2\ Z_{ab1} + a\ Z_{ab2} \\ Z_{ca} &= Z_{ab0} + a\ Z_{ab1} + a^2\ Z_{ab2} \end{aligned}\right\} \qquad (9)$$

There are similar relations for n-phase systems.

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EXPLANATION OF THEORY AND USE OF SEQUENCE OPERATOR

Let us define the symmetrical sequences of nth roots of unity in the following manner:

$$\left.\begin{array}{l}
S^0 = 1, \quad 1, \quad 1 \ldots 1 \\
S^1 = 1, \quad a^{-1}, \quad a^{-2} \ldots a^{-(n-1)} \\
S^2 = 1, \quad a^{-2}, \quad a^{-4} \ldots a^{-2(n-1)} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
S^r = 1, \quad a^{-r}, \quad a^{-2r} \ldots a^{-(n-1)r} \\
S^{(r+1)} = 1, \; a^{-(r+1)}, \; a^{-2(r+1)} \ldots a^{-(n-1)(r+1)} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
S^{(n-1)} = 1, \; a^{-(n-1)}, \quad a^{-2(n-1)} \ldots a^{-(n-1)^2}
\end{array}\right\} \tag{10}$$

Consider the sequence obtained by the products of similar terms of S^r and S^1. It will be

$$S^{(r+1)} = 1, \; a^{-(r+1)}, \quad a^{-2(r+1)} \ldots a^{-(n-1)(r+1)} \tag{11}$$

Similarly

$$S^k = 1, \quad a^{-k}, \quad a^{-2k} \ldots a^{-(n-1)k} \tag{12}$$

and the sequence obtained by products of like terms of this sequence and S^r is

$$S^{(r+k)} = 1, \; a^{-(r+k)}, \quad a^{-2(r+k)} \ldots a^{-(n-1)(r+k)} \tag{13}$$

We may therefore apply the law of indices to the products of sequences to obtain the resulting sequence.

In the case of the three-phase system we shall have the following sequences only to consider, viz.:

$$\left.\begin{array}{l}
S^0 = 1, \quad 1, \quad 1 \\
S^1 = 1, \quad a^2, \quad a \\
S^2 = 1, \quad a, \quad a^2
\end{array}\right\} \tag{14}$$

The complete system of currents $\check{I}_a$ $\check{I}_b$ $\check{I}_c$ are defined by

$$S\,(\check{I}_a) = S^0\,\check{I}_{a0} + S^1\,\check{I}_{a1} + S^2\,\check{I}_{a2} \tag{15}$$

Similarly the impedances Z_{aa} Z_{bb} Z_{cc} may be expressed in symmetrical form

$$S\,(Z_{aa}) \equiv S^0\,Z_{aa0} + S^1\,Z_{aa1} + S^2\,Z_{aa2} \tag{16}$$

and the mutual impedances Z_{ab}, Z_{bc}, Z_{ca} are expressed by

$$S\,(Z_{ab}) \equiv S^0\,Z_{ab0} + S^1\,Z_{ab1} + S^2\,Z_{ab2} \tag{17}$$

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Attention is called to the importance of preserving the cyclic order of self and mutual impedances, otherwise the rule for the sequence operator will not hold. Thus, Z_{ab}, Z_{bc} and Z_{ca} are in proper sequence as also are Z_{ca}, Z_{ab}, Z_{bc}.

When it is desired to change the first term in the sequence of polyphase vectors the resulting expression will be

$$\left.\begin{aligned} S\,(\check{I}_b) &= S^0\,\check{I}_{a0} + S^1\,a^2\,\check{I}_{a1} + S^2\,a\,\check{I}_{a2} \\ S\,(\check{I}_c) &= S^0\,\check{I}_{a0} + S^1\,a\,\check{I}_{a1} + S^2\,a^2\,\check{I}_{a2} \end{aligned}\right\} \qquad (18)$$

Similarly in the case of the operators $S\,(Z_{ab})$ we have

$$\left.\begin{aligned} S\,(Z_{bc}) &= S^0\,Z_{ab0} + S^1\,a^2\,Z_{ab1} + S^2\,a\,Z_{ab2} \\ S\,(Z_{ca}) &= S^0\,Z_{ab0} + S^1\,a\,Z_{ab1} + S^2\,a^2\,Z_{ab2} \end{aligned}\right\} \qquad (19)$$

Similar rules apply to the e.m.fs. E_a E_b E_c

$$\left.\begin{aligned} S\,(\check{E}_a) &= S^0\,\check{E}_{a0} + S^1\,\check{E}_{a1} + S^2\,\check{E}_{a2} \\ S\,(\check{E}_b) &= S^0\,\check{E}_{a0} + S^1\,a^2\,\check{E}_{a1} + S^2\,a\,\check{E}_{a2} \\ S\,(\check{E}_c) &= S^0\,\check{E}_{a0} + S^1\,a\,E_{a1} + S^2\,a^2\,\check{E}_{a2} \end{aligned}\right\} \qquad (20)$$

It should be kept in mind that any one of the several expressions $S\,(\check{I}_a)$ $S\,(\check{I}_b)$ $S\,(\check{I}_c)$, etc., completely specifies the system, and each of the members of the groups of equations given above is a complete statement of the system of vectors or operators and their relation.

Application to Self and Mutual Impedance Operations

We may now proceed with the current systems $S\,(\check{I}_a)$, $S\,(\check{I}_b)$, $S\,(\check{I}_c)$ and the operating groups $S\,(Z_{aa})$ $S\,(Z_{bb})$ $S\,(Z_{cc})$ etc. and the electromotive forces in exactly the same manner as for simple a-c. circuits. Thus,

$$\begin{aligned} S\,(\check{E}_a) &= S\,(Z_{aa})\,S\,(\check{I}_a) + S\,(Z_{ab})\,S\,(\check{I}_b) + S\,(Z_{ca})\,S\,(\check{I}_c) \qquad (21) \\ &= (S^0\,Z_{aa0} + S^1\,Z_{aa1} + S^2\,Z_{aa2})\,(S^0\,\check{I}_{a0} + S^1\,\check{I}_{a1} + S^2\,I_{a2}) \\ &\quad + (S^0\,Z_{ab0} + S^1\,Z_{ab1} + S^2\,Z_{ab2}) \\ &\qquad\qquad (S^0\,\check{I}_{a0} + S^1\,a^2\,\check{I}_a + S^2\,a\,\check{I}_{a2}) \\ &\quad + (S^0\,Z_{ab0} + S^1\,a\,Z_{ab1} + S^2\,a^2\,Z_{ab2}) \\ &\qquad\qquad (S^0\,\check{I}_{a0} + S^1\,a\,\check{I}_{a1} + S^2\,a^2\,\check{I}_{a2}) \\ &= S^0\,(Z_{aa0} + 2\,Z_{ab0})\,\check{I}_{a0} + S^0\,\{Z_{aa2} + (1 + a^2)\,Z_{ab2}\}\,\check{I}_{a1} \end{aligned}$$

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$$
\begin{aligned}
&+ S^0 \{Z_{aa1} + (1 + a) Z_{ab1}\} \check{I}_{a2} \\
&+ S^1 \{Z_{aa1} + (1 + a) Z_{ab1}\} \check{I}_{a0} \\
&+ S^1 \{Z_{aa0} + (a + a^2) Z_{ab0}\} \check{I}_{a1} \\
&+ S^1 \{Z_{aa2} + 2\, a\, Z_{ab2}\} \check{I}_{a2} \\
&+ S^2 \{Z_{aa2} + (1 + a^2) Z_{ab2}\} \check{I}_{a0} \\
&+ S^2 \{Z_{aa1} + 2\, a^2 Z_{ab1}\} \check{I}_{a1} \\
&+ S^2 \{Z_{aa0} + (a + a^2) Z_{ab0}\} \check{I}_{a2} \qquad (22)
\end{aligned}
$$

Or since $1 + a + a^2 = 0$, $1 + a = -a^2$, $1 + a^2 = -a$ and $a + a^2 = -1$

$$
\begin{aligned}
S(\check{E}_a) = {} & S^0 (Z_{aa0} + 2\, Z_{ab0}) \check{I}_{a0} + S^0 (Z_{aa2} - a\, Z_{ab2}) \check{I}_{a1} \\
& + S^0 (Z_{aa1} - a^2 Z_{ab1}) \check{I}_{a2} + S^1 (Z_{aa1} - a^2 Z_{ab1}) \check{I}_{a0} \\
& + S^1 (Z_{aa0} - Z_{ab0}) \check{I}_{a1} + S^1 (Z_{aa2} + 2\, a\, Z_{ab2}) \check{I}_{a2} \\
& + S^2 (Z_{aa2} - a\, Z_{ab2}) \check{I}_{a0} + S^2 (Z_{aa1} + 2\, a^2 Z_{ab1}) \check{I}_{a1} \\
& + S^2 (Z_{aa0} - Z_{ab0}) \check{I}_{a2} \qquad (23)
\end{aligned}
$$

Or since

$$
\begin{aligned}
S(Z_{bc}) &= S^0 Z_{bc0} + S^1 Z_{bc1} + S^2 Z_{bc2} \\
&= S^0 Z_{ab0} + S^1 a^2 Z_{ab1} + S^2 a\, Z_{ab2}
\end{aligned}
$$

we may write (23) in the form

$$
\begin{aligned}
S(\check{E}_a) = {} & S^0 (Z_{aa0} + 2\, Z_{bc0}) \check{I}_{a0} + S^0 (Z_{aa2} - Z_{bc2}) \check{I}_{a1} \\
& + S^0 (Z_{aa1} - Z_{bc1}) \check{I}_{a2} + S^1 (Z_{aa1} - Z_{bc1}) \check{I}_{a0} \\
& + S^1 (Z_{aa0} - Z_{bc0}) \check{I}_{a1} + S^1 (Z_{aa2} + 2\, Z_{bc2}) \check{I}_{a2} \\
& + S^2 (Z_{aa2} - Z_{bc2}) \check{I}_{a0} + S^2 (Z_{aa1} + 2\, Z_{bc1}) \check{I}_{a1} \\
& + S^2 (Z_{aa0} - Z_{bc0}) \check{I}_{a2} \qquad (24)
\end{aligned}
$$

which is the more symmetrical form. We have therefore from (24) by expressing $S(\check{E}_a)$ in terms of symmetrical co-ordinates the three symmetrical equations

$$
\left.
\begin{aligned}
S^0 \check{E}_{a0} &= S^0 \{(Z_{aa0} + 2\, Z_{bc0}) \check{I}_{a0} + (Z_{aa2} - Z_{bc2}) \check{I}_{a1} \\
&\qquad + (Z_{aa1} - Z_{bc1}) \check{I}_{a2}\} \\
S^1 E_{a1} &= S^1 \{(Z_{aa1} - Z_{bc1}) \check{I}_{a0} + (Z_{aa0} - Z_{bc0}) \check{I}_{a1} \\
&\qquad + (Z_{aa2} + 2\, Z_{bc2}) \check{I}_{a2}\} \\
S^2 E_{a2} &= S^2 \{(Z_{aa2} - Z_{bc2}) \check{I}_{a0} + (Z_{aa1} + 2\, Z_{bc1}) \check{I}_{a1} \\
&\qquad + (Z_{aa0} - Z_{bc0}) \check{I}_{a2}\}
\end{aligned}
\right\} \qquad (25)
$$

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An important case to which we must next give consideration is that of mutual inductance between a primary polyphase circuit and a secondary polyphase circuit. The mutual impedances may be arranged in three sets. Let the currents in the secondary windings be I_u I_v and I_w, we may then express the generalized mutual impedances as follows:

$$\left.\begin{array}{r} \text{(I) } Z_{au}\ Z_{bv}\ Z_{cw} \\ \text{(II) } Z_{bw}\ Z_{cu}\ Z_{av} \\ \text{(III) } Z_{cv}\ Z_{aw}\ Z_{bu} \end{array}\right\} \tag{26}$$

Each set may be resolved into three symmetrical groups, so that

$$\left.\begin{aligned} S\,(Z_{au}) &= S^0\,Z_{au0} + S^1\,Z_{au1} + S^2\,Z_{au2} \\ S\,(Z_{bw}) &= S^0\,Z_{bw0} + S^1\,Z_{bw1} + S^2\,Z_{bw2} \\ S\,(Z_{cv}) &= S^0\,Z_{cv0} + S^1\,Z_{cv1} + S^2\,Z_{cv2} \end{aligned}\right\} \tag{27}$$

and we have for $S\,(\check{E}_a)$ the primary induced e.m.f. due to the secondary currents $S\,(\check{I}_u)$

$$S\,(\check{E}_a) = S\,(Z_{au})\,S\,(\check{I}_u) + S\,(Z_{av})\,S\,(\check{I}_v) + S\,(Z_{aw})\,S\,(\check{I}_w). \tag{28}$$

Substituting for $S\,(\check{I}_u)$, $S\,(\check{I}_v)$ and $S\,(\check{I}_w)$ and $S\,(Z_{au})$, $S\,(Z_{av})$ $S\,(Z_{aw})$ their symmetrical equivalents we have

$$\begin{aligned} S\,(\check{E}_a) = {} & S^0\,(Z_{au0} + Z_{bw0} + Z_{cv0})\,I_{u0} \\ & + S^0\,(Z_{au2} + a\,Z_{bw2} + a^2\,Z_{cv2})\,\check{I}_{u1} \\ & + S^0\,(Z_{au1} + a^2\,Z_{bw1} + a\,Z_{cv1})\,\check{I}_{u2} \\ & + S^1\,(Z_{au1} + a\,Z_{bw1} + a^2\,Z_{cv1})\,\check{I}_{u0} \\ & + S^1\,(Z_{au0} + a^2\,Z_{bw0} + a\,Z_{cv0})\,\check{I}_{u1} \\ & + S^1\,(Z_{au2} + Z_{bw2} + Z_{cv2})\,\check{I}_{u2} \\ & + S^2\,(Z_{au2} + a^2\,Z_{bw2} + a\,Z_{v2})\,\check{I}_{u0} \\ & + S^2\,(Z_{au1} + Z_{bw1} + Z_{cv1})\,\check{I}_{u1} \\ & + S^2\,(Z_{au0} + a\,Z_{bw0} + a^2\,Z_{cv0})\,\check{I}_{u2} \end{aligned} \tag{29}$$

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On expressing $S\ (\check{E}_a)$ in symmetrical form we have the following three symmetrical equations

$$\left.\begin{aligned}
S^0\ \check{E}_{a0} &= S^0\ \{(Z_{au0} + Z_{bw0} + Z_{cv0})\ \check{I}_{u0} \\
&\qquad + (Z_{au2} + a\ Z_{bw2} + a^2\ Z_{cv2})\ \check{I}_{u1} \\
&\qquad + (Z_{au1} + a^2\ Z_{bw1} + a\ Z_{cv1})\ \check{I}_{u2}\} \\
S^1\ \check{E}_{a1} &= S^1\ \{(Z_{au1} + a\ Z_{bw1} + a^2\ Z_{cv1})\ \check{I}_{u0} \\
&\qquad + (Z_{au0} + a^2\ Z_{bw0} + a\ Z_{cv0})\ \check{I}_{u1} \\
&\qquad + (Z_{au2} + Z_{bw2} + Z_{cv2})\ \check{I}_{u2}\} \\
S^2\ E_{a2} &= S^2\ \{(Z_{au2} + a^2\ Z_{bw2} + a\ Z_{cv2})\ \check{I}_u{}^0 \\
&\qquad + (Z_{au1} + Z_{bw1} + Z_{cv1})\ \check{I}_{u1} \\
&\qquad + (Z_{au0} + a\ Z_{bw0} + a^2\ Z_{cv0})\ \check{I}_{u2}\}
\end{aligned}\right\} \quad (30)$$

For the e.m.f. $S\ (\check{E}_u)$ induced in the secondary by the primary currents $S\ (\check{I}_a)$ we have

$$S\ (\check{E}_u) = S\ (Z_{au})\ S\ (\check{I}_a) + S\ (Z_{bu})\ S\ (\check{I}_b) + S\ (Z_{cu})\ S\ (\check{I}_c) \quad (31)$$

Since $S\ (Z_{bu})$ bears the same relation to $S\ (Z_{cv})$ as $S\ (Z_{av})$ does to $S\ (Z_{bw})$ and $S\ (Z_{au})$ bears the same relation to $S\ (Z_{bw})$ as $S\ (Z_{aw})$ does to $S\ (Z_{cv})$ to obtain $S\ (\check{E}_u)$ all that will be necessary will be to interchange Z_{bw} and Z_{cv} in (29) and change $\check{I}_{u0}\ \check{I}_{u1}\ \check{I}_{u2}$ to $\check{I}_{a0}\ I_{a1}$ and $\check{I}_{a2}$ respectively, this gives

$$\begin{aligned}
S\ (\check{E}_u) &= S^0\ (Z_{au0} + Z_{bw0} + Z_{cv0})\ \check{I}_{a0} \\
&\quad + S^0\ (Z_{au2} + a^2\ Z_{bw2} + a\ Z_{cv2})\ \check{I}_{a1} \\
&\quad + S^0\ (Z_{au1} + a\ Z_{bw1} + a^2\ Z_{cv1})\ \check{I}_{a2} \\
&\quad + S^1\ (Z_{au1} + a^2\ Z_{bw1} + a\ Z_{cv1})\ \check{I}_{a0} \\
&\quad + S^1\ (Z_{au0} + a\ Z_{bw0} + a^2\ Z_{cv0})\ \check{I}_{a1} \\
&\quad + S^1\ (Z_{au2} + Z_{bw2} + Z_{cv2})\ \check{I}_{a2} \\
&\quad + S^2\ (Z_{au2} + a\ Z_{bw2} + a^2\ Z_{cv2})\ \check{I}_{a0} \\
&\quad + S^2\ (Z_{au1} + Z_{bw1} + Z_{cv1})\ \check{I}_{a1} \\
&\quad + S^2\ (Z_{au0} + Z_{bw0} + Z_{cv0})\ \check{I}_{a2}
\end{aligned} \quad (32)$$

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and the three symmetrical equations will be

$$\left.\begin{aligned}
S^0 \check{E}_{u0} &= S^0 \{(Z_{au0} + Z_{bw0} + Z_{cv0}) \check{I}_{a0} \\
&\quad + (Z_{au2} + a^2 Z_{bw2} + a Z_{bw2} + a Z_{cv2}) \check{I}_{a1} \\
&\quad + (Z_{au1} + a Z_{bw1} + a^2 Z_{cv1}) \check{I}_{a2}\} \\
S^1 \check{E}_{a1} &= S^1 \{(Z_{au1} + a^2 Z_{bw1} + a Z_{cv1}) \check{I}_{a0} \\
&\quad + (Z_{au0} + a Z_{bw0} + a^2 Z_{cv0}) \check{I}_{a1} \\
&\quad + (Z_{au2} + Z_{bw2} + Z_{cv2}) \check{I}_{a2}\} \\
S^2 \check{E}_{u2} &= S^2 \{(Z_{au2} + a Z_{bw2} + a^2 Z_{cv2}) \check{I}_{a0} \\
&\quad + (Z_{au1} + Z_{bw1} + Z_{cv1}) \check{I}_{a1} \\
&\quad + (Z_{au0} + a^2 Z_{bw0} + a Z_{cv0}) \check{I}_{a2}\}
\end{aligned}\right\} \quad (33)$$

The same methods may be applied to polyphase systems of any number of phases. When the number of phases is not prime the system may sometimes be dealt with as a number of polyphase systems having mutual inductance between them:—For example, a nine-phase system may be treated as three three-phase systems, a twelve-phase system as three four-phase or four three-phase systems. In certain forms of dissymmetry this method is of great practical value, and its application will be taken up later.

For the present part of the paper we shall confine ourselves to the three-phase system, and dissymmetries of several different kinds.

The operators Z_{au} Z_{aa}, etc., must be interpreted in the broadest sense. They may be simple complex quantities or they may be functions of the differential operator $\frac{d}{dt}$. For if

$$i = \Sigma\,(A_n \cos n\,w\,t + B_n \sin n\,w\,t)$$

it may be expressed in the form

$$\left.\begin{aligned}
i &= \Sigma \left(\frac{A_n - j\,B_n}{2}\,\epsilon^{jnwt} + \frac{A_n + j\,B_n}{2}\,\epsilon^{-jnwt}\right) \\
&= \frac{\check{I}}{2} + \frac{\hat{I}}{2} \\
&= \text{real part of } \check{I}
\end{aligned}\right\} \quad (34)$$

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and any linear algebraic operation performed on $\check{I}/2$ will give a result which will be conjugate to that obtained by carrying out the same operation on $\hat{I}/2$ and since the true solution is the sum of these results, it may also be obtained by taking the real part of the result of performing the operation on $\check{I}$.

Modification of the General Case Met With in Practical Networks

Several symmetrical arrangements of the operator Z_{au} etc.' are frequently met with in practical networks which result in a much simpler system of equations than those obtained for the general case as in equations (**29**) to (**33**). Thus for example if all the operators in (**26**) are equal, all the operators in (**27**), except $S^0 Z_{au0}$ $S^0 Z_{bw0}$ and $S^0 Z_{cv0}$ are equal to zero, and these three quantities are also equal to one another so that equation (**30**) becomes

$$\left.\begin{aligned} S^0 \check{E}_{a0} &= S^0 (Z_{au0} + Z_{bw0} + Z_{cv0}) \check{I}_{u0} \\ S^1 \check{E}_{a1} &= 0 \\ S^2 E_{a2} &= 0 \end{aligned}\right\} \qquad (35)$$

and equation (**33**)

$$\left.\begin{aligned} S^0 \check{E}_{u0} &= S^0 (Z_{au0} + Z_{bw0} + Z_{cv0}) \check{I}_{a0} \\ S^1 E_{u1} &= 0 \\ S^2 E_{u2} &= 0 \end{aligned}\right\} \qquad (36)$$

This is the statement in symmetrical co-ordinates that a symmetrically disposed polyphase transmission line will produce no electromagnetic induction in a second similar polyphase system so disposed with respect to the first that mutual inductions between all phases of the two are equal except that due to single-phase currents passing through the conductors.

If in (**26**) the quantities in each group only are equal, equations (**30**) and (**33**) become

$$\left.\begin{aligned} S^0 \check{E}_{a0} &= S^0 (Z_{au0} + Z_{bw0} + Z_{cv0}) \check{I}_{u0} \\ S^1 \check{E}_{a1} &= S^1 (Z_{au0} + a^2 Z_{bw0} + a Z_{cv0}) \check{I}_{u1} \\ S^2 \check{E}_{a2} &= S^2 (Z_{au0} + a Z_{bw0} + a^2 Z_{cv0}) \check{I}_{u2} \end{aligned}\right\} \qquad (37)$$

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$$\left.\begin{aligned} S^0\,\check{E}_{u0} &= S^0\,(Z_{au0} + Z_{bw0} + Z_{cv0})\,\check{I}_{a0} \\ S^1\,\check{E}_{a1} &= S^1\,(Z_{au0} + a\,Z_{bw0} + a^2\,Z_{cv0})\,\check{I}_{a1} \\ S^2\,\check{E}_{a2} &= S^1\,(Z_{au0} + a^2\,Z_{bw0} + a\,Z_{cv0})\,\check{I}_{a2} \end{aligned}\right\} \quad (38)$$

Symmetrical Forms of Common Occurrence

A symmetrical form which is of importance because it is of frequent occurrence in practical polyphase networks has the terms in group (I) equation (**26**) all equal and those in group (II) $\cos\frac{2\pi}{3}$ times those in group (I) and those in group (III) $\cos\frac{4\pi}{3}$ times those in group (I).

Since $\cos\frac{2\pi}{3} = \frac{a+a^2}{2} = \cos\frac{4\pi}{3}$ we have on substituting the values of the impedances in this case,

$$\left.\begin{aligned} S^0\,\check{E}_{a0} &= S^0\,\{Z_{au0}\,(1 + a + a^2)\}\,\check{I}_{u0} = 0 \\ S^1\,\check{E}_{a1} &= S^1\,1\tfrac{1}{2}\,Z_{au0}\,\check{I}_{u1} \\ S^2\,E_{a2} &= S^2\,1\tfrac{1}{2}\,Z_{au0}\,\check{I}_{u2} \end{aligned}\right\} \quad (39)$$

$$\left.\begin{aligned} S^0\,E_{u0} &= S^0\,\{Z_{au0}\,(1 + a + a^2)\}\,\check{I}_{a0} = 0 \\ S^1\,\check{E}_{u1} &= S^1\,1\tfrac{1}{2}\,Z_{au0}\,\check{I}_{a1} \\ S^2\,\check{E}_{u2} &= S^2\,1\tfrac{1}{2}\,Z_{au0}\,\check{I}_{a2} \end{aligned}\right\} \quad (40)$$

The elements in group I may be unequal but groups II and III may be obtained from group I by multiplying by $\cos\frac{4\pi}{3}$ and $\cos\frac{2\pi}{3}$ respectively.

The members of the three groups will then be related as follows, the same sequence being used as before,

$$\left.\begin{aligned} &\text{(I)} \quad Z_{au},\quad Z_{bv},\quad Z_{cw} \\ &\text{(II)} \quad \frac{a+a^2}{2}\,Z_{cw},\quad \frac{a+a^2}{2}\,Z_{au},\quad \frac{a+a^2}{2}\,Z_{bv} \\ &\text{(III)} \quad \frac{a+a^2}{2}\,Z_{bv},\quad \frac{a+a^2}{2}\,Z_{cw},\quad \frac{a+a^2}{2}\,Z_{au} \end{aligned}\right\} \quad (41)$$

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Consequently the following relations are true:

$$\left.\begin{aligned}
S^0 Z_{bw0} &= \frac{a + a^2}{2} S^0 Z_{au0} \\
S^0 Z_{cv0} &= \frac{a + a^2}{2} S^0 Z_{au0} \\
S^1 Z_{bw1} &= \frac{1 + a^2}{2} S^1 Z_{au1} \\
S^2 Z_{bw2} &= \frac{1 + a}{2} S^2 Z_{au2} \\
S^1 Z_{cv1} &= \frac{1 + a}{2} S^1 Z_{au1} \\
S^2 Z_{cv2} &= \frac{1 + a^2}{2} S^2 Z_{au2}
\end{aligned}\right\} \tag{42}$$

Substituting these relations in (30) and (33) we have for this system of mutual impedances

$$\left.\begin{aligned}
Z_{au0} + Z_{bw0} + Z_{cv0} &= 0 \\
Z_{au0} + a\, Z_{bw0} + a^2 Z_{cv0} &= 1\tfrac{1}{2}\, Z_{cv0} \\
Z_{au0} + a^2 Z_{bw0} + a\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{aw0}
\end{aligned}\right\} \tag{43}$$

$$\left.\begin{aligned}
Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \\
Z_{au1} + a\, Z_{bw1} + a^2 Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \\
Z_{au1} + a^2 Z_{bw1} + a\, Z_{cv1} &= 0
\end{aligned}\right\} \tag{44}$$

$$\left.\begin{aligned}
Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2} \\
Z_{au2} + a\, Z_{bw2} + a^2 Z_{cv2} &= 0 \\
Z_{au2} + a^2 Z_{bw2} + a\, Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2}
\end{aligned}\right\} \tag{45}$$

which on substitution in (30) and (33) gives

$$\left.\begin{aligned}
S^0 \check{E}_{a0} &= 0 \\
S^1 \check{E}_{a1} &= S^1 \{1\tfrac{1}{2}\, Z_{au1} \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au0} \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2} \check{I}_{u2}\} \\
S^2 \check{E}_{a2} &= S^2 \{1\tfrac{1}{2}\, Z_{au2} \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au1} \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0} \check{I}_{u2}\}
\end{aligned}\right\} \tag{46}$$

$$\left.\begin{aligned}
S^0 \check{E}_{u0} &= S^0 \{1\tfrac{1}{2}\, Z_{au2} \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au1} \check{I}_{a2}\} \\
S^1 \check{E}_{u1} &= S^1 \{1\tfrac{1}{2}\, Z_{au0} \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au2} \check{I}_{a2}\} \\
S^2 \check{E}_{u2} &= S^2 \{1\tfrac{1}{2}\, Z_{au1} \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au0} \check{I}_{a2}\}
\end{aligned}\right\} \tag{47}$$

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The above symmetrical forms in which the factors $\cos \frac{2\pi}{3}$ and $\cos \frac{4\pi}{3}$ occur apply particularly to electromagnetic induction between windings distributed over the surfaces of co-axial cylinders; where if the plane of symmetry of one winding be taken as the datum plane, the mutual impedance between this winding and any other is a harmonic function of the angle between its plane of symmetry and the datum plane. In other words, the mutual impedances are functions of position on the circumference of a circle and may therefore be expanded by Fourier's theorem in a series of integral harmonics of the angle made by the planes of symmetry with the datum plane. Since the same procedure applies to all the terms of the expansion it is necessary only to consider the simple harmonic case. In the partially symmetrical cases of mutual induction, such as that taken up in the preceding discussion, there will be a difference between two possible cases, viz:—Symmetrical primary, unsymmetrical secondary, which is the case just considered, and unsymmetrical primary and symmetrical secondary in which the impedances of (**26**) will have the following values

$$\left.\begin{array}{lll} \text{(I)} & Z_{au}, \quad Z_{bc}, \quad Z_{cw} & \\ \text{(II)} & \frac{a+a^2}{2} Z_{bc}, & \frac{a+a^2}{2} Z_{cw}, \quad \frac{a+a^2}{2} Z_{au} \\ \text{(III)} & \frac{a+a^2}{2} Z_{cw}, & \frac{a+a^2}{2} Z_{au}, \quad \frac{a+a^2}{2} Z_{bc} \end{array}\right\} \qquad (48)$$

The results may be immediately set down by symmetry from equations (**46**) and (**47**), but the difference between the two cases will be better appreciated by setting down the component symmetrical impedances, thus we have

$$\left.\begin{array}{l} S^0 Z_{bw0} = \frac{a+a^2}{2} S^0 Z_{au0} \\ S^0 Z_{cv0} = \frac{a+a^2}{2} S^0 Z_{au0} \\ S^1 Z_{bw1} = \frac{1+a}{2} S^1 Z_{au1} \\ S^2 Z_{bw2} = \frac{1+a^2}{2} S^2 Z_{au2} \\ S^1 Z_{cv1} = \frac{1+a^2}{2} S^1 Z_{au1} \\ S^2 Z_{cv2} = \frac{1+a}{2} S^2 Z_{au2} \end{array}\right\} \qquad (49)$$

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Substituting these relations in the impedances used in (30) and (33) they become

$$\left.\begin{aligned} Z_{au0} + Z_{bw0} + Z_{cv0} &= 0 \\ Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0} \\ Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0} \end{aligned}\right\} \quad (50)$$

$$\left.\begin{aligned} Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \\ Z_{au1} + a\, Z_{bw1} + a^2\, Z_{cv1} &= 0 \\ Z_{au1} + a^2\, Z_{bw1} + a\, Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1} \end{aligned}\right\} \quad (51)$$

$$\left.\begin{aligned} Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2} \\ Z_{au2} + a\, Z_{bw2} + a^2\, Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2} \\ Z_{au2} + a^2\, Z_{bw2} + a\, Z_{cv2} &= 0 \end{aligned}\right\} \quad (52)$$

And we have from (30) and (33), or by symmetry

$$\left.\begin{aligned} S^0\, \check{E}_{a0} &= S^0\, \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u2}\} \\ S^1\, \check{E}_{a1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u2}\} \\ S^2\, \check{E}_{a2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2} \end{aligned}\right\} \quad (53)$$

$$\left.\begin{aligned} S^0\, \check{E}_{u0} &= 0 \\ S^1\, \check{E}_{u1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{a0} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{u2}\} \\ S^2\, \check{E}_{u2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au2}\, \check{I}_{a0} + 1\tfrac{1}{2}\, Z_{au1}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \check{I}_{u2}\} \end{aligned}\right\} \quad (54)$$

If the angle between the planes of symmetry of the coils and the datum plane are subject to changes, $\cos \frac{2\pi}{3}$ and $\cos \frac{4\pi}{3}$ in the preceding discussion must be replaced by

$$\left.\begin{aligned} \cos\left(\frac{2\pi}{3} + \theta\right) &= \frac{a}{2}\, \epsilon^{j\theta} + \frac{a^2}{2}\, \epsilon^{-j\theta} \\ \cos\left(\frac{4\pi}{3} + \theta\right) &= \frac{a}{2}\, \epsilon^{-j\theta} + \frac{a^2}{2}\, \epsilon^{j\theta} \end{aligned}\right\} \quad (55)$$

where θ is measured from the datum plane

In the strictly symmetrical case of co-axial cylindrical surface windings in which the members of each group of mutual

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impedances are equal, the result of substituting (**55**) in the equations for induced e.m.f. will be

$$\left.\begin{aligned} S^0\,\check{E}_{a0} &= 0 \\ S^1\,\check{E}_{a1} &= S^1\,(1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{j\theta}\;\check{I}_{u1}) \\ S^2\,\check{E}_{a2} &= S^2\,(1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{-j\theta}\,\check{I}_{u2}) \end{aligned}\right\} \qquad (56)$$

$$\left.\begin{aligned} S^0\,\check{E}_{u0} &= 0 \\ S^1\,\check{E}_{u1} &= S^1\,(1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{-j\theta}\,\check{I}_{a1}) \\ S^2\,\check{E}_{u2} &= S^2\,(1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{j\theta}\;\check{I}_{a2}) \end{aligned}\right\} \qquad (57)$$

In the case having symmetrical primary and unsymmetrical secondary in which members of each group are different, but in which there are harmonic relations between corresponding members of the different groups, the impedances are

$$\left.\begin{aligned} &\text{(I) } Z_{au},\quad Z_{bv},\quad Z_{cw} \\ &\text{(II) } \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) Z_{cw} \\ &\left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\epsilon^{-j\theta}\right)\,Z_{au},\ \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) Z_{bv} \\ &\text{(III) } \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\,\epsilon^{-j\theta}\right) Z_{bv}, \\ &\left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\,\epsilon^{-j\theta}\right) Z_{cw},\ \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\epsilon^{-j\theta}\right) Z_{au} \end{aligned}\right\} \qquad (58)$$

The symmetrical component mutual impedances will have the following values in terms of $Z_{au0}\ Z_{au1}\ Z_{au2}$

$$\left.\begin{aligned} S^0\,Z_{bw0} &= \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) S^0\,Z_{au0} \\ S^0\,Z_{cv0} &= \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\,\epsilon^{-j\theta}\right) S^0\,Z_{au0} \\ S^1\,Z_{bw1} &= \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{\epsilon^{-j\theta}}{2}\right) S^1\,Z_{au1} \\ S^2\,Z_{bw2} &= \left(\frac{\epsilon^{j\theta}}{2} + \frac{a}{2}\epsilon^{-j\theta}\right) S^2\,Z_{au2} \\ S^1\,Z_{cv1} &= \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{\epsilon^{-j\theta}}{2}\right) S^1\,Z_{au1} \\ S^2\,Z_{cv2} &= \left(\frac{\epsilon^{j\theta}}{2} + \frac{a^2}{2}\epsilon^{-j\theta}\right) S^2\,Z_{au2} \end{aligned}\right\} \qquad (59)$$

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Substituting these relations in the impedances of equations (30) and (33) they become

$$\left.\begin{aligned} Z_{au0} + Z_{bw0} + Z_{cv0} &= 0 \\ Z_{au0} + a\, Z_{bw0} + a^2\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{-j\theta} \\ Z_{au0} + a^2\, Z_{bw0} + a\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{j\theta} \end{aligned}\right\} \tag{60}$$

$$\left.\begin{aligned} Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{-j\theta} \\ Z_{au1} + a\, Z_{bw1} + a^2\, Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{j\theta} \\ Z_{au1} + a^2\, Z_{bw1} + a\, Z_{cv1} &= 0 \end{aligned}\right\} \tag{61}$$

$$\left.\begin{aligned} Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{j\theta} \\ Z_{au2} + a\, Z_{bw2} + a^2\, Z_{cv2} &= 0 \\ Z_{au2} + a^2\, Z_{bw2} + a\, Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au}\, \epsilon^{-j\theta} \end{aligned}\right\} \tag{62}$$

which on substitution in (30) and (33) give

$$\left.\begin{aligned} S^0\, \check{E}_{a0} &= 0 \\ S^1\, \check{E}_{a1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{j\theta}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{j\theta}\, \check{I}_{u1} \\ &\qquad + 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{j\theta}\, \check{I}_{u2}\} \\ S^2\, \check{E}_{a2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{-j\theta}\, \check{I}_{u0} + 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{-j\theta}\, \check{I}_{u1} \\ &\qquad + 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{-j\theta}\, \check{I}_{u2}\} \end{aligned}\right\} \tag{63}$$

$$\left.\begin{aligned} S^0\, \check{E}_{u0} &= S^0\, \{1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{-j\theta}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{j\theta}\, \check{I}_{a2}\} \\ S^1\, \check{E}_{u1} &= S^1\, \{1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{-j\theta}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{j\theta}\, \check{I}_{a2}\} \\ S^2\, \check{E}_{u2} &= S^2\, \{1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{-j\theta}\, \check{I}_{a1} + 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{j\theta}\, \check{I}_{a2}\} \end{aligned}\right\} \tag{64}$$

In the case of unsymmetrical primary and symmetrical secondary, we have for the value of the impedance in terms of Z_{au0} Z_{au1} and Z_{au2}

$$\left.\begin{aligned} &\text{(I)}\ Z_{au},\quad Z_{bv},\quad Z_{cw} \\ &\text{(II)}\ \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) Z_{bc}, \\ &\left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) Z_{cw},\ \left(\frac{a}{2}\,\epsilon^{j\theta} + \frac{a^2}{2}\,\epsilon^{-j\theta}\right) Z_{au} \\ &\text{(III)}\ \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\,\epsilon^{-j\theta}\right) Z_{cw}, \\ &\left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\,\epsilon^{-j\theta}\right) Z_{au},\ \left(\frac{a^2}{2}\,\epsilon^{j\theta} + \frac{a}{2}\,\epsilon^{-j\theta}\right) Z_{bc} \end{aligned}\right\} \tag{65}$$

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The symmetrical component mutual impedances in terms of Z_{au0}, Z_{au1}, Z_{au2} are

$$\left.\begin{aligned}
S^0 Z_{bw0} &= \left(\frac{a}{2}\epsilon^{j\theta} + \frac{a^2}{2}\epsilon^{-j\theta}\right) S^0 Z_{au0} \\
S^0 Z_{cv0} &= \left(\frac{a^2}{2}\epsilon^{j\theta} + \frac{a}{2}\epsilon^{-j\theta}\right) S^0 Z_{au0} \\
S^1 Z_{bw1} &= \left(\frac{\epsilon^{j\theta}}{2} + \frac{a}{2}\epsilon^{-j\theta}\right) S^1 Z_{au1} \\
S^2 Z_{bw2} &= \left(\frac{a^2}{2}\epsilon^{j\theta} + \frac{\epsilon^{-j\theta}}{2}\right) S^2 Z_{au2} \\
S^1 Z_{cv1} &= \left(\frac{\epsilon^{j\theta}}{2} + \frac{a^2}{2}\epsilon^{-j\theta}\right) S^1 Z_{au1} \\
S^2 Z_{cv2} &= \left(\frac{a}{2}\epsilon^{j\theta} + \frac{\epsilon^{-j\theta}}{2}\right) S^2 Z_{au2}
\end{aligned}\right\} \quad (66)$$

And the impedances of equations (**30**) and (**33**) become

$$\left.\begin{aligned}
Z_{au0} + Z_{bw0} + Z_{cv0} &= 0 \\
Z_{au0} + a\, Z_{bw0} + a^2 Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{-j\theta} \\
Z_{au0} + a^2 Z_{bw0} + a\, Z_{cv0} &= 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{j\theta}
\end{aligned}\right\} \quad (67)$$

$$\left.\begin{aligned}
Z_{au1} + Z_{bw1} + Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{j\theta} \\
Z_{au1} + a\, Z_{bw1} + a^2 Z_{cv1} &= 0 \\
Z_{au1} + a^2 Z_{bw1} + a\, Z_{cv1} &= 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{-j\theta}
\end{aligned}\right\} \quad (68)$$

$$\left.\begin{aligned}
Z_{au2} + Z_{bw2} + Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{-j\theta} \\
Z_{au2} + a\, Z_{bw2} + a^2 Z_{cv2} &= 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{j\theta} \\
Z_{au2} + a^2 Z_{bw} + a\, Z_{cv2} &= 0
\end{aligned}\right\} \quad (69)$$

And on substitution in (**30**) and (**33**), or by symmetry from (**63**) and (**64**), we have

$$\left.\begin{aligned}
S^0 \check{E}_{a0} &= S^0 \{1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{j\theta}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{-j\theta}\, \check{I}_{u2}\} \\
S^1 \check{E}_{a1} &= S^1 \{1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{j\theta}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au2}\, \epsilon^{-j\theta}\, \check{I}_{u2}\} \\
S^2 \check{E}_{a2} &= S^2 \{1\tfrac{1}{2}\, Z_{au1}\, \epsilon^{j\theta}\, \check{I}_{u1} + 1\tfrac{1}{2}\, Z_{au0}\, \epsilon^{-j\theta}\, \check{I}_{u2}
\end{aligned}\right\} \quad (70)$$

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$$\left.\begin{aligned}
S^0\,\check{E}_{u0} &= 0 \\
S^1\,\check{E}_{u1} &= S^1\,\{1\tfrac{1}{2}\,Z_{au1}\,\epsilon^{-j\theta}\,\check{I}_{a0} + 1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{-j\theta}\,\check{I}_{u1} \\
&\qquad + 1\tfrac{1}{2}\,Z_{au2}\,\epsilon^{-j\theta}\,\check{I}_{a2}\} \\
S^2\,\check{E}_{u2} &= S^2\,\{1\tfrac{1}{2}\,Z_{au2}\,\epsilon^{j\theta}\,\check{I}_{a0} + 1\tfrac{1}{2}\,Z_{au1}\,\epsilon^{j\theta}\,\check{I}_{a1} \\
&\qquad + 1\tfrac{1}{2}\,Z_{au0}\,\epsilon^{j\theta}\,\check{I}_{a2}\}
\end{aligned}\right\} \tag{71}$$

A fuller discussion of self and mutual impedances of co-axial cylindrical windings will be found in the Appendix. It will be sufficient to note here that in the case of self inductance and mutual inductance of stationary windings symmetrically disposed if they are equal

$$\left.\begin{aligned}
M_{ab} = M_{bc} = M_{ca} &= \Sigma\left(A_n \cos\frac{2\,n\,\pi}{3}\right) \\
L_{aa} = L_{bb} = L_{cc} = M_{aa} = M_{bb} = M_{cc} &= \Sigma\,A_n
\end{aligned}\right\} \tag{72}$$

If the windings are symmetrically disposed but have different number of turns

$$\left.\begin{aligned}
L_{aa} &= M_{aa} = \Sigma\,A_n \\
L_{bb} &= M_{bb} = \Sigma\,B_n \\
L_{cc} &= M_{cc} = \Sigma\,C_n
\end{aligned}\right\} \tag{73}$$

$$\left.\begin{aligned}
M_{ab} &= \Sigma\left(\sqrt{A_n\,B_n}\cos\frac{2\,n\,\pi}{3}\right) \\
M_{bc} &= \Sigma\left(\sqrt{B_n\,C_n}\cos\frac{2\,n\,\pi}{3}\right) \\
M_{ca} &= \Sigma\left(\sqrt{C_n\,A_n}\cos\frac{2\,n\,\pi}{3}\right)
\end{aligned}\right\} \tag{74}$$

If the coils are alike but unsymmetrically spaced $L_{aa}\,L_{bb}\,L_{cc}$ have the same values, namely $\Sigma\,A_n$ and

$$\left.\begin{aligned}
M_{ab} &= \Sigma\left\{(A_n \cos n\,\theta_1)\cos\frac{2\,n\,\pi}{3} + (A_n \sin n\,\theta_1)\sin\frac{2\,n\,\pi}{3}\right\} \\
M_{bc} &= \Sigma\left\{(A_n \cos n\,\theta_2)\cos\frac{2\,n\,\pi}{3} + (A_n \sin n\,\theta_2)\sin\frac{2\,n\,\pi}{3}\right\} \\
M_{ca} &= \Sigma\left\{(A_n \cos n\,\theta_3)\cos\frac{2\,n\,\pi}{3} + (A_n \sin n\,\theta_3)\sin\frac{2\,n\,\pi}{3}\right\}
\end{aligned}\right\} \tag{75}$$

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If they are unequal as well as unsymmetrically disposed but are otherwise similar L_{aa} L_{bb} L_{cc} have values as in (**64**) and

$$\left.\begin{aligned}
M_{ab} &= \Sigma \left\{ (\sqrt{A_n B_n} \cos n\,\theta_1) \cos \frac{2\,n\,\pi}{3} \right. \\
&\qquad \left. + (\sqrt{A_n B_n} \sin n\,\theta_1) \sin \frac{2\,n\,\pi}{3} \right\} \\
M_{bc} &= \Sigma \left\{ (\sqrt{B_n C_n} \cos n\,\theta_2) \cos \frac{2\,n\,\pi}{3} \right. \\
&\qquad \left. + (\sqrt{B_n C_n} \sin n\,\theta_2) \sin \frac{2\,n\,\pi}{3} \right\} \\
M_{ca} &= \Sigma \left\{ (\sqrt{C_n A_n} \cos n\,\theta_1) \cos \frac{2\,n\,\pi}{3} \right. \\
&\qquad \left. + (\sqrt{C_n A_n} \sin n\,\theta_3) \sin \frac{2\,n\,\pi}{3} \right\}
\end{aligned}\right\} \qquad (76)$$

Where the windings are dissimilar in every respect the expressions become more complicated. A short outline of this subject is given in the Appendix.

In the case of mutual inductance between two coaxial cylindrical systems, one of which A, B, C is the primary and the other U, V, W the secondary, the following conventions should be followed:

(a) All angles are measured, taking the primary planes of symmetry as data in a positive direction.

(b) The datum plane for all windings is the plane of symmetry of the primary A phase.

(c) All mechanical motions unless otherwise stated shall be considered as positive rotations of the secondary cylinder about its axis.

(d) The conventional disposition of the phases and the direction of rotation of the secondary winding are indicated in Fig. 3.

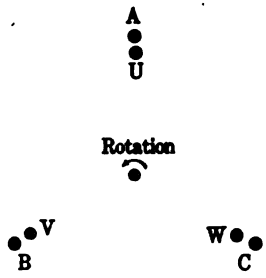

FIG. 3—CONVENTIONAL DISPOSITION OF PHASES AND DIRECTION OF ROTATION.

We shall consider five cases; Case 1 being the completely symmetrical case and the rest being symmetrical in one winding, the other winding being unsymmetrical in magnitude and phase, or both, but all windings having the same form and distribution of coils.

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Case I. All Windings Symmetrical.

$$\left.\begin{aligned} M_{au} &= M_{bv} = M_{cw} = \Sigma A_n \cos n\theta \\ M_{bw} &= M_{cu} = M_{av} = \Sigma A_n \cos n\left(\frac{2\pi}{3} + \theta\right) \\ M_{cv} &= M_{aw} = M_{bu} = \Sigma A_n \cos n\left(\frac{4\pi}{3} + \theta\right) \end{aligned}\right\} \quad (77)$$

Case II. Primary Windings equal and Symmetrical, Secondary Windings unequal but otherwise Symmetrical.

$$\left.\begin{aligned} M_{au} &= \Sigma A_n \cos n\theta, \quad M_{bv} = \Sigma B_n \cos n\theta, \\ &\qquad\qquad M_{cw} = \Sigma C_n \cos n\theta \\ M_{bw} &= \Sigma C_n \cos n\left(\frac{2\pi}{3} + \theta\right), \\ M_{cu} &= \Sigma A_n \cos n\left(\frac{2\pi}{3} + \theta\right), \\ M_{av} &= \Sigma B_n \cos n\left(\frac{2\pi}{3} + \theta\right) \\ M_{cv} &= \Sigma B_n \cos n\left(\frac{4\pi}{3} + \theta\right), \\ M_{aw} &= \Sigma C_n \cos n\left(\frac{4\pi}{3} + \theta\right), \\ M_{bu} &= \Sigma A_n \cos n\left(\frac{4\pi}{3} + \theta\right) \end{aligned}\right\} \quad (78)$$

Case III. Primary Windings Unequal but Otherwise Symmetrical, Secondary Winding Equal and Symmetrical.

$$\left.\begin{aligned} M_{au} &= \Sigma A_n \cos n\theta, M_{bv} = \Sigma B_n \cos n\theta, M_{cw} = \Sigma C_n \cos n\theta \\ M_{bw} &= \Sigma B_n \cos n\left(\frac{2\pi}{3} + \theta\right), \\ M_{cu} &= \Sigma C_n \cos n\left(\frac{2\pi}{3} + \theta\right) \\ M_{av} &= \Sigma A_n \cos n\left(\frac{2\pi}{3} + \theta\right) \\ M_{cv} &= \Sigma C_n \cos n\left(\frac{4\pi}{3} + \theta\right), \\ M_{aw} &= \Sigma A_n \cos n\left(\frac{4\pi}{3} + \theta\right), \\ M_{bu} &= \Sigma B_n \cos n\left(\frac{4\pi}{3} + \theta\right) \end{aligned}\right\} \quad (79)$$

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Case IV. Same as Case II except in addition to inequality Secondary Windings are Displaced from Symmetry by angles α_1 α_2 and α_3 whose sum is zero.

$$\left.\begin{aligned}
M_{au} &= \Sigma\,(A_n \cos\alpha_1 \cos n\,\theta + A_n \sin\alpha_1 \sin n\,\theta)\\
M_{bv} &= \Sigma\,(B_n \cos\alpha_2 \cos n\,\theta + B_n \sin\alpha_2 \sin n\,\theta)\\
M_{cw} &= \Sigma\,(C_n \cos\alpha_3 \cos n\,\theta + C_n \sin\alpha_3 \sin n\,\theta)\\
M_{bw} &= \Sigma\left\{C_n \cos\alpha_3 \cos n\left(\frac{2\pi}{3}+\theta\right)\right.\\
&\qquad \left. + C_n \sin\alpha_3 \sin n\left(\frac{2\pi}{3}+\theta\right)\right\}\\
M_{cu} &= \Sigma\left\{A_n \cos\alpha_1 \cos n\left(\frac{2\pi}{3}+\theta\right)\right.\\
&\qquad \left. + A_n \sin\alpha_1 \sin n\left(\frac{2\pi}{3}+\theta\right)\right\}\\
M_{av} &= \Sigma\left\{B_n \cos\alpha_2 \cos n\left(\frac{2\pi}{3}+\theta\right)\right.\\
&\qquad \left. + B_n \sin\alpha_2 \sin n\left(\frac{2\pi}{3}+\theta\right)\right\}\\
M_{cv} &= \Sigma\left\{B_n \cos\alpha_2 \cos n\left(\frac{4\pi}{3}+\theta\right)\right.\\
&\qquad \left. + B_n \sin\alpha_2 \sin n\left(\frac{4\pi}{3}+\theta\right)\right\}\\
M_{aw} &= \Sigma\left\{C_n \cos\alpha_3 \cos n\left(\frac{4\pi}{3}+\theta\right)\right.\\
&\qquad \left. + C_n \sin\alpha_3 \sin n\left(\frac{4\pi}{3}+\theta\right)\right\}\\
M_{bu} &= \Sigma\left\{A_n \cos\alpha_1 \cos n\left(\frac{4\pi}{3}+\theta\right)\right.\\
&\qquad \left. + A_n \sin\alpha_1 \sin n\left(\frac{4\pi}{3}+\theta\right)\right\}
\end{aligned}\right\} \quad (80)$$

Case V. Same as Case III except that the Primary Windings are Unsymmetrically disposed with respect to one another as well as being unequal.

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$$
\left.
\begin{aligned}
M_{au} &= \Sigma\,(A_n \cos \alpha_1 \cos n\,\theta + A_n \sin \alpha_1 \sin n\,\theta) \\
M_{bv} &= \Sigma\,(B_n \cos \alpha_2 \cos n\,\theta + B_n \sin \alpha_2 \sin n\,\theta) \\
M_{cw} &= \Sigma\,(C_n \cos \alpha_3 \cos n\,\theta + C_n \sin \alpha_3 \sin n\,\theta) \\
M_{bw} &= \Sigma \left\{ B_n \cos \alpha_2 \cos n \left(\frac{2\pi}{3} + \theta\right) \right. \\
&\qquad \left. + B_n \sin \alpha_2 \sin n \left(\frac{2\pi}{3} + \theta\right) \right\} \\
M_{cu} &= \Sigma \left\{ A_n \cos \alpha_1 \cos n \left(\frac{2\pi}{3} + \theta\right) \right. \\
&\qquad \left. + A_n \sin \alpha_1 \sin n \left(\frac{2\pi}{3} + \theta\right) \right\} \\
M_{av} &= \Sigma \left\{ C_n \cos \alpha_3 \cos n \left(\frac{2\pi}{3} + \theta\right) \right. \\
&\qquad \left. + C_n \sin \alpha_3 \sin n \left(\frac{2\pi}{3} + \theta\right) \right\} \\
M_{cv} &= \Sigma \left\{ C_n \cos \alpha_3 \cos n \left(\frac{4\pi}{3} + \theta\right) \right. \\
&\qquad \left. + C_n \sin \alpha_3 \sin n \left(\frac{4\pi}{3} + \theta\right) \right\} \\
M_{aw} &= \Sigma \left\{ A_n \cos \alpha_1 \cos n \left(\frac{4\pi}{3} + \theta\right) \right. \\
&\qquad \left. + A_n \sin \alpha_1 \sin n \left(\frac{4\pi}{3} + \theta\right) \right\} \\
M_{bu} &= \Sigma \left\{ B_n \cos \alpha_2 \cos n \left(\frac{4\pi}{3} + \theta\right) \right. \\
&\qquad \left. + B_n \sin \alpha_2 \sin n \left(\frac{4\pi}{3} + \theta\right) \right\}
\end{aligned}
\right\} \quad (81)
$$

The expressions for dissymmetry in both windings and for unsymmetrically wound coils, etc., are more complicated and will be dealt with in the Appendix.

The impedances Z_{aa} Z_{bb}, etc., Z_{au} Z_{bv}, etc., are functions of M_{aa} M_{bb}, etc., M_{au} M_{bv}, etc., and the resistances of the system. The component of e. m. f. proportional to the current due to

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mutual impedance is so small that it may generally be neglected so that Z_{au} becomes $\frac{d}{dt} M_{au}$, $Z_{bv} = \frac{d}{dt} M_{bv}$ and so forth.

If the secondary winding is rotating at an angular velocity α, θ in equation (**55**) becomes αt and the operators Z_{aa}, etc. operate on such products as $\epsilon^{j\alpha t} \check{I}_{u1} \quad \epsilon^{j\alpha t} \check{I}_{u2}$ where $\check{I}_{u1}$ and $\check{I}_{u2}$ are the variables.

The following relations will be found useful in the application of the method in actual examples.

If D denotes the operator $\frac{d}{dx}$ and $\varphi(Z)$ is a rational algebraic function of Z

$$\left.\begin{aligned} \varphi(D)\, e^{ax} &= \varphi(a)\, e^{ax} \\ \varphi(D)\, \{e^{ax}\ X\} &= e^{ax}\ \varphi(D+a)\, X \\ \varphi(D)\, Y &= e^{ax} \varphi(D+a)\, Y\, e^{-ax} \end{aligned}\right\} \quad (82)$$

Where X and Y may be any function of x.

Star and Delta e.m.fs. and Currents in Terms of Symmetrical Components

It has been shown in the preceding portion of this paper that the e. m. fs. $\check{E}_a$ $\check{E}_b$ and $\check{E}_c$ and the currents $\check{I}_a$ $\check{I}_b$ and $\check{I}_c$ whatever their distortion, may be represented by the sum of symmetrical systems of e. m. fs. or currents so that the two expressions

$$\left.\begin{aligned} S(\check{E}_a) &= S^0\, \check{E}_{a0} + S^1\, \check{E}_{a1} + S^2\, \check{E}_{a2} \\ S(\check{I}_a) &= S^0\, \check{I}_{a0} + S^1\, \check{I}_{a1} + S^2\, \check{I}_{a2} \end{aligned}\right\} \quad (83)$$

completely define these two systems.

If we take the delta e. m. fs. and currents corresponding to $S^0\, \check{E}_{a0}$, $S^1\, \check{E}_{a1}$ and $S^2\, \check{E}_{a2}$, $S^1\, \check{I}_{a1}$, $S^2\, \check{I}_{a2}$, we have, since $\check{E}_{bc1}$ leads $\check{E}_{a1}$ by $\frac{\pi}{2}$ and $\check{E}_{bc2}$ lags behind $\check{E}_{a2}$ by the same angle

$$\left.\begin{aligned} S^0\, \check{E}_{bc0} &= 0 \\ S^1\, \check{E}_{bc1} &= j\sqrt{3}\, S^1\, \check{E}_{a1} \\ S^2\, \check{E}_{bc2} &= -j\sqrt{3}\, S^2\, \check{E}_{a2} \\ S^0\, \check{I}_{bc0} &= \text{indeterminate from } S(\check{I}_a) \\ S^1\, \check{I}_{bc1} &= j\, \frac{1}{\sqrt{3}}\, S^1\, \check{I}_{a1} \\ S^2\, \check{I}_{bc2} &= -j\, \frac{1}{\sqrt{3}}\, S^2\, \check{I}_{a2} \end{aligned}\right\} \quad (84)$$

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And therefore if we take $\check{E}_{ab}$ as the principal vector

$$\left.\begin{aligned} S^0\,\check{E}_{ab0} &= 0 \\ S^1\,E_{ab1} &= \quad j\,a\,s^1\,\sqrt{3}\,\check{E}_{a1} \\ S^2\,\check{E}_{ab2} &= -j\,a^2\,s^2\,\sqrt{3}\,\check{E}_{a2} \\ S\,(\check{E}_{ab}) &= S^1\,\check{E}_{ab1} + S^2\,\check{E}_{ab2} \end{aligned}\right\} \quad (85)$$

The last equation of group (85) when expanded gives

$$\left.\begin{aligned} \check{E}_{ab} &= j\,\sqrt{3}\,(a\,\check{E}_{a1} - a^2\,\check{E}_{a2}) \\ \check{E}_{bc} &= j\,\sqrt{3}\,(\check{E}_{a1} - \check{E}_{a2}) \\ \check{E}_{ca} &= j\,\sqrt{3}\,(a^2\,\check{E}_{a1} - a\,\check{E}_{a2}) \end{aligned}\right\} \quad (86)$$

which may also be obtained direct from (83) by means of the relations

$$\check{E}_{ab} = \check{E}_b - \check{E}_a$$

$$\check{E}_{bc} = \check{E}_c - \check{E}_b$$

$$\check{E}_{ca} = \check{E}_a - \check{E}_0$$

Similarly

$$S^0\,\check{I}_{ab} = \text{indeterminate from } S\,(\check{I}_a)$$

$$S^1\,\check{I}_{ab1} = j\,a\,\frac{1}{\sqrt{3}}\,\check{I}_{a1}$$

$$S^2\,\check{I}_{ab2} = j\,a^2\,\frac{1}{\sqrt{3}}\,\check{I}_{a2} \quad (87)$$

$$S\,(\check{I}_{ab}) = S^0\,\check{I}_{ab0} + S^1\,I_{ab1} + S^2\,\check{I}_{ab2}$$

with similar expression for $\check{I}_{ab}$ $\check{I}_{bc}$ and $\check{I}_{ca}$ which may be verified by means of the relations

$$\check{I}_a = \check{I}_{ca} - \check{I}_{ab} + \check{I}_{a0}$$

$$\check{I}_b = \check{I}_{ab} - \check{I}_{bc} + \check{I}_{a0}$$

$$\check{I}_c = \check{I}_{bc} - \check{I}_{ca} + \check{I}_{a0}$$

Conversely to (84) we have the following relations

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$$\left.\begin{aligned}
S^0\,\check{E}_{a0} &= \text{indeterminate from } S\,(\check{E}_{ab}) \\
S^1\,\check{E}_{a1} &= -\,j\frac{1}{\sqrt{3}}\,S^1\,\check{E}_{bc1} = -j\,\frac{a^2}{\sqrt{3}}\,S^1\,\check{E}_{ab1} \\
S^2\,\check{E}_{a2} &= \quad j\,\frac{1}{\sqrt{3}}\,S^2\,E_{bc2} = \quad j\,\frac{a}{\sqrt{3}}\,S^2\,E_{ab2} \\
S^0\,\check{I}_{a0} &= \text{indeterminate from } S\,(\check{I}_{ab}) \\
S^1\,\check{I}_{a1} &= -\,j\,\sqrt{3}\,S^1\,I_{bc1} = -\,j\,a^2\,\sqrt{3}\,S^1\,\check{I}_{ab1} \\
S^2\,\check{I}_{a2} &= \quad j\,\sqrt{3}\,S^2\,\check{I}_{bc} = \quad j\,a\,\sqrt{3}\,S^2\,\check{I}_{ab2}
\end{aligned}\right\} \qquad (88)$$

It will be sufficient in order to illustrate the application of the principle of symmetrical coordinates to simple circuits to apply it to a few simple cases of transformer connections before proceeding to its application to rotating polyphase systems to which it is particularly adapted.

Unsymmetrical Bank of Delta-Delta Transformers Operating on a Symmetrical Circuit Supplying a Balanced System

Let the transformer effective impedances be Z_{AB} Z_{BC} Z_{CA} and let the secondary load currents be $\check{I}_U$ $\check{I}_V$ and $\check{I}_W$ and let the star load impedance be Z. One to one ratio of transformation will be assumed, and the effect of the magnetizing current will be neglected. The symmetrical equations are

$$\left.\begin{aligned}
O &= S^0\,(Z_{AB0}\,\check{I}_{ab0} + Z_{AB2}\,\check{I}_{ab1} + Z_{AB1}\,\check{I}_{ab2}) \\
S^1\,\check{E}_{uv1} &= S^1\,\check{E}_{ab1} - S^1\,(Z_{AB1}\,\check{I}_{ab0} + Z_{AB0}\,\check{I}_{ab1} + Z_{AB2}\,\check{I}_{ab2}) \\
S^2\,\check{E}_{uv2} &= 0 - S^2\,(Z_{AB2}\,\check{I}_{ab0} + Z_{AB1}\,\check{I}_{ab1} + Z_{AB0}\,\check{I}_{ab2}) \\
S^0\,\check{I}_{u0} &= 0 \\
S^1\,Z\,\check{I}_{u1} &= \check{E}_{u1} \\
S^2\,Z\,\check{I}_{u2} &= \check{E}_{u2}
\end{aligned}\right\} \qquad (89)$$

Since the transformation ratio is unity and the effects of magnetizing currents are negligible $S^1\,\check{I}_{ab1} = S^1\,\check{I}_{UV1}$, $S^2\,\check{I}_{ab2} = S^2\,\check{I}_{UV2}$. And therefore by means of the relations (85), the last two equations may be expressed

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$$\left.\begin{aligned} S^1 \check{E}_{uv1} &= S^1\, 3\, Z\, \check{I}_{ab1} \\ S^2 \check{E}_{uv2} &= S^2\, 3\, Z\, \check{I}_{ab2} \end{aligned}\right\} \quad (90)$$

in other words, the symmetrical components appear in the secondary as independent systems, $3\,Z$ being the delta load impedance equivalent to the star impedance Z.

Substituting from (90) in the second and third equation and eliminating $\check{I}_{ab0}$ by means of the first equation, and we have

$$\left.\begin{aligned} S^1 \check{E}_{ab1} &= S^1 \left\{ \left(3\,Z + Z_{AB0} - \frac{Z_{AB1}\, Z_{AB2}}{Z_{AB0}} \right) \check{I}_{ab1} \right. \\ &\qquad \left. + \left(Z_{AB2} - \frac{Z^2_{AB1}}{Z_{AB0}} \right) \check{I}_{ab2} \right\} \\ S^2 O &= S^2 \left\{ \left(Z_{AB1} - \frac{Z^2_{AB2}}{Z_{AB0}} \right) \check{I}_{ab1} \right. \\ &\qquad \left. + \left(3\,Z + Z_{AB0} - \frac{Z_{AB1}\, Z_{AB2}}{Z_{AB0}} \right) \check{I}_{ab2} \right\} \end{aligned}\right\} \quad (91)$$

which, when S^1 and S^2 are removed, give two simultaneous equations in $\check{I}_{ab1}$ and $\check{I}_{ab2}$.

A modification of the problem may occur even when the load impedances are symmetrical, as they may have symmetrical but unequal impedances Z_1 and Z_2, to the two components $\check{I}_{u1}$ and $\check{I}_{u2}$ respectively, as in the case of a load consisting of a symmetrical rotating machine. The equations corresponding to (89), (90) and (91) then become

$$\left.\begin{aligned} O &= S^0 \left(Z_{AB0}\, \check{I}_{ab0} + Z_{AB2}\, \check{I}_{ab1} + Z_{AB1}\, \check{I}_{ab2} \right) \\ S^1 \check{E}_{uv1} &= S^1 \check{E}_{ab1} - S^1 \left(Z_{AB1}\, \check{I}_{ab0} + Z_{AB0}\, \check{I}_{ab1} + Z_{AB2}\, \check{I}_{ab2} \right) \\ S^2 E_{uv2} &= O - S^2 \left(Z_{AB2}\, \check{I}_{ab0} + Z_{AB1}\, \check{I}_{ab1} + Z_{AB0}\, \check{I}_{ab2} \right) \\ S^0 \check{I}_{u0} &= O \\ S^1 Z_1\, \check{I}_{u1} &= \check{E}_{u1} \\ S^2 Z_2\, \check{I}_{u2} &= \check{E}_{u2} \end{aligned}\right\} \quad (92)$$

$$\left.\begin{aligned} S^1 \check{E}_{uv1} &= S^1\, 3\, Z_1\, \check{I}_{ab1} \\ S^2 \check{E}_{uv2} &= S^2\, 3\, Z_2\, \check{I}_{ab2} \end{aligned}\right\} \quad (93)$$

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$$\left.\begin{aligned} S^1 \check{E}_{ab1} &= S^1 \left\{ \left(3 Z_1 + Z_{AB0} - \frac{Z_{AB1}\, Z_{AB2}}{Z_{AB0}} \right) \check{I}_{ab1} + \right. \\ &\qquad \left. \left(Z_{AB2} - \frac{Z^2_{AB1}}{Z_{AB0}} \right) \check{I}_{ab2} \right\} \\ S^2 O &= S^2 \left\{ \left(Z_{AB1} - \frac{Z^2_{AB1}}{Z_{AB0}} \right) \check{I}_{ab1} + \right. \\ &\qquad \left. \left(3 Z_2 + Z_{AB0} - \frac{Z_{AB1}\, Z_{AB2}}{Z_{AB0}} \right) \check{I}_{ab2} \right\} \end{aligned}\right\} \quad (94)$$

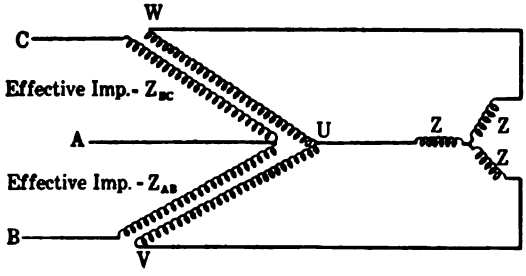

FIG. 4—OPEN DELTA OR V CONNECTION.

In an open delta system $Z_{AB1} = Z_{AB2} = Z_{AB0} - Z_{AB}$ the transformers in this case being both the same. Equation (91) becomes in this particular case where Z_{AB0} is infinite

$$\left.\begin{aligned} S^1 \check{E}_{ab1} &= S^1 \{ (3 Z + 2 Z_{AB}) \check{I}_{ab} + Z_{AB} \check{I}_{ab2} \} \\ S^2 O &= S^2 \{ Z_{AB} \check{I}_{ab1} + (3 Z + 2 Z_{AB}) \check{I}_{ab2} \} \end{aligned}\right\} \quad (95)$$

and we have

$$\check{I}_{ab0} = -\check{I}_{ab1} - I_{ab2} \quad (96)$$

Similarly, instead of (94) we have

$$\left.\begin{aligned} S^1 \check{E}_{ab1} &= S \{ (3 Z_1 + 2 Z_{AB}) \check{I}_{ab1} + Z_{AB} \check{I}_{ab2} \} \\ S^2 O &= S^2 \{ Z_{AB} \check{I}_{ab1} + (3 Z_2 + 2 Z_{AB}) \check{I}_{ab2} \} \end{aligned}\right\} \quad (97)$$

The secondary voltages are obtained from (90) and (93) for this latter case.

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The solution of (95) gives

$$\left.\begin{aligned} \check{I}_{ab1} &= \frac{3\,Z_1 + 2\,Z_{AB}}{(3\,Z_1 + 3\,Z_{AB})\,(3\,Z_1 + Z_{AB})}\,\check{E}_{ab} \\ \check{I}_{ab2} &= -\frac{Z_{AB}}{(3\,Z_1 + 3\,Z_{AB})\,(3\,Z_1 + Z_{AB})}\,\check{E}_{ab} \\ \check{I}_{ab0} &= -\frac{1}{3\,Z_1 + 3\,Z_{AB}}\,\check{E}_{ab} \end{aligned}\right\} \quad (98)$$

And we have

$$\left.\begin{aligned} S^1\,\check{I}_{a1} &= S^1\,\frac{3\,Z_1 + 2\,Z_{AB}}{3\,(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\,\check{E}_a \\ S^2\,\check{I}_{a2} &= S^2\,\frac{Z_{AB}}{3\,(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\,\check{E}_b \end{aligned}\right\} \quad (99)$$

And therefore

$$\left.\begin{aligned} \check{I}_a &= \frac{\check{E}_a}{Z_1 + \dfrac{Z_{AB}}{3}} + \frac{\frac{1}{3}\,Z_{AB}}{(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\,\check{E}_{ab} \\ \check{I}_b &= \frac{\check{E}_b}{Z_1 + \dfrac{Z_{AB}}{3}} - \frac{\frac{1}{3}\,Z_{AB}}{(Z_1 + Z_{AB})\left(Z_1 + \dfrac{Z_{AB}}{3}\right)}\,\check{E}_{ab} \\ \check{I}_c &= \frac{\check{E}_c}{Z_1 + \dfrac{Z_{AB}}{3}} \end{aligned}\right\} \quad (100)$$

Three-Phase System with Symmetrical Waves Having Harmonics

We may express $\check{E}_a$ in the following form:

$$\left.\begin{aligned} \check{E}_a &= E_1\,\epsilon^{jwt} + E_2\,\epsilon^{j2wt} + E_3\,\epsilon^{j3wt} + \ldots \\ &= \Sigma\,E_n\,\epsilon^{jnwt} \end{aligned}\right\} \quad (101)$$

where E_n is in general a complex number.

If the system is symmetrical three-phase $\check{E}_b$ is obtained by displacing the complete wave by the angle $-\dfrac{2\,\pi}{3}$ or

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$$\check{E}_b = \epsilon^{-j\frac{2\pi}{3}} E_1 \epsilon^{jwt} + \epsilon^{-j\frac{4\pi}{3}} E_2 \epsilon^{j2wt} + \epsilon^{-j\frac{6\pi}{3}} E_3 \epsilon^{j3wt} + \ldots$$

$$E_c = \epsilon^{j\frac{2\pi}{3}} E_1 \epsilon^{jwt} + \epsilon^{j\frac{4\pi}{3}} E_2 \epsilon^{j2wt} \; \epsilon^{j\frac{6\pi}{3}} E_3 \epsilon^{j3wt} + \ldots$$

or since $\epsilon^{-j\frac{2\pi}{3}} = a^2$, $\epsilon^{j\frac{2\pi}{3}} = a$ etc.

$$\left.\begin{aligned} \check{E}_a &= E_1 \epsilon^{jwt} + E_2 \epsilon^{j2wt} + E_3 \epsilon^{j3wt} + \ldots \\ \check{E}_b &= a^2 E_1 \epsilon^{jwt} + a E_2 \epsilon^{j2wt} + E_3 \epsilon^{j3wt} + \ldots \\ \check{E}_c &= a E_1 \epsilon^{jwt} + a^2 E_2 \epsilon^{j2wt} + E_3 \epsilon^{j3wt} + \ldots \end{aligned}\right\} \quad (102)$$

or

$$\left.\begin{aligned} S(\check{E}_a) = S^0 \{E_3 \epsilon^{j3wt} + E_6 \epsilon^{j6wt} + E_9 \epsilon^{j9wt} + \ldots\} \\ + S^1 \{E_1 \epsilon^{jwt} + E_4 \epsilon^{j4wt} + E_7 \epsilon^{j7wt} + \ldots\} \\ + S^2 \{E_2 \epsilon^{j2wt} + E_5 \epsilon^{j5wt} + E_8 \epsilon^{j8wt} + \ldots\} \end{aligned}\right\} \quad (103)$$

$$S(\check{E}_a) = S^0 \Sigma (E_{3n} \epsilon^{j3nwt}) + S^1 \Sigma (E_{3n-2} \epsilon^{j(3n-2)wt}) + S^2 \Sigma (E_{3n-1} \epsilon^{j(3n-1)wt}) \quad (104)$$

This shows that a symmetrical three-phase system having harmonics is made up of positive and negative phase sequence harmonic systems and others of zero phase sequence, that is to say of the same phase in all windings, which comprise the group of third harmonics. These facts are not generally appreciated though they are factors that may have an appreciable influence in the performance of commercial machines. It should be particularly noted that in three-phase generators provided with dampers the fifth, eleventh, seventeenth, and twenty-third harmonics produce currents in the damper windings.

In dealing with the complex variable it will be convenient to use for the amplitude the root mean square value for each harmonic. When instantaneous values are required, the real part of the complex variable should be multiplied by $\sqrt{2}$. In the remainder of this paper this convention will be adopted.

Power Representation in Symmetrical Co-ordinates

Since the power in an alternating-current system is also a harmonically varying scalar quantity, it may therefore be represented in the same manner as the current or electromotive force,

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that is to say by a complex variable which we shall denote by $(P + j\,Q) + (P_H + j\,Q_H)$, $P + j\,Q$ being the mean value, is the term of the complex variable of zero frequency, P representing the real power and Q the wattless power, $\sqrt{P^2 + Q^2}$ will be the volt-amperes.

The value of the complex variable $(P + j\,Q) + (P_H + j\,Q_H)$ may be taken as

$$(P + j\,Q) + (P_H + j\,Q_H) = \check{E}\,\hat{I} + \check{E}\,\check{I} \tag{105}$$

with the provision that for all terms having negative indices the conjugate terms must be substituted, these terms being present in the product $\hat{E}\,\check{I} + \hat{E}\,\hat{I}$, which is the conjugate of the product (105). A similar rule holds good for the symmetrical vector system

$$\left.\begin{aligned} S(\check{E}_a) &= S^0\,\check{E}_{a0} + S^1\,\check{E}_{a1} + \quad . \quad . \quad . \quad + S^{n-1}\,\check{E}_{a(n-1)} \\ S(\check{I}_a) &= S^0\,\check{I}_{a0} + S^1\,\check{I}_{a1} + \quad . \quad . \quad . \quad + S^{n-1}\,\check{I}_{a(n-1)} \end{aligned}\right\} \tag{106}$$

The conjugate of $S\,\check{I}_a$ is

$$S(\hat{I}_a) = S^0\,\hat{I}_{a0} + S^{(n-1)}\,\hat{I}_{a1} + \quad . \quad . \quad . \quad + S^1\,\hat{I}_{a(n-1)} \tag{107}$$

and the power is represented by

$$(P + P_b) + j\,(Q + Q_b) = \Sigma\,\{S(\check{E}_a)\,S(\hat{I}_a) + S(\check{E}_a)\,S(\check{I}_a)\} \tag{108}$$

with the same provision for terms having negative indices. The sign Σ signifies that all the products in each sequence are added together.

$$\left.\begin{aligned} \Sigma\,\{S(\hat{I}_a)\,S(\check{E}_a)\} &= \Sigma\,S^0\,\{\hat{I}_{a0}\,\check{E}_{a0} + \hat{I}_{a1}\,\check{E}_{a1} + \quad . \quad . \\ &\qquad\qquad + \hat{I}_{a(n-1)}\,\check{E}_{a(n-1)}\} \\ &+ \Sigma\,S^1\,\{\hat{I}_{a0}\,\check{E}_{a1} + \hat{I}_{a1}\,\check{E}_{a2} + \hat{I}_{a2}\,\check{E}_{a3} + \quad . \quad . \quad . \\ &\qquad\qquad + \hat{I}_{a(n-1)}\,\check{E}_{a0}\} \\ &+ \Sigma\,S^2\,\{\hat{I}_{a0}\,\check{E}_{a2} + \hat{I}_{a1}\,\check{E}_{a3} + \hat{I}_{a2}\,\check{E}_4 + \quad . \quad . \quad . \\ &\qquad\qquad + \check{I}_{a(n-1)}\,\check{E}_{a1}\} \\ &\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ &+ \Sigma\,S^{(n-1)}\,\{\hat{I}_{a0}\,\check{E}_{a(n-1)} + \hat{I}_{a1}\,\check{E}_{a0} + \quad . \quad . \quad . \\ &\qquad\qquad + \hat{I}_{a(n-1)}\,\check{E}_{a(n-2)}\} \end{aligned}\right\} \tag{109}$$

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The terms prefixed by S^1, S^2, S^3 . . . $S^{(n-1)}$ all become zero and since S^0 becomes n

$$\Sigma S(\hat{I}_a) S(\check{E}_a) = n \{\hat{I}_{a0} \check{E}_{a0} + \hat{I}_{a1} \check{E}_{a1} + \ . \ . \ . \ + \hat{I}_{a(n-1)} \check{E}_{a(n-1)}\} \tag{110}$$

In a similar manner it may be shown that

$$\Sigma S(\check{I}_a) S(\check{E}_a) = n \{\check{I}_{a0} \check{E}_{a0} + \check{I}_{a1} \check{E}_{a(n-1)} + \check{I}_{a2} \check{E}_{a(n-2)} + \ . \ . \ . \ + \check{I}_{a(n-1)} \check{E}_{a1}\} \tag{111}$$

and therefore

$$\begin{aligned}(P + jQ) + (P_H + jQ_H) &= n \{\hat{I}_{a0} \check{E}_{a0} + \hat{I}_{a1} \check{E}_{a1} + \ . \ . \ . \ + \hat{I}_{a(n-1)} \check{E}_{a(n-1)}\} \\ &+ n \{\check{I}_{a0} \check{E}_{a0} + \check{I}_{a1} \check{E}_{a(n-1)} + \ . \ . \ . \ + \check{I}_{a(n-1)} \check{E}_{a1}\}\end{aligned} \tag{112}$$

For a three-phase system the expression reduces to

$$\begin{aligned}(P + jQ) + (P_H + jQ_b) &= 3(\hat{I}_{a0} \check{E}_{a0} + \hat{I}_{a1} \check{E}_{a1} + \hat{I}_{a2} \check{E}_{a2}) \\ &+ 3(\check{I}_{a0} \check{E}_{a0} + \check{I}_{a1} \check{E}_{a2} + \check{I}_{a2} \check{E}_{a1})\end{aligned} \tag{113}$$

In the above expression $P + P_H$ is the value of the instantaneous power on the system, P being the mean value and P_H the harmonic portion. When the currents are simple sine waves, Q may be interpreted to be the mean wattless power of the circuit or the sum of the wattless volt-amperes of each circuit. In rotating machinery since the coefficients of mutual induction may be complex harmonic functions of the angular velocity, this is not strictly true for all cases; but if the effective impedances to the various frequencies of the component currents be used, it will be found to be equal to the mean wattless volt-amperes of the system with each harmonic considered independent.

In a balanced polyphase system P_H and Q_H both become zero.

The instantaneous power is a quantity of great importance in polyphase systems because the instantaneous torque is proportional to it and this quantity enters into the problem of vibrations which is at times a matter of great importance, especially when caused by unbalanced e. m. fs. A system of currents and e. m. fs. may be transformed to balanced polyphase by means of transformers alone, provided that the value of P_H is zero, while on the other hand polyphase power cannot be supplied from a pulsating power system without means for

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supplying the necessary storage to make a continuous flow of energy.

PART II

Application of the Method to Rotating Polyphase Networks

The methods of determining the constants Z_a Z_u, M, etc., of co-axial cylindrical networks is taken up in Appendix I of this paper. It will be assumed that the reader has familiarized himself with these quantities and understands their significance. We shall first consider the case of symmetrically wound machines taking up the simple cases first and proceeding to more complex ones.

Symmetrically Wound Induction Motor Operating on Unsymmetrical Polyphase Circuit

Denoting the pole-pitch angle by π let the synchronous angular velocity be ω_0 and let the angular slip velocity be ω_1. And let $S^1 E_{a1}$ $S^2 E_{a2}$ be the symmetrical components of impressed polyphase e. m. f. Let R_a be the primary resistance and R_u the secondary resistance. The primary self-inductance being M_{aa}, that of the secondary being M_{uu} and corresponding symbols being used to denote the mutual inductances between the different pairs of windings. Then by means of (**39**), (**40**), (**56**) and (**57**)

$$\left.\begin{aligned}
S^1 \check{E}_{a1} &= S^1 \left\{ R_a \check{I}_{a1} + 1\tfrac{1}{2} M_{aa} \frac{d}{dt} \check{I}_{a1} \right. \\
&\qquad \left. + 1\tfrac{1}{2} M_{au} \frac{d}{dt} \epsilon^{j(w_0 - w_1)t} \check{I}_{u1} \right\} \\
S^2 \check{E}_{a2} &= S^2 \left\{ R_a \check{I}_{a2} + 1\tfrac{1}{2} M_{aa} \frac{d}{dt} \check{I}_{a2} \right. \\
&\qquad \left. + 1\tfrac{1}{2} M_{au} \frac{d}{dt} \epsilon^{-j(w_0 - w_1)t} \check{I}_{u2} \right\} \\
S^1 \check{E}_{u1} = O &= S^1 \left\{ R_u \check{I}_{u1} + 1\tfrac{1}{2} M_{uu} \frac{d}{dt} \check{I}_{u1} \right. \\
&\qquad \left. + 1\tfrac{1}{2} M_{au} \frac{d}{dt} \epsilon^{-j(w_0 - w_1)t} \check{I}_{a1} \right\} \\
S^2 \check{E}_{u2} = O &= S^2 \left\{ R_u \check{I}_{u2} + 1\tfrac{1}{2} M_{uu} \frac{d}{dt} \check{I}_{u2} \right. \\
&\qquad \left. + 1\tfrac{1}{2} M_{au} \frac{d}{dt} \epsilon^{j(w_0 - w_1)t} \check{I}_{a2} \right\}
\end{aligned}\right\} \quad (114)$$

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denote $1\frac{1}{2}\ M_{aa}$ by L_a and $1\frac{1}{2}\ M_{uu}$ by L_u, $1\frac{1}{2}\ M_{au}$ by M, the equations (1) become

$$\left.\begin{aligned}
S^1\,\check{E}_{a1} &= S^1\left\{\left(R_a + L_a\frac{d}{dt}\right)\check{I}_{a1}\right. \\
&\qquad \left. + M\frac{d}{dt}\,\epsilon^{j(w_0-w_1)t}\,\check{I}_{u1}\right\} \\
S^2\,\check{E}_{a2} &= S^2\left\{\left(R_a + L_a\frac{d}{dt}\right)\check{I}_{a2}\right. \\
&\qquad \left. + M\frac{d}{dt}\,\epsilon^{-j(w_0-w_1)t}\,\check{I}_{u2}\right\} \\
S^1\,\check{E}_{u1} = O &= S^1\left\{\left(R_u + L_u\frac{d}{dt}\right)\check{I}_{u1}\right. \\
&\qquad \left. + M\frac{d}{dt}\,\epsilon^{-j(w-w_1)t}\,\check{I}_{a1}\right\} \\
S^2\,\check{E}_{u2} = O &= S^2\left\{\left(R_u + L_u\frac{d}{dt}\right)\check{I}_{u2}\right. \\
&\qquad \left. + M\frac{d}{dt}\,\epsilon^{j(w_0-w_1)t}\,\check{I}_{a2}\right\}
\end{aligned}\right\} \tag{115}$$

From the last two equations we have

$$\check{I}_{u1} = -\frac{M\dfrac{d}{dt}}{R_u + L_u\dfrac{d}{dt}}\,\epsilon^{-j(w_0-w_1)t}\,\check{I}_{a1} \tag{116}$$

$$\check{I}_{u2} = -\frac{M\dfrac{d}{dt}}{R_u + L_u\dfrac{d}{dt}}\,\epsilon^{j(w_0-w_1)t}\,\check{I}_{a2} \tag{117}$$

Substituting these in the first two equations of (115) we obtain

$$S'\,\check{E}_{a1} = S^1\left[\left(R_a + L_a\frac{d}{dt}\right) - \frac{M^2\dfrac{d}{dt}\left\{\dfrac{d}{dt} - j(w_0-w_1)\right\}}{R_u + L_u\left\{\dfrac{d}{dt} - j(w_0-w_1)\right\}}\right]\check{I}_{a1} \tag{118}$$

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$$S^2 \check{E}_{a2} = S^2 \left[\left(R_a + L_a \frac{d}{dt}\right) - \frac{M^2 \frac{d}{dt} \left\{ \frac{d}{dt} + (j\, w_0 - w_1) \right\}}{R_u + L_u \left\{ \frac{d}{dt} - j\,(w_0 - w_1) \right\}} \right] \check{I}_{a2} \quad (119)$$

If $\check{E}_{a1} = E_{a1}\, \epsilon^{jwt}$ and $\check{E}_{a2} = E_{a2}\, \epsilon^{jwt}$ the solution for $\check{I}_{a1}$ and $\check{I}_{a2}$ will be

$$\check{I}_{a1} = \frac{\check{E}_{a1}}{Z_1} \quad (120)$$

$$\check{I}_{a2} = \frac{\check{E}_{a2}}{Z_2} \quad (121)$$

Where

$$Z_1 = R_a + j\, w_0\, L_a + \frac{w_0\, w_1\, M^2}{R_u^2 + w_1^2\, L_u^2} (R_u - j\, w_1\, L_u) \quad (122)$$

$$Z_2 = R_a + j\, w_0\, L_a + \frac{w_0\,(2\, w_0 - w_1)\, M^2}{R_u^2 + (2\, w_0 - w_1)^2\, L_u^2} \{R_u - j\,(2\, w_0 - w_2)\, L_u\} \quad (123)$$

The impedances Z_1 and Z_2 will be found more convenient to use in the form

$$Z_1 = (R_a + K_1^2\, R_u) + j\, w_0\, (L_a - K_1^2\, L_u) + \frac{w_0 - w_1}{w_1} K_1^2\, R_u \quad (124)$$

$$Z_2 = (R_a + K_2^2\, R_u) + j\, w_0\, (L_a - K_2^2\, L_u) - \frac{w_0 - w_1}{Z\, w_0 - w_1} K_2^2\, R_u \quad (125)$$

Where, as we will see later, K_1^2 and K_2^2 are the squares of the transformation ratios between primary and secondary currents of positive and negative phase sequence.

The last real term in each expression is the virtual resistance due to mechanical rotation and when combined with the mean square current represents mechanical work performed, the positive sign representing work performed and the negative sign work required.

Thus, for example, to enable the currents $S^2\, \check{I}_{a2}$ to flow, the mechanical work $3\, I_{a2}^2\, \frac{w_0 - w_1}{2\, w_0 - w_1} K_1^2\, R_u$ must be applied to the shaft of the motor.

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The phase angles of the symmetrical systems $S^1 \check{I}_{a1}$ $S^2 \check{I}_{a2}$ with respect to their impressed e. m. f., $S^1 \check{E}_{a1}$ and $S^2 \check{E}_{a2}$ are given by these impedances so that the complete solution of the primary circuit is thus obtained.

The secondary currents are given by equations (**116**) and (**117**) and are

$$\check{I}_{u1} = -\frac{j\,w_1\,M}{R_u + j\,w_1\,L_u}\,I_{a1}\,\epsilon^{jw_1t} = \check{K}_1\,I_{a1}\epsilon^{jw_1t} \tag{126}$$

$$\check{I}_{u2} = -\frac{j\,(2\,w_0 - w_1)\,M}{R_u + j\,(2\,w_0 - w_1)\,L_u}\,I_{a2}\,\epsilon^{j(2w_0 - w_1)t} = \check{K}_2\,I_{a2}\,\epsilon^{j(2w_0 - w_1)t} \tag{127}$$

In the results just given, M is not the maximum value of mutual inductance between a pair of primary and secondary windings but is equal to the total mutual inductance due to a current passing through the two coils W and V through the coil

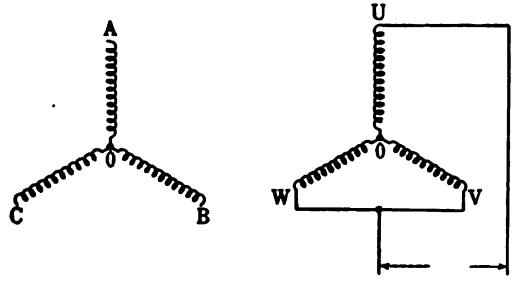

FIG. 5

U as shown in the sketch Fig 5 and the winding "A" when A and U have their planes of symmetry coincident.

Where the windings are symmetrical the induced e. m. f. is independent of the division of current between W and V, but this quantity must not be used in unsymmetrical windings, or with star windings having a neutral point connection so that $\check{I}_{a0}$ is not zero.

The appearance of M in this equation follows from the equation

$$\check{I}_u + \check{I}_v + \check{I}_w = O$$

so that

$$\check{I}_u = -\,(\check{I}_v + \check{I}_w)$$

The power delivered by the motor is

$$P_0 = 3\left\{\frac{w_0 - w_1}{w_1}\,K_1^2\,I_{a1}^2\,R_u - \frac{w_0 - w_1}{2\,w_0 - w_1}\,K_2^2\,I_{a2}^2\,R_u\right\} \tag{128}$$

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The copper losses are given by

$$P_L = 3\,\{I_{a1}^2\,(R_p + K_1^2\,R_u) + I_{a2}^2\,(R_p + K_2^2\,R_u)\} \qquad (129)$$

The iron loss is independent of the copper loss and power output. The iron loss and windage may be taken as

$$P_F = \text{Iron loss and windage} \qquad (130)$$

The power input as

$$P_I = P_O + P_L + P_F \qquad (131)$$

The mechanical power output is P_O less friction and windage losses.

$$\text{Torque} = 3\left\{\frac{1}{w_1}\,K_1^2\,I_{a1}^2\,R_u - \frac{1}{2\,w_0 - w_1}\,K_2^2\,I_{a2}^2\,R_u\right\} \times 10^7 \text{ dyne-cm.} \qquad (132)$$

The kv-a. at the terminals is

$$\sqrt{P_I^2 + Q_I^2} = \text{The effective value of } 3\,(E_{a1}\,I_{a1} + E_{a2}\,I_{a2}) \qquad (133)$$

This last result may be arrived at in the following way

$$\left.\begin{aligned} S\,(\check{E}_a) &= S^1\,\check{E}_{a1} + S^2\,\check{E}_{a2}) \\ S\,(\hat{I}_a) &= S^2\,\hat{I}_{a1} + S^1\,\hat{I}_{a2} \end{aligned}\right\} \qquad (134)$$

Since $S^2\hat{I}_{a1}$ is conjugate to $S^1\,\check{I}_{a1}$, etc.

The product of $\check{E}_{a2}$ and $\hat{I}_{a2}$ is the power product of the two vectors, $S\,(\check{E}_a)$ and $S\,(\check{I}_a)$ and omits the harmonic variation as a double frequency quantity, the average wattless appears as an imaginary non-harmonic quantity.

$$P_I + j\,Q_I = \Sigma\,(S^0\,\check{E}_{a1}\,\hat{I}_{a1} + S^0\,\check{E}_{a2}\,\hat{I}_{a2} + S^1\,\check{E}_{a2}\,\hat{I}_{a1} + S^2\,\check{E}_{a1}\,\hat{I}_{a2}) \qquad (135)$$

The S^1 and S^2 products have zero values, since the sum of the terms of each sequence is zero, hence—

$$P_I + j\,Q_I = 3\,(\check{E}_{a1}\,\hat{I}_{a1} + \check{E}_{a2}\,\hat{I}_{a2}) \qquad (136)$$

$$\sqrt{P_I^2 + Q_I^2} = \text{The effective value of } 3\,(\check{E}_{a1}\,\hat{I}_{a1} + \check{E}_{a2}\,\hat{I}_{a2}) \qquad (137)$$

The solution for the general case of symmetrical motor operating on an unsymmetrical circuit is not of as much interest as

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certain special cases depending thereon. Some of the most important of these will be taken up in the following paragraphs.

Case I. Single-phase e. m. f. impressed across one phase of three-phase motor.

Assuming the single-phase voltage to be $\check{E}_{bc}$ impressed across the terminals B C. The known data or constraints are

$$\left.\begin{aligned} \check{E}_{bc} &= j\sqrt{3}\,(\check{E}_{a1} - \check{E}_{a2}) \\ \check{I}_a &= O,\ \check{I}_b = -\check{I}_c \end{aligned}\right\} \tag{138}$$

and therefore

$$\check{I}_{a1} = -\check{I}_{a2} \tag{139}$$

$$\frac{\check{E}_{a1}}{Z_1} = -\frac{\check{E}_{a2}}{Z_2}$$

$$\check{E}_{a2} = -\frac{Z_2}{Z_1}\check{E}_{a1} \tag{140}$$

Substituting in (138)

$$\left.\begin{aligned} \check{E}_{a1} &= -j\frac{\check{E}_{bc}}{\sqrt{3}}\cdot\frac{Z_1}{Z_1+Z_2} \\ \check{E}_{a2} &= \quad j\frac{\check{E}_{bc}}{\sqrt{3}}\cdot\frac{Z_2}{Z_1+Z_2} \end{aligned}\right\} \tag{141}$$

and therefore

$$\left.\begin{aligned} \check{I}_{a1} &= -j\frac{\check{E}_{bc}}{\sqrt{3}}\cdot\frac{1}{Z_1+Z_2} \\ \check{I}_{a2} &= \quad j\frac{\check{E}_{bc}}{\sqrt{3}}\cdot\frac{1}{Z_1+Z_2} \end{aligned}\right\} \tag{142}$$

Since $\check{I}_b = \check{I}_{b1} + \check{I}_{b2} = a^2\check{I}_{a1} + a\check{I}_{a2}$

$$\check{I}_b = -I_c = -\frac{\check{E}_{bc}}{Z_1+Z_2} \tag{143}$$

$$P_0 = \left(\frac{w_0 - w_1}{w_1}K_2^2 R_u - \frac{w_0 - w_1}{2w_0 - w_1}K_2^2 R_u\right)I_0^2 \tag{144}$$

$$P_1 + jQ_1 = I_b^2(Z_1 + Z_2) + P_F \tag{145}$$

The power factor is obtained from (145) by the formula

$$\cos\alpha = \frac{P_1}{\sqrt{P_1^2 + Q_1^2}} \tag{146}$$

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Substituting from (**142**) in equation (**126**) and (**127**) of the general case we obtain for the secondary currents

$$\left.\begin{aligned} \check{I}_{u1} &= -\,j\,\check{K}_1 \frac{E_{bc}}{Z_1 + Z_2}\,\epsilon^{jw_1 t} \\ \check{I}_{u2} &= \quad j\,\check{K}_2 \frac{E_{bc}}{Z_1 + Z_2}\,\epsilon^{j(2w_0 - w_1)t} \end{aligned}\right\} \qquad (\mathbf{147})$$

Many unsymmetrical cases may be expressed in terms of the operation of coupled symmetrical motors operating on symmetrical systems. This is invariably the case with symmetrical polyphase motors operating on single-phase circuits. Since the physical interpretations are useful in impressing the facts on ones memory they will be given whenever they appear to be useful.

Equations (**141**) and (**142**) show that single-phase operation is exactly equivalent to operating two duplicate motors in series with a symmetrical polyphase e. m. f. $S^1\,E_{ab}$ impressed across one motor, the other being connected in series with the first but with phase sequence reversed, the two motors being directly coupled.

Case II. B and C connected together e. m. f. impressed across A B.

The data given by the conditions of constraint are

$$\left.\begin{aligned} \check{E}_{ab} &= -\,\check{E}_{ca} \\ \check{E}_{bc} &= O = j\,\sqrt{3}\,(\check{E}_{a1} - \check{E}_{a2}) \end{aligned}\right\} \qquad (\mathbf{148})$$

We therefore have

$$\check{E}_{a1} = \check{E}_{a2} = -\,\frac{\check{E}_{ab}}{3} \qquad (\mathbf{149})$$

and

$$\left.\begin{aligned} \check{I}_{a1} &= -\,\frac{\check{E}_{ab}}{3\,Z_1} \\ \check{I}_{a2} &= -\,\frac{\check{E}_{ab}}{3\,Z_2} \end{aligned}\right\} \qquad (\mathbf{150})$$

The remainder follows from the general solution and need not be repeated here.

(**150**) shows that a motor operated in this manner is the exact equivalent in all respects to two duplicate mechanically coupled polyphase motors, one of which has sequence reversed, operating in parallel on a balanced three-phase circuit of e. m. f. $S^1\,\frac{E_{ab}}{\sqrt{3}}$.

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The secondary currents follow from substitution of (**150**) in equations (**126**) and (**127**) of the general case.

Case III. B and C connected together by the terminals of a balance coil, the impressed e. m. f. E_{AD} *applied between A and the middle point of the balance coil. Resistance and reactance of balance coil negligible.*

The data furnished by the connection in this case is

$$\check{I}_b = \check{I}_c = -\frac{\check{I}_a}{2} \tag{151}$$

and therefore

$$\check{I}_{a1} = \frac{\check{I}_a - a\,\frac{\check{I}_a}{2} - a^2\,\frac{\check{I}_a}{2}}{3} = \frac{\check{I}_a}{2}$$

$$\check{I}_{a2} = \check{I}_{a1} = \frac{\check{I}_a}{2}$$

We therefore have

$$\left.\begin{aligned} \check{E}_{a1} &= \frac{Z_1\,\check{I}_a}{2} \\ \check{E}_{a2} &= \frac{Z_2\,\check{I}_a}{2} \end{aligned}\right\} \tag{152}$$

we have

$$\begin{aligned} \check{E}_{ab} &= j\sqrt{3}\,(a\,\check{E}_{a1} - a^2\,\check{E}_{a2}) \\ &= j\sqrt{3}\,\frac{\check{I}_a}{2}\,(a\,Z_1 - a^2\,Z_2) \end{aligned}$$

$$\check{E}_{bc} = j\sqrt{3}\,\frac{\check{I}_a}{2}\,(Z_1 - Z_2)$$

$$\left.\begin{aligned} \check{E}_{ad} &= \left(\check{E}_{ab} + \frac{\check{E}_{bc}}{2}\right) \\ &= j\sqrt{3}\,\frac{\check{I}_a}{2}\,\{(a + \tfrac{1}{2})\,Z_1 - (a^2 + \tfrac{1}{2})\,Z_2\} \\ &= -\tfrac{3}{4}\,\check{I}_a\,(Z_1 + Z_2) \end{aligned}\right\} \tag{153}$$

and therefore,

$$\check{I}_a = -1\tfrac{1}{2}\,\frac{\check{E}_{ad}}{Z_1 + Z_2} \tag{154}$$

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$$P_0 = \tfrac{3}{4}\left\{\frac{w_0 - w_1}{w_1} K_1^2 - \frac{w_0 - w_1}{2\,w_0 - w_1} K_2^2\right\} I_a^2 R_s \qquad (155)$$

$$P_1 + j\,Q_1 = \tfrac{3}{4} I_a^2 (Z_1 + Z_2) + P_F \qquad (156)$$

$$\cos\alpha = \frac{P_1}{\sqrt{P_1^2 + Q_1^2}} \qquad (157)$$

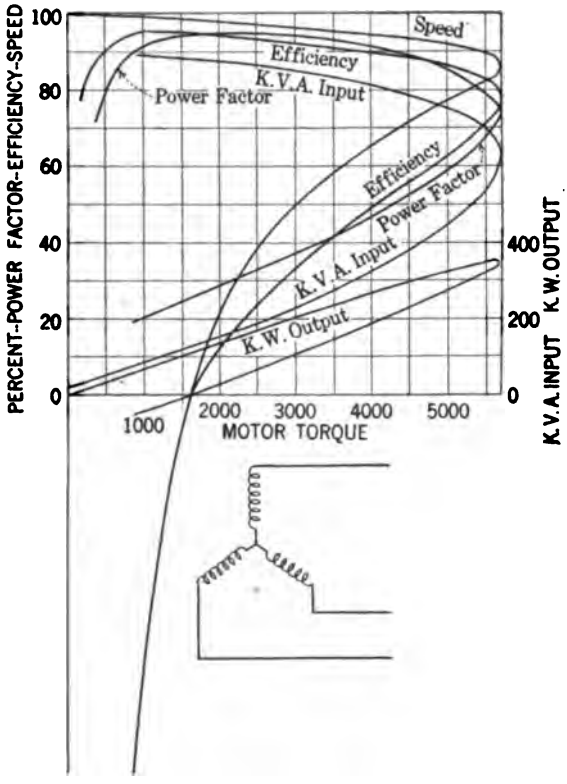

FIG. 6—CHARACTERISTICS OF THREE-PHASE INDUCTION MOTOR—BALANCED THREE-PHASE

Evidently (155), (156) and (157) are identical to (144), (145) and (146) if I_a is equal to $I_b \div \frac{\sqrt{3}}{2}$. This will be the case if the value of $E_{ad} = \frac{\sqrt{3}}{2}$ times that of E_{bc}. The total heating of

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the motors will be the same in each case but the heating in one phase for Case III will be one-third greater than for Case I.

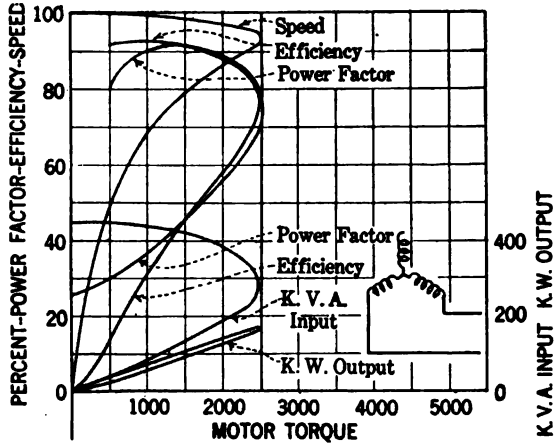

FIG. 7—CHARACTERISTICS OF THREE-PHASE INDUCTION MOTOR—SINGLE-PHASE OPERATION —ONE LEAD OPEN

This method of operation is therefore, as far as total losses, etc. are concerned, the exact counterpart of two polyphase

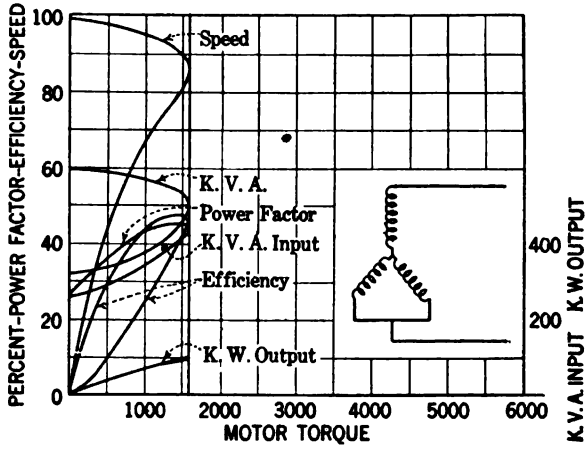

FIG. 8—CHARACTERISTICS OF THREE-PHASE INDUCTION MOTOR—SINGLE-PHASE OPERATION IN MANNER INDICATED

motors connected in series with shafts mechanically connected, one of which has its phase sequence reversed.

Figs. 6, 7 and 8 show characteristic curves of a three-phase

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induction motor operating respectively on a symmetrical circuit, according to Case I and according to Case II.

Synchronous Machinery

The Symmetrical Three-Phase Generator Operating on Unsymmetrically Loaded Circuit

The polyphase salient pole generator is not strictly a symmetrical machine, the exciting winding is not a symmetrical polyphase winding and it therefore sets up unsymmetrical trains of harmonics in exactly the same way as they are set up in an induction motor with unsymmetrical secondary winding. These cases will therefore be taken up later on. A three-phase generator may however be wound with a distributed polyphase winding to serve both as exciting and damper winding and if properly connected will be perfectly symmetrical. Such a machine will differ from an induction motor only in respect to the fact that it operates in synchronism and has internally generated symmetrical e. m. fs. which we will denote by $S^1 \check{E}_{a1}$, $S^2 \check{E}_{a2}$ the negative phase sequence component being zero; an e. m. f. $S^0 \check{E}_{a0}$ may exist but since in all the connections that will be considered there will be no neutral connection its value may be ignored. If the load impedances be Z_a', Z_b' and Z_c' they may be expressed by

$$Z_{aa}^1 = S^0 Z_{a0}^1 + S^1 Z_{a1}' + S^2 Z_{a2}'$$

and the equations of the generator will be

$$\left.\begin{aligned}
S^1 \check{E}_{a1} &= S^1 \left\{ \left(R_a + L_a \frac{d}{dt}\right) \check{I}_{a1}' + Z_{a0}' I_{a1}' \right. \\
&\qquad \left. + Z_{a2}' I_{a2}' + M \frac{d}{dt} \epsilon^{jwt} \check{I}_{u1}' \right\} \\
O &= S^2 \left\{ \left(R_a + L_a \frac{d}{dt}\right) \check{I}_{a2}' + Z_{a0}' I_{a2}' \right. \\
&\qquad \left. + Z_{a1}^1 \check{I}_{a1}' + M \frac{d}{dt} \epsilon^{-jwt} \check{I}_{u2}' \right\} \\
O &= \left(R_u + L_u \frac{d}{dt}\right) \check{I}_{u1}' + M \frac{d}{dt} \epsilon^{-jwt} \check{I}_{a1}' \\
O &= \left(R_u + L_u \frac{d}{dt}\right) \check{I}_{u2}' + M \frac{d}{dt} \epsilon^{jwt} \check{I}_{a2}'
\end{aligned}\right\} \quad (158)$$

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The last two equations give

$$\left.\begin{aligned} I_{u1}' &= -\frac{M\dfrac{d}{dt}}{R_u + L_u\dfrac{d}{dt}}\,\epsilon^{-jw_0t}\,\check{I}_{a1}' \\ I_{u2}' &= -\frac{M\dfrac{d}{dt}}{R_u + L_u\dfrac{d}{dt}}\,\epsilon^{jw_0t}\,I_{a2}' \end{aligned}\right\} \tag{159}$$

which on substitution in the first two equations of (158) give the equations

$$\left.\begin{aligned} &\left\{R_a + L_a\frac{d}{dt} - \frac{M^2\dfrac{d}{dt}\left(\dfrac{d}{dt} - j\,w_0\right)}{R_u + L_u\left(\dfrac{d}{dt} - j\,w_0\right)}\right\}\check{I}_{a1}' \\ &\qquad + Z_{a0}^1\,\check{I}_{a1}' + Z_{a2}'\,\check{I}_{a2}' = \check{E}_{a1} \\ &Z_{a1}^1\,\check{I}_{a1}' + \left\{R_a + L_a\frac{d}{dt}\right. \\ &\left. - \frac{M^2\dfrac{d}{dt}\left(\dfrac{d}{dt} + j\,w_0\right)}{R_u + L_u\left(\dfrac{d}{dt} + j\,w_0\right)}\right\}\check{I}_{a2}' + Z_{a0}^1\,\check{I}_{a2}' = 0 \end{aligned}\right\} \tag{160}$$

or if

$$\check{E}_{a1} = E_{a1}\,\epsilon^{jw_0t} \tag{161}$$

the impedances Z_{a0}, Z_{a1}, Z_{a2} become ordinary impedance for an electrical angular velocity w_0 and equations (160) become

$$\left.\begin{aligned} &(R_a + j\,w\,L_a + Z_{a0}^1)\,\check{I}_{a1}' + Z_{a2}'\,\check{I}_{a'2} = \check{E}_{a1} \\ &Z_{a1}'\,\check{I}_{a1}' + \{Z_{a0}' + (R_a + K_2^2\,R_u) + j\,2\,w_0\,(L_a - K_2^2\,L_u) - \\ &\qquad\qquad \tfrac{1}{2}\,K_2^2\,R_u\}\,\check{I}_{a2}' = 0 \end{aligned}\right\} \tag{162}$$

It is apparent that in the generator the impedances

$$R_a + j\,w_0\,L_a = Z_1'$$

and $\{(R_a + K_2^2\,R_u) + j\,2\,w_0\,(L_a - K_2^2\,L_u) - \tfrac{1}{2}\,K^2R_u\} = Z_2'$

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take the place of Z_1 and Z_2 in the symmetrical induction motor operating on an unsymmetrical circuit, and we may express equation (**162**)

$$\left.\begin{aligned}(Z_{a0}' + Z_1')\,\check{I}_{a1}' + Z_{a2}'\,\check{I}_{a2}' &= \check{E}_{a1}\\ Z_a'\,\check{I}_{a1}' + (Z_{a0}' + Z_2')\,I_{a2}' &= O\end{aligned}\right\}\quad(\mathbf{163})$$

which gives

$$\check{I}_{a2}' = -\frac{Z_{a1}'}{Z_{a0} + Z_2'}\,\check{I}_{a1}'$$

$$\check{I}_{a1}' = \frac{\check{E}_{a1}}{(Z_{a0}' + Z_1') - \dfrac{Z_{a1}'\,Z_{a2}'}{Z_{a0}' + Z_2'}}$$

Or in more symmetrical form

$$\left.\begin{aligned}\check{I}_{a1}' &= \frac{Z_{a0}' + Z_2'}{(Z_{a0}' + Z_1')\,(Z_{a0}' + Z_2') - Z_{a1}'\,Z_{a2}'}\,\check{E}_{a1}\\ \check{I}_{a2}' &= -\frac{Z_{a1}{}^1}{(Z_{a0}' + Z_1')\,(Z_{a0}' + Z_2') - Z_{a1}'\,Z_{a2}'}\,\check{E}_{a1}\end{aligned}\right\}\quad(\mathbf{164})$$

From (**159**) we have for the damper currents

$$\left.\begin{aligned}\check{I}_{u1}' &= O \text{ if } R_u > O\\ \check{I}_{u2}' &= -\check{K}_2\,I_{a2}\,\epsilon^{j2w_0t}\\ &\text{where } \check{K}_2 = j\,\frac{2\,w_0\,M}{R_u + j\,2\,w_0\,L_u}\end{aligned}\right\}\quad(\mathbf{165})$$

A particular case of interest is when the load is a Synchronous Motor or Induction Motor with unsymmetrical line impedances in series—Equation (**163**) becomes

$$\left.\begin{aligned}(Z_{a0}' + Z_1' + Z_1)\,\check{I}_{a1}' + Z_{a2}'\,I_{a2}' &= \check{E}_{a2}\\ Z_{a1}\,\check{I}_{a1}' + (Z_{a0}' + Z_2' + Z_2)\,I_{a2}' &= O\\ I_{a1}' = \frac{Z_{a0}' + Z_2' + Z_2}{(Z_{a0}'+Z_1'+Z_1)\,(Z_{a0}'+Z_2'+Z_2) - Z_{a1}\,Z_{a2}}\,\check{E}_{a1}\\ I_{a2}' = \frac{Z_{a1}}{(Z_{a0}'+Z_1'+Z_1)\,(Z_{a0}'+Z_2'\,Z_2) - Z_{a1}\,Z_{a2}}\,\check{E}_{a1}\end{aligned}\right\}\quad(\mathbf{166})$$

An important case is that of a generator feeding into a symmetrical motor and an unsymmetrical load. Let the motor currents be

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$\check{I}_a$, $\check{I}_b$, $\check{I}_c$, those of the load I_a', I_b', I_c' and the load impedances Z_a', Z_b', Z_c'. The equations of this system will be

$$\left.\begin{aligned} S^1\,\check{E}_{a1} &= S^1\{Z_1'(\check{I}_{a1}+\check{I}_{a1}')+Z_{a0}'\,\check{I}_{a1}'+Z_{a2}'\,\check{I}_{a2}'\} \\ S^1\,\check{E}_{a1} &= S^1\{Z_1'(\check{I}_{a1}+\check{I}_{a1}')+Z_1\,\check{I}_{a1}\} \\ S^2\,O &= S^2\{Z_2'(\check{I}_{a2}+\check{I}_{a2}')+Z_{a0}'\,\check{I}_{a2}+Z_{a1}'\,\check{I}_{a1}'\} \\ S^2\,O &= S^2\{Z_2'(\check{I}_{a2}+\check{I}_{a2}')+Z_2\,\check{I}_{a2}\} \end{aligned}\right\} \quad (167)$$

Or, omitting the sequence symbols and re-arranging—

$$\left.\begin{aligned} \check{E}_{a1} &= Z_1'\,\check{I}_{a1}+(Z_1'+Z_{a0}')\,\check{I}_{a1}'+Z_{a2}'\,\check{I}_{a2}' \\ \check{E}_{a1} &= (Z_1'+Z_1)\,\check{I}_{a1}+Z_1'\,\check{I}_{a1}^1 \\ O &= Z_2'\,\check{I}_{a2}+Z_{a1}'\,\check{I}_{a1}^1+(Z_2'+Z_{a0}')\,\check{I}_{a2}' \\ O &= (Z_2'+Z_2)\,\check{I}_{a2}+Z_2'\,\check{I}_{a2}' \end{aligned}\right\} \quad (168)$$

These equations can be further simplified as follows:

$$\left.\begin{aligned} O &= (Z_2'+Z_2)\,\check{I}_{a2}+Z_2'\,\check{I}_{a2}' \\ O &= -\,Z_2\,\check{I}_{a2}+Z_{a1}'\,\check{I}_{a1}'+Z_{a0}'\,\check{I}_{a2}' \\ O &= -\,Z_1\,\check{I}_{a1}+Z_{a0}'\,\check{I}_{a0}'+Z_{a2}'\,I_{a2}' \\ \check{E}_{a1} &= (Z_1'+Z_1)\,\check{I}_{a1}+Z_1'\,\check{I}_{a1}' \end{aligned}\right\} \quad (169)$$

A set of simultaneous equations which may be easily solved.

The Single-Phase Generator is an Important Case of the Three-Phase Generator Operated on an Unbalanced Load

Let the impedance of the single-phase load be Z and let us suppose it to be made up of three star connected impedances

$$Z_a' = 3\,Z_s + \frac{Z}{2}$$

$$Z_b' = \frac{Z}{2}$$

$$Z_c' = \frac{Z}{2}$$

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the value of Z_x in the limit being infinity. Then we have

$$\left.\begin{aligned} Z_{a0}' &= Z_x + \frac{Z}{2} \\ Z_{a1}' &= Z_x \\ Z_{a2}' &= Z_x \end{aligned}\right\} \quad (170)$$

Equation (164) in the limit when Z_x becomes infinite reduces to

$$\begin{aligned} \check{I}_{a1}' &= \frac{\check{E}_{a1}}{Z + Z_1' + Z_2'} \\ \check{I}'_{a2} &= -\frac{\check{E}_{a1}}{Z + Z_1' + Z_2'} \end{aligned} \quad (171)$$

The single-phase load being across the phase $B\ C$, the single-phase current I will therefore be equal to $\check{I}_c$ or

$$\left.\begin{aligned} \check{I} &= \frac{j\sqrt{3}\,\check{E}_{a1}}{Z + Z_1' + Z_2'} \\ &= \frac{\check{E}_{bc}}{Z + Z_1' + Z_2'} \end{aligned}\right\} \quad (172)$$

$$\left.\begin{aligned} \check{I}_{u1} &= O \text{ if } R_u > O \\ \check{I}_{u2} &= -j\,\frac{1}{\sqrt{3}}\,\check{K}_2\,\check{I}\,\epsilon^{jwt} \\ \check{I}_{u2} &= -\frac{j\,\check{K}_2}{\sqrt{3}}\,I\,\epsilon^{j2wt} \end{aligned}\right\} \quad (173)$$

$\check{I}_{u2}$ is double normal frequency

$$\left.\begin{aligned} P_I + j\,Q_I &= 3\,I^2\,Z \\ P_L + j\,Q_L &= 3\,I^2\,(Z_1' + Z_2') \\ (P + j\,Q) + (P_H + j\,Q_H) &= 3\,\check{E}_{bc}\,(\check{I} + \check{I}) \end{aligned}\right\} \quad (174)$$

In the case of the generally unbalanced three-phase load

$$\left.\begin{aligned} P_1 + j\,Q_1 &= 3\,\{(I_{a1}^2 + I_{a2}^2)\,Z_{a0}' \\ &\qquad + \check{I}_{a1}\,\check{I}_{a2}\,Z_{a2}' + \check{I}_{a1}\,\check{I}_{a2}\,Z_{a1}'\} \\ P_L + j\,Q_L &= 3\,\{I_{a1}^2\,Z_1' + I_{a2}^2\,Z_2'\} \\ (P + j\,Q) + (P_H + j\,Q_H) &= 3\,\check{E}_{a1}\,(\check{I}_{a2} + \check{I}_{a2}) \end{aligned}\right| \quad (175)$$

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When the generator has harmonics in its wave form equations (**162**) must be written

$$\left.\begin{aligned}(R_a + j\,w\,L_a + Z_{a0}')\,\check{I}_{a1}' + Z_{a2}'\,I_{a2}' &= \check{E}_{a1}\\ Z_{a1}'\,I_{a1}' + \{Z_{a0}' + (R_a + K_2^2\,R_u)\\ + j\,2\,w\,(L_a - K_2^2\,L_u) - \tfrac{1}{2}\,a^2\,R_u\}\,I_{a2}' &= \check{E}_{a2}\end{aligned}\right\} \quad (176)$$

Where $\check{E}_{a1}$ is finite, $\check{E}_{a2}$ is zero and vice versa, the frequencies being different in each case, we have therefore a solution for each frequency depending on the phase and amplitude and phase sequence of the e. m. f. of this frequency generated. Of course the values of Z_1' and Z_2' change with each frequency on account of the change in the reactance with frequency, and a value must be taken for w conforming with the frequency of the harmonic under consideration.

Symmetrical Synchronous Motor, Synchronous Condenser, Etc.

As in the case of the generator, the synchronous motor has two impedances, one to the positive phase sequence current of a given frequency and the other to the negative phase sequence current of the same frequency. But, since there is no quantity in the positive phase sequence impedance corresponding to the virtual resistance which indicates mechanical work in an induction motor, its equivalent is furnished by the excitation of the field. Let us denote the e. m. f. due to the field excitation by $S^1\,\check{E}_{a1}'$ assuming it to be for the present a simple harmonic three-phase system. Let P_0 be the output of the motor which will include the windage and iron losses assumed to be constant. Then for the synchronous motor on a balanced circuit of e. m. f. $S^1\,\check{E}_{a1}$ we have

$$S^1\,\check{E}_{a1} = S^1\,\{\check{I}_{a1}'(R_a' + j\,w\,L_a') + \check{E}_{a1}'\} \quad (177)$$

$$S^0\,\check{E}_{a1}\,\hat{I}_{a1} = S^0\left\{I_{a1}^2\,(R_a' + j\,w\,L_a') + \frac{P_0}{3} - j\,\frac{Q_0}{3}\right\} \quad (178)$$

Where Q_0 is the imaginary part of the product, $\check{E}_{a1}'\,\hat{I}_{a1}$. (**178**) reduces to

$$E_{a1}\,I_{a1}\cos\alpha = I_{a1}^2\,R_a' + \frac{P_0}{3} \quad (179)$$

Where $\cos\alpha$ is the required operating power factor. Solving for I_{a1}

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$$I_{a1} = \frac{E_{a1} \cos \alpha}{2 R_a^1} \left\{ 1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 E_{a1}^2 \cos^2 \alpha}} \right\} \tag{180}$$

$$\check{I}_{a1} = \check{E}_{a1} \frac{\cos \alpha}{2 R_{a1}} \left\{ 1 \pm \sqrt{1 - \frac{4 R_{a1}' P_0}{3 E_{a1}^2 \cos^2 \alpha}} \right\} (\cos \alpha - j \sin \alpha) \tag{181}$$

The apparent impedance of the motor is

$$\frac{2 R_1 \sec \alpha}{1 \pm \sqrt{1 - \frac{4 P_0}{3 E_a^2 \cos^2 \alpha}}} (\cos \alpha + j \sin \alpha) \tag{182}$$

and

$$\check{E}_{a1}^1 = \check{E}_{a1} \left[1 - \frac{\cos \alpha}{2 R_a^1} \left\{ 1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 E_{a1}^2 \cos^2 \alpha}} \right\} (\cos \alpha - j \sin \alpha) (R_a' + j w L_a') \right] \tag{183}$$

The same equations apply to the case of the synchronous condenser with the difference that the mechanical work is that required to overcome the iron and windage losses only.

If we take

$$\left.\begin{aligned} \check{E}_{a1} &= E_{a1} (\cos \alpha + j \sin \alpha) \epsilon^{jwt} = (A_1 + j B_1) \epsilon^{jwt} \\ \check{E}_{a1}' &= (A_1' + j B_1') \epsilon^{jwt} \end{aligned}\right\} \tag{184}$$

we have

$$\check{I}_{a1} = \frac{A_1}{2 R_a^1} \left(1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 A_1^2}} \right) \epsilon^{jwt} \tag{185}$$

$$A_1' = \frac{A_1}{2} \left(1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 A_1^2}} \right) \epsilon^{jwt} \tag{186}$$

$$B_1' = \left\{ B_1 - \frac{j w L_a' A_1}{2 R_a'} \left(1 \pm \sqrt{1 - \frac{4 R_a' P_0}{3 A_1^2}} \right) \right\} \epsilon^{jwt} \tag{187}$$

Since α may be a positive or negative angle, the sine may be positive or negative for a positive cosine, and therefore the power factor will be leading or lagging accordingly as B_1 is negative or positive respectively. The double signs throughout are due to the fact that for any given load and power factor there are always two theoretically possible running conditions. However, since

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we are concerned only with that one which will give the max. operating efficiency, that is the condition that gives I_{a1} the lesser value, for a given value of P_0 the equations may be written

$$\left.\begin{aligned} \check{I}_{a1} &= \frac{A_1}{2\,R_a^1}\left(1-\sqrt{1-\frac{4\,R_a'\,P_0}{3\,A_1^2}}\right)\epsilon^{jw_0t} \\ A_1' &= \frac{A_1}{2}\left(1+\sqrt{1-\frac{4\,R_a'\,P_0}{3\,A_1^2}}\right)\epsilon^{jw_0t} \\ B_1' &= \left\{B_1-\frac{j\,w_0\,L_a'\,A_1}{2\,R_a'}\left(1-\sqrt{1-\frac{4\,R_a'\,P_0}{3\,A_1^2}}\right)\right\} \end{aligned}\right\} \quad (188)$$

And corresponding values for (**180**), (**181**), (**182**) and (**183**) may be obtained by omitting the positive sign in these equations.

Another condition of operation is obtained by inspection of (**180**), due to the fact that I_{a1} must be a real quantity

$$\frac{4\;R_a'\;P_0}{3\,E_{a1}^2\cos^2\alpha} \text{ must be } > 1 \qquad (189)$$

this is the condition of stability. In terms of (**184**) it becomes

$$\frac{4\,R_a'\,P_0}{3\,A_1^2} \text{ must be } > 1 \qquad (190)$$

The same conditions apply to the synchronous condenser, the total mechanical load in this case being the iron loss and windage and friction losses.

Proceeding now to operation with unbalanced circuits having sine waves the motor also having a sine wave. In addition to equation (**177**) we shall have

$$S^2\,\check{E}_{a2} = S^2\,Z_2'\,\check{I}_{a2} \qquad (191)$$

The mechanical power delivered through the operation of this negative phase sequence e. m. f. is given by P_N where

$$P_N = -\,3\;I_{a2}^2\,\frac{R_a'}{2} \qquad (192)$$

this quantity must therefore be subtracted from the value of P_0 in all the equations in which P_0 appears when unbalanced circuits are used in connection with equations (**177**) to (**190**) inclusive. These equations, however, give the conditions for maintaining a given mechanical load and a given power factor in the positive phase sequence component, but in practise what is re-

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quired is the combined power factor of the whole system, or the conditions to give a certain combined factor while delivering a given mechanical load; this may be obtained as follows:

The negative phase sequence component is a perfectly definite impedance and is independent of the load, and therefore the zero frequency part of the product $E_{a2}\, I_{a2}$ may be set down as

$$\check{E}_{a2}\, \hat{I}_{a2} = \frac{P_2}{3} + j\,\frac{Q_2}{3} \tag{193}$$

we have also for the positive phase sequence power delivered

$$(A_1 + j\,B_1)\; I_{a1} = I_{a1}^2\, R_a' + \frac{P_0}{3} - \frac{P_N}{3} + j\,(w\, I_{a1}\, L_a' + B_1^1)\; I_{a1} \tag{194}$$

And the power factor is given by $\cos\alpha$, where

$$\tan\alpha = \frac{I_{a1}\, B_1 + \dfrac{Q_2}{3}}{I_{a1}\, A_1 + \dfrac{P_2}{3}} \tag{195}$$

From (**194**) we have

$$A_1\, I_{a1} = I_{a1}^2\, R_a^1 + \frac{P_0}{3} - \frac{P_N}{3} \tag{196}$$

$$B_1 = w\, I_{a1}\, L_a' + B_1' \tag{197}$$

$$A_1^2 + B_1^2 = E_{a1}^2 \tag{198}$$

The simplest method of solving these equations is by means of curves. Taking arbitrary values of I_{a1}, B_1 and A_1 are chosen consistent with (**198**) so as to satisfy (**195**), $\frac{P_0}{3}$ A_1' and B_1' are then obtained from (**196**) and (**197**). If there are harmonics in the impressed e. m. f. but there are none in the wave form of the machine, the machine will have a definite impedance to the positive and negative phase sequence components of each harmonic, so that there will be a definite amount of mechanical work contributed by each harmonic which must be subtracted from the total work to be done to give the amount of work contributed by the positive phase sequence fundamental component, the equations will be identical to (**193**), (**194**), (**195**), (**196**),

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(197) and (198), if we take P_N to mean the total mechanical work done by the harmonics both positive and negative phase sequence and P_2 and Q_2 to represent the products

$$\Sigma\ ({}_n\check{E}_{a1}\ \ {}_n\hat{I}_{a1} + {}_n\check{E}_{a2}\ \ {}_n\hat{I}_{a2})$$

the zero frequency part only being taken into account.

When harmonics are present both in the impressed wave and in the generated wave, the problem becomes too complicated to treat generally, but specific cases can be worked out without much difficulty.

Phase Converters and Balancers

The phase converter is a machine to transform energy from single-phase or pulsating form to polyphase or non-pulsating form or vice versa to transform energy from polyphase to single-phase. The transformation may not be complete, that is to say, the polyphase system may not be perfectly balanced when supplied from a single-phase source through the medium of a phase converter. Phase converters may be roughly divided into two classes, namely—shunt type and series type.

Induction Motor or Synchronous Condenser Operating as a Phase Converter of the Shunt Type to Supply a Symmetrical Induction Motor or Synchronous Motor

Let Z_1 and Z_2 be the positive and negative phase sequence impedances of the motor, Z_1', Z_2' those of the phase converter. Let $S^1\ \check{E}_{a1}$ and $S^2\ \check{E}_{a2}$ be the positive and negative phase sequence components of the star e. m. f. impressed on the motor as a result of the operation. The single-phase supply will be one side of the delta e. m. f. $S\ \check{E}_{bc}$ which has positive and negative phase sequence components $S^1\ E_{bc1}$ and $S^2\ E_{bc2}$ the single-phase supply being $\check{E}_{bc} = \check{E}_{bc1} + \check{E}_{bc2}$.

The value of Z_2' may be considered fixed for all practical purposes and since in the induction motor phase converter the speed is practically no-load speed, Z' is practically the no-load impedance plus a real part obtained by increasing the real part of the no-load impedance by the ratio of the normal no-load losses to these same losses plus ½ the secondary losses due to the phase converter currents. The latter may be calculated roughly as even a large error in its value will have an inappreciable effect on the actual results. We have therefore

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$$\left.\begin{aligned} S^1 \check{E}_{a1} &= -S^1 j \frac{\check{E}_{bc}}{\sqrt{3}} \\ S^2 \check{E}_{a2} &= S^2 j \frac{\check{E}_{bc2}}{\sqrt{3}} \end{aligned}\right\} \quad (199)$$

$$\left.\begin{aligned} S_1{}^1 \check{I}_{a1}' &= -S^1 j \frac{\check{E}_{bc1}}{\sqrt{3}\, Z_1'} \\ S^1 \check{I}_{a1} &= -S^1 j \frac{\check{E}_{bc1}}{\sqrt{3}\, Z_1} \end{aligned}\right\} \quad (200)$$

$$\left.\begin{aligned} S^2 \check{I}_{a2}' &= S^2 j \frac{\check{E}_{bc2}}{\sqrt{3}\, Z_2'} \\ S^2 \check{I}_{a2} &= S^2 j \frac{\check{E}_{bc2}}{\sqrt{3}\, Z_2} \end{aligned}\right\} \quad (201)$$

In the common lead of motor and converter we have

$$\check{I}_{a1}' + \check{I}_{a2}' + \check{I}_{a1} + \check{I}_{a2} = O \quad (202)$$

or, substituting from (200) and (201)

$$\check{E}_{bc2}\left(\frac{1}{Z_2'} + \frac{1}{Z_2}\right) = \check{E}_{bc1}\left(\frac{1}{Z_1'} + \frac{1}{Z_1}\right) \quad (203)$$

$$\frac{\check{E}_{bc1}}{\check{E}_{bc2}} = \frac{\dfrac{1}{Z_2'} + \dfrac{1}{Z_2}}{\dfrac{1}{Z_1'} + \dfrac{1}{Z_1}} \quad (204)$$

$$\check{E}_{bc1} = \frac{\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}}{\left(\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}\right) + \left(\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}\right)} \check{E}_{bc} \quad (205)$$

$$\check{E}_{bc2} = \frac{\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}}{\left(\dfrac{1}{Z_1} + \dfrac{1}{Z_1'}\right) + \left(\dfrac{1}{Z_2} + \dfrac{1}{Z_2'}\right)} \check{E}_{bc} \quad (206)$$

which give the complete solution for all the quantities required with the aid of equations (200) and (201). For the supply current $\check{I}$

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$$\left.\begin{aligned} \check{I} &= \check{I}_{bc1} + \check{I}_{bc2} + I_{bc1}' + I_{bc2}' \\ S\,\check{I}_{bc} &= S^1\,\check{I}_{bc1} + S^2\,\check{I}_{bc2} \\ S\,\check{E}_{bc} &= S^1\,\check{E}_{bc1} + S^2\,E_{bc2} \end{aligned}\right\} \quad (207)$$

$$P_1 + j\,Q_1 = \check{E}_{bc}\,\hat{I} \quad (208)$$

In order to obtain a perfect balance we may consider the addition of an e. m. f. $S^2\,j\,\frac{E_{x2}}{\sqrt{3}}$ in series with the phase converter whose value must be a function of the load and the phase converter impedances, and therefore equation (**201**) will be replaced by

$$S^2\,\check{I}_{a2}' = S^2\left(j\,\frac{\check{E}_{bc2}}{\sqrt{3}\,Z_2'} + j\,\frac{\check{E}_{x2}}{\sqrt{3}\,Z_2'}\right) \quad (209)$$

$$S^2\,\check{I}_{a2} = S^2\,j\,\frac{\check{E}_{bc}}{\sqrt{3}\,Z_2}$$

and since the balance is perfect $\check{E}_{bc2}$ is zero, and therefore

$$S^2\,j\,\frac{\check{E}_{x2}}{\sqrt{3}} = S^2\,Z_2'\,\check{I}_{a2}' \quad (210)$$

An e. m. f. equal and of opposite phase to the negative phase sequence drop through the phase converter is required to produce a perfect balance.

Carrying out the solution in the same manner as in the imperfect converter, we obtain

$$\check{E}_{bc2} = \frac{\frac{1}{Z_1} + \frac{1}{Z_1'}}{\frac{1}{Z_2} + \frac{1}{Z_2'}}\,E_{bc} - \frac{\frac{1}{Z_2'}}{\frac{1}{Z_2} + \frac{1}{Z_2'}}\,\check{E}_{x2} \quad (211)$$

and since $\check{E}_{bc2}$ is zero and $\check{E}_{bc1} = \check{E}_{bc}$ the single-phase impressed e. m. f., we obtain

$$\check{E}_{x2} = Z_2'\left(\frac{1}{Z_1} + \frac{1}{Z_1^1}\right)\check{E}_{bc} \quad (212)$$

and therefore from (**210**)

$$S^2\,\check{I}_{a2}' = S^2\,j\left(\frac{1}{Z_1} + \frac{1}{Z_1'}\right)\frac{\check{E}_{bc}}{\sqrt{3}} \quad (213)$$

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$$S^1 \check{I}_{a1}{}^1 = - S^1 j \frac{\check{E}_{bc}}{\sqrt{3}\, Z_1'} \tag{214}$$

$$S^2 \check{I}_{a2} = O \tag{215}$$

$$S^1 \check{I}_{a1} = - S^1 j \frac{\check{E}_{bc}}{\sqrt{3}\, Z_1} \tag{216}$$

Figs. 9, 10, 11 and 12 are vector diagrams of some of the principal compensated shunt-type phase converters. There will be no difficulty in following out these diagrams if the principles of this paper have been grasped.

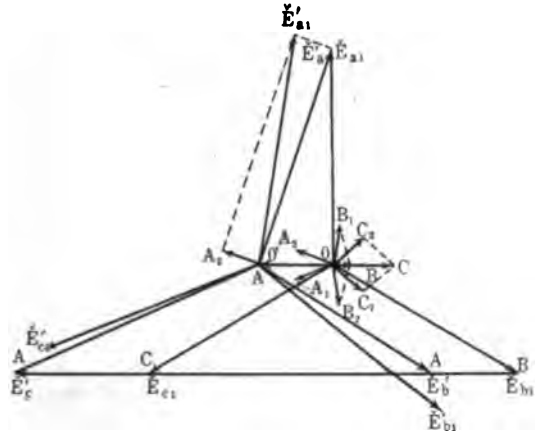

FIG. 9—VECTOR DIAGRAM OF SHUNT-TYPE PHASE CONVERTER OPERATED FROM TRANSFORMER SO AS TO DELIVER BALANCED CURRENTS

Terminal voltages of phase converter $S\check{E}'_a$

Terminal voltages of motor $S^1\check{E}_{a1}$

Negative phase sequence e.m.fs. in phase converter S^2 (OA_2)

The Phase Balancer is a device to maintain symmetry of e. m. fs. at a given point in a polyphase system. It may consist of an induction motor or synchronous condenser with an auxiliary machine connected in series to supply an e. m. f. always proportional to the product of the negative phase sequence current passing through the machine and the negative phase sequence impedance of the balancer. It therefore has the effect of annulling the impedance of the machine to the flow of negative phase sequence current. Thus, in a symmetrical polyphase network, where we have an unbalanced system of currents due to certain conditions

$$S\, \check{I}_a = S^1\, \check{I}_{a1} + S^2\, \check{I}_{a2} \tag{217}$$

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If a balancer be placed at the proper point the component $S^2 \check{I}_{a2}$ will circulate between the loads and the phase balancer, the other component $S^1 I_{a1}$ being furnished from the power house. On the other hand, if there be a dissymmetry in the impedance of the system up to the phase balancer, the latter will draw a negative phase sequence current sufficient to counteract the unbalance due to any symmetrical load by causing the proper amount of negative phase sequence current to flow to produce a balance.

The balancer may be made inherently self-balancing by inserting in series with it a machine which is self-exciting and is able

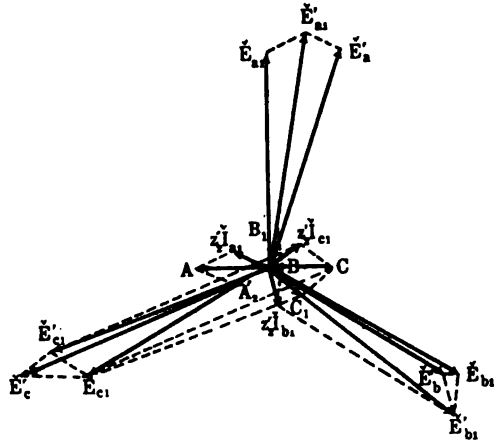

FIG. 10—VECTOR DIAGRAM SHOWING RELATIONS BETWEEN MOTOR TERMINAL E.M.F'S., CONVERTER TERMINAL E.M.FS., AND SYMMETRICAL GENERATED E.M.F'S., SAME CONNECTION AS FOR FIG. 9.

Negative phase sequence drops in phase converter $S^2 Z_2' \check{I}_{a1}$
Conjugate positive phase sequence e.m.fs. $S^1(ABC)$

to furnish an e. m. f. equal to the negative phase sequence impedance drop. The combination thus has zero impedance to negative phase sequence currents. If in the neighborhood of a phase balancer the loads have impedances

$$S\,Z_a = S^0\,Z_{a0} + S^1\,Z_{a1} + S^2\,Z_{a2}$$

The equations of the system are

$$\left.\begin{aligned} S^1\,\check{E}_{a1} &= S^1\,Z_{a0}\,\check{I}_{a1} + S^1\,Z_{a2}\,\check{I}_{a2} \\ S^2\,E_{a2} &= O = S^2\,Z_{a0}\,\check{I}_{a2} + S^2\,Z_{a1}\,\check{I}_{a1} \end{aligned}\right\} \quad (218)$$

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The currents in the phase balancer are

$$- S^2 \check{I}_{a2} \text{ and } S^1 \frac{\check{E}_{a1}}{Z_1'}$$

The solution of (218) gives $S^2 \check{I}_{a2}$ and $S^1 \check{I}_{a1}$, the former of which are the phase balancer currents. The solution is

$$\left.\begin{aligned} \check{I}_{a1} &= \frac{Z_{a0}}{Z_{a0}^2 - Z_{a1} Z_{a2}} \check{E}_{a1} \\ \check{I}_{a2} &= - \frac{Z_{a1}}{Z_{a0}^2 - Z_{a1} Z_{a2}} \check{E}_{a1} \end{aligned}\right\} \quad (219)$$

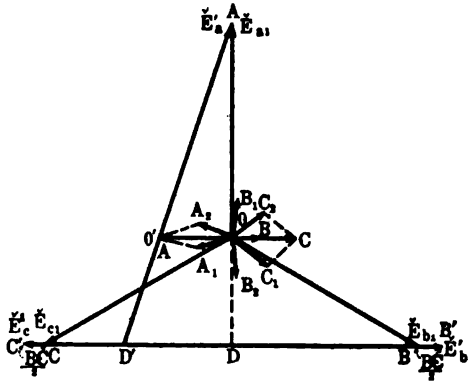

FIG. 11—VECTOR DIAGRAM OF SHUNT-TYPE PHASE CONVERTER SCOTT CONNECTED WITH COMPENSATION BY TRANSFORMER TAPS
Terminal voltages of converter $D'A$ and $B'C^1$
Terminal voltages of motor $S^1\check{E}_{a1}$

The phase balancer is a voltage balancer and will maintain balanced e. m. f. for any condition of impedance, and if the impedance of the mains is unsymmetrical it will draw a sufficient amount of wattless negative phase sequence current through these mains to produce an e. m. f. balance at its terminals. Hence the complete solution requires consideration of all the connections in the network between the supply point and the balancer. Two equations for each mesh and connection are required, one of the positive phase sequence e. m. fs. and the other of the negative phase sequence e. m. f., and these equations may be solved in the usual way.

Series Phase Converter. In discussing the various reactions in rotating machines we have made use of the terms "positive phase

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sequence impedance" and "negative phase sequence impedance." These terms are definite enough when dealing with relations between machines whose generated e. m. fs. all have the same phase sequence, but require further definition when we are dealing with relations between machines whose e. m. fs. have different phase sequence. We shall retain the symbols Z_1 and Z_2 for the values of the positive and negative phase sequence impedances, depending upon the sequence symbol S to define whether these impedances apply to a negative or positive phase sequence current. Thus, the phase sequence of the currents and

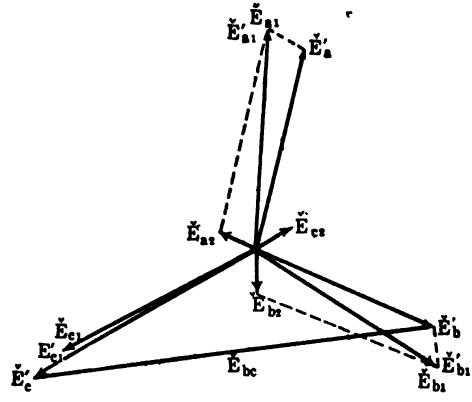

FIG. 12—VECTOR DIAGRAM OF SHUNT-TYPE PHASE CONVERTER WITH AUXILIARY ROTATING COMPENSATOR TO EFFECT A PERFECT BALANCE

Terminal voltages of phase converter $S\,\check{E}_a'$
Terminal voltages of motor $S^1\check{E}_{a1}$
Terminal voltages of compensator $S^2\check{E}_{a2}$

e. m. f. will be defined by the apparatus supplying and receiving power and the impedances of the transmitting apparatus will be defined in relation to these currents. As an example a motor series connected in counter phase sequence relation in a circuit and driven in a positive direction will have impedances

$$\begin{aligned} &\text{positive phase sequence } Z_2 \\ &\text{negative phase sequence } Z_1 \end{aligned} \tag{220}$$

Where an auxiliary machine is defined as being of negative phase sequence relation to other machines, it will have imped-

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ances as given above to the positive and negative phase sequence currents passing through the other machines.

A single-phase transformer winding tapped at the middle point may be regarded as an unbalanced three-phase system where

$$\check{E}_a = O, \check{E}_b = + \check{E}_s, \check{E}_c = - \check{E}_s$$

$2\check{E}_s$ being the single-phase e. m. f The system may be represented by the equation

$$S\check{E}_a = S^1\check{E}_{a1} + S^2\check{E}_{a2}$$

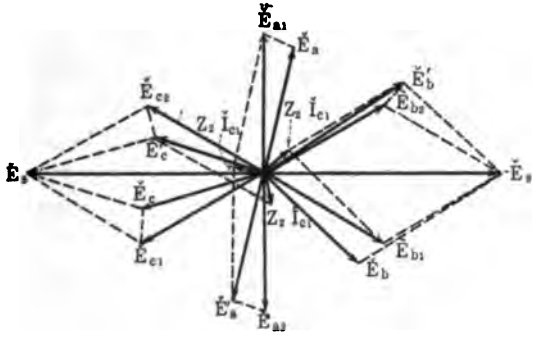

FIG. 13—VECTOR DIAGRAM OF SERIES-TYPE CONVERTER.
NO LOAD E.M.F'S. ACROSS MOTOR TERMINALS $S_1\check{E}_{a1}$
NO LOAD E.M.F'S. ACROSS CONVERTER TERMINALS $S^2\check{E}_{a2}$
SINGLE-PHASE E.M.F'S. $2\check{E}_s$
E.M.F. ACROSS TERMINAL OF MOTOR UNDER LOAD $\check{E}_a\check{E}_b\check{E}_c$
E.M.F. ACROSS TERMINAL OF CONVERTER UNDER LOAD $\check{E}'_a\check{E}'_b\check{E}'_c$

$$\left.\begin{aligned} \text{where } \check{E}_{a1} &= j\frac{\check{E}_s}{\sqrt{3}} \\ \check{E}_{a2} &= -j\frac{\check{E}_s}{\sqrt{3}} \end{aligned}\right\} \quad (221)$$

If, therefore between the single-phase source of power and the load we interpose a polyphase machine with e. m. f. $- S^2\check{E}_{a2}$, we shall have at the load terminals the e. m. f. $S^1\check{E}_{a1}$. If we use an induction-type phase converter it will have impedances to motor currents as follows

$$\left.\begin{aligned} &\text{To positive phase sequence } Z_2' \\ &\text{To negative phase sequence } Z_1' \end{aligned}\right\} \quad (222)$$

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we therefore have the relations

$$S^1 \check{E}_{a1} = S^1 \check{I}_{a1} (Z_1 + Z_2') \tag{223}$$

$$S^2 \check{E}_{a2} = S^2 \check{I}_{a2} (Z_2 + Z_1') \tag{224}$$

If the converter is doing no mechanical work, Z_1' is large compared with Z_2' or Z_2, and therefore the component of negative phase sequence is small in the motor. The value of Z_1' depends upon the slip of the phase converter which will depend on the mechanical load it carries as well as on the load carried by the motors. Approximately the load currents due to the motors produce the equivalent at the phase converter of a mechanical load equal to one-half the rotor loss of the phase converter due to these load currents. Substituting the values given in (**221**) for $S^1 \check{E}_{a1}$ and $S^2 \check{E}_{a2}$, we obtain

$$\left.\begin{aligned} S^1 j \frac{\check{E}_s}{\sqrt{3}} &= S^1 \check{I}_{a1} (Z_1 + Z_2') \\ - S^2 j \frac{E_s}{\sqrt{3}} &= S^2 \check{I}_{a2} (Z_2 + Z_1') \end{aligned}\right\} \tag{225}$$

$$\begin{aligned} S^1 \check{I}_{a1} &= S^1 j \frac{\check{E}_s}{\sqrt{3}\,(Z_1 + Z_2')} \\ S^2 \check{I}_{a2} &= - S^2 j \frac{\check{E}_s}{\sqrt{3}\,(Z_2 + Z_1')} \end{aligned} \tag{226}$$

If instead of an induction-type phase converter a synchronous phase converter is used an e. m. f. of negative phase sequence $S^2 \check{E}_{a2}$ the generated e. m. f. of the phase converter must be introduced in equations (**224**) *and* (**225**) *and the value and phase of these e. m. fs. will depend upon the load on the phase converter shaft as well as the load carried by the motors.* The equations will be

$$S^1 \check{E}_{a1} = S^1 \check{I}_{a1} (Z_1 + Z_2') \tag{227}$$

$$S^2 \check{E}_{a2} = S^2 \check{I}_{a2} (Z_2 + Z_1') + S^2 \check{E}_{a2}' \tag{228}$$

or

$$\left.\begin{aligned} S^1 j \frac{\check{E}_s}{\sqrt{3}} &= S^1 \check{I}_{a1} (Z_1 + Z_2') \\ - S^2 j \frac{\check{E}_s}{\sqrt{3}} &= S^2 \check{I}_{a2} (Z_2 + Z_1') + S^2 \check{E}_{a2}' \end{aligned}\right\} \tag{229}$$

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The last member of equations (229) is the equation of a synchronous condenser. Assuming its windage, iron loss and increased losses due to secondary reactions to be P_0, we have by equation (160) of the Section on Synchronous Motors

$$\frac{E_s}{\sqrt{3}} I_{a2} \cos \alpha = I_{a2}^2 (R_2 + R_1') + \frac{P_0}{3} \tag{230}$$

Let

$$\check{I}_{a2} = a_2 + j\, b_2 \tag{231}$$

then (230) becomes

$$\frac{E_s}{\sqrt{3}} a_2 = (a_2^2 + b_2^2)(R_2 + R_1') + \frac{P_0}{3} \tag{232}$$

Of the two quantities a_2 and b_2, b_2 alone is arbitrary and depends upon the excitation, a_2 will depend upon the value of b_2 and also upon the losses. Solving therefore for a_2 in terms of b_2, we have

$$a_2 = \frac{E_s}{2\sqrt{3}(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4(R_2 + R_1')\{3 b_2^2 (R_2 + R_1') + P_0\}}{E_s^2}} \right\} \tag{233}$$

Since b_2 is arbitrary we may now determine $\cos \alpha_2 = \frac{a_2}{\sqrt{a_2^2 + b_2^2}}$ and the value of $\check{I}_{a2}$ in terms of the impressed e. m. f. will be by (181) of Section on Synchronous Motors

$$S^2 \check{I}_{a2} = - S^2 \left[j \frac{\check{E}_s}{\sqrt{3}} \frac{\cos \alpha_2}{2(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4(R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha_2}} \right\} \epsilon^{j\alpha} \right] \tag{234}$$

The effective value of $\check{I}_{a2}$ in terms of the effective value of $\check{E}_s$ will then be

$$I_{a2} = \frac{E_s}{\sqrt{3}} \frac{\cos \alpha_2}{2(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4(R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha_2}} \right\} \tag{235}$$

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and since the component of the e.m.f. generated in phase with the current is determined only by the magnitude of $\check{I}_{a2}$ and the motor losses, if we define its value by A_2' the quadrature component being B_2' we shall have

$$A_2' = \frac{E_s}{\sqrt{3}} \frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4 (R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha_2}}\right) \tag{236}$$

and

$$B_2' = - \frac{E_s}{\sqrt{3}} \sin \alpha_2 - \frac{w (L_2 + L_2')}{A_2^1} \tag{237}$$

$$= - \frac{E_s}{\sqrt{3}} \left\{ \sin \alpha_2 + \frac{3 w (L_2 + L_1')}{P_0} \cdot \frac{\cos \alpha_2}{2 (R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4 (R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha_2}}\right) \right\} \tag{238}$$

and therefore we have

$$\check{E}_2' = - j \frac{E_s}{\sqrt{3}} \left[\frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4 (R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha_2}}\right) - j \left\{ \sin \alpha_2 + \frac{3 w (L_2 + L_1') \cos \alpha_2}{2 P_0 (R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4 (R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha_2}}\right) \right\} \right] (\epsilon^{j\omega t}) \tag{239}$$

The impedance of the phase converter to the flow of negative phase sequence current is

$$\frac{2 (R_2 + R_1') \sec \alpha}{1 - \sqrt{1 - \frac{4 (R_2 + R_1') P_0}{E_s^2 \cos^2 \alpha}}} \tag{240}$$

The balance will be at its best when $\check{I}_{a2}$ is a minimum with $\cos \alpha_2$ as the independent variable. This will be the case when $\cos \alpha_2$ is unity; that is to say when b_2 is zero.

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Recapitulating the results given above, we have for the general case taking the single-phase e. m. f. $\check{E}_s$ as reference

$$S^1 \check{I}_{s1} = S^1 j \frac{\check{E}_s}{\sqrt{3}\,(Z_1 + Z_2')} \tag{241}$$

$$S^2 \check{I}_{s2} = -\,j\,(a_2 + j\,b_2) \tag{242}$$

where b_2 is arbitrary and

$$a_2 = \frac{E_s}{2\sqrt{3}\,(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,\{3\,b_2^2\,(R_2 + R_1') + P_0\}}{E_s^2}} \right\} \tag{243}$$

Since b_2 is arbitrary $\cos\alpha_2$ is determined by

$$\cos\alpha_2 = \frac{a_2}{\sqrt{a_2^2 + b_2^2}} \tag{244}$$

we may express $\check{I}_{s2}$ in terms of $\check{E}_s$ by

$$S^2 \check{I}_{s2} = -\,S^2 j \frac{\check{E}_s}{\sqrt{3}} \frac{\cos\alpha_2}{2\,(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2\alpha_2}} \right\} \epsilon^{j\alpha_2} \tag{245}$$

The effective value of $\check{I}_{s2}$ will be

$$I_{s2} = \sqrt{a_2^2 + b_2^2} = \frac{E_s}{\sqrt{3}} \frac{\cos\alpha_2}{2\,(R_2 + R_1')} \left\{ 1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2\alpha_2}} \right\} \tag{246}$$

If A_2' and B_2' are components of $\check{E}_{s2}'$ these being the generated e. m. f. in phase and in quadrature with the current $\check{I}_{s2}$ we shall have

$$\check{E}_{s2}' = -\,j\,(A_2' + j\,B_2') \tag{247}$$

and A_2' and B_2' will have the following values

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$$A_2' = \frac{E_s}{\sqrt{3}} \; \frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) \tag{248}$$

$$B_2' = -\frac{E_s}{\sqrt{3}} \left\{ \sin \alpha_2 + \frac{3\,w\,(L_2 + L_1')}{P_0} \cdot \frac{\cos \alpha_2}{2\,(R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4\,(R_2 + Q_1')\,P_0}{E_s^2 \cos \alpha^2}}\right) \right\} \tag{249}$$

and $\check{E}_{s2}'$ expressed in terms of $\check{E}_s$ becomes

$$\check{E}_2' = -j\,\frac{\check{E}_s}{\sqrt{3}} \left[\frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) - j \left\{ \sin \alpha_2 + \frac{3\,w\,(L_2 + L')\cos \alpha_2}{2\,P_0\,(R_2 + R_1')} \left(1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) \right\} \right] \tag{250}$$

The effective impedance of the phase converter to the flow of negative phase sequence currents is

$$\frac{2\,(R_2 + R_1')\sec \alpha_2}{1 - \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha}}} \,(\cos \alpha_2 - j \sin \alpha_2) \tag{251}$$

or

$$\frac{E_s^2}{P_0} \cdot \frac{\cos \alpha_2}{2} \left(1 + \sqrt{1 - \frac{4\,(R_2 + R_1')\,P_0}{E_s^2 \cos^2 \alpha_2}}\right) \epsilon^{-j\alpha_2} \tag{252}$$

In the above equations $\cos \alpha_2$ is arbitrary or b_2 may be considered arbitrary and $\cos \alpha_2$ will then be determined.

Minimum Unbalance is obtained when $\cos \alpha_2$ is made unity or when b_2 is made zero in equations (**241**) and (**252**).

Perfect Balance is obtained by driving the phase converter mechanically so as to supply the mechanical power P_0 from a separate or symmetrical source. Under this condition a_2 and b_2 both become zero when $\cos \alpha_2$ is unity. The only equation of the system is then (**241**).

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Currents and Power Factor in the Single-Phase Supply Circuit of Series Phase Converter.

The e. m. f. is $2\check{E}_s$ and the current supplied is

$$\check{I}_s = \frac{\check{I}_b - \check{I}_c}{2}$$

$$= \frac{\check{I}_{b1} - \check{I}_{c1}}{2} + \frac{\check{I}_{b2} - \check{I}_{c2}}{2} \tag{253}$$

If we take

$$S^1 \check{I}_{s1} = S^1 j\,(a_1 - j\,b_1) \tag{254}$$

$$\frac{\check{I}_{b1} - \check{I}_{c1}}{2} = \frac{\sqrt{3}}{2}\,(a_1 - j\,b_1) \tag{255}$$

Similarly, since under the same conditions

$$S^2 \check{I}_{s2} = -\,S^2 j\,(a_2 + j\,b_2) \tag{256}$$

$$\frac{\check{I}_{b2} - \check{I}_{c2}}{2} = \frac{\sqrt{3}}{2}\,(a_2 + j\,b_2) \tag{257}$$

and therefore

$$\check{I}_s = \frac{\sqrt{3}}{2}\,\{(a_1 + a_2) - j\,(b_1 - b_2)\} \tag{258}$$

where a_1, b_1, a_2, b_2 are to be obtained by means of equations (**243**) to (**254**). The single-phase power factor is given by

$$\tan\theta = \frac{b_1 - b_2}{a_1 + a_2} \tag{259}$$

of these quantities a_2 is usually the smallest and its value may be obtained approximately by assigning to b_2 a value which will make the ratio $\frac{b_1 - b_2}{a_1}$ equal to $\tan\theta$, and obtaining the corresponding value of a_2 by (**242**), the value of b_2 may then be recalculated from (**259**) by substituting the tentative value obtained for a_2. This procedure may be repeated until sufficient accuracy has been obtained.

Single-Phase Power Factor in Shunt-Type Phase Converter.

The simplest procedure is to obtain a curve of admittances for varying excitation of the converter and plot the power factor obtained by varying the admittance with a fixed load. The true

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and wattless power is obtained easily by means of (**208**) whether the system is balanced or unbalanced.

Figs. 14, 15, 16 and 17 are vector diagrams of several methods of using phase converters to supply a balanced 3-phase e. m. f. to a symmetrical load such as an induction motor. The diagrams are all based on a main machine having the same negative phase sequence impedance and the system in each case is

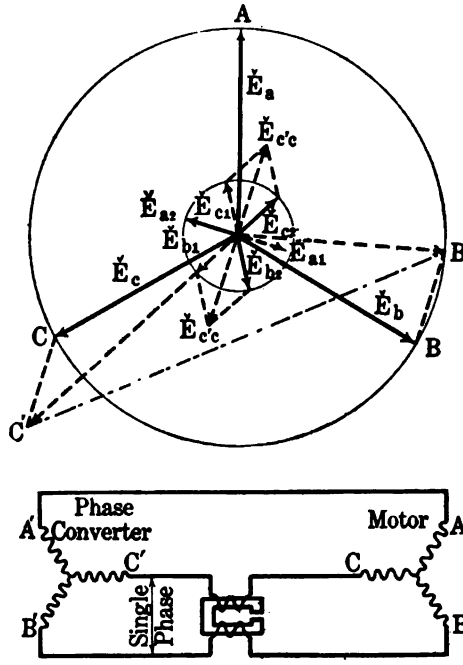

FIG. 14

SINGLE-PHASE IMPRESSED E.M.F. = $B'C'$
MOTOR E.M.F. = BC
NEGATIVE PHASE SEQUENCE E.M.FS. $\check{E}_{a2}\check{E}_{b2}\check{E}_{c2}$
CONJUGATE POSITIVE PHASE SEQUENCE E.M.FS. $\check{E}_{a1}\check{E}_{b1}\check{E}_{c1}$
PHASE CONVERTER TERMINAL E.M.F. $AB'C'$

delivering the same amount of power at the same voltage and 3-phase power factor without supplying any wattless power. It will be noted that the scheme Fig. 14 has the lowest single-phase power factor, Fig. 16 the highest and the rest arcing alike. It may be remarked, however, that with the shunt-type schemes adjustments can be made for power factor correction which will result also in better regulation.

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APPENDIX I

Cylindrical Fields in Fourier Harmonics

When we have a diametrical coil around a cylinder concentric with another cylinder which forms the return magnetic path, and the length of the gap is uniform and the coil dimensions are very small, the field across the gap takes the form of a square topped

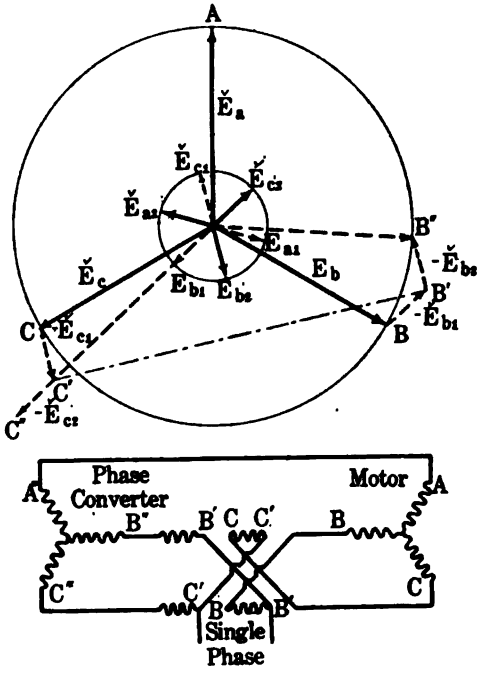

FIG. 15

SINGLE-PHASE IMPRESSED E.M.F. = $B'C'$
MOTOR E.M.F. = BC
PHASE CONVERTER E.M.F. = $B''C''$
NEGATIVE PHASE SEQUENCE E.M.F $\check{E}_{a2}\check{E}_{b2}\check{E}_{c2}$
CONJUGATE POSITIVE PHASE SEQUENCE E.M.F. $\check{E}_a{}^1E_{b1}E_{c1}$
PHASE CONVERTER TERMINAL E.M.F. $AB''C''$

wave, which may be expressed in the form of a Fourier series with the plane of symmetry of the coil as reference plane, and its Fourier expansion is

$$\mathfrak{B} = \frac{4B}{\pi}\left(\cos\theta - \tfrac{1}{3}\cos 3\theta + \frac{1}{S}\cos S\theta - \ldots + \ldots\right) \qquad (1)$$

where B is the average induction in the air gap.

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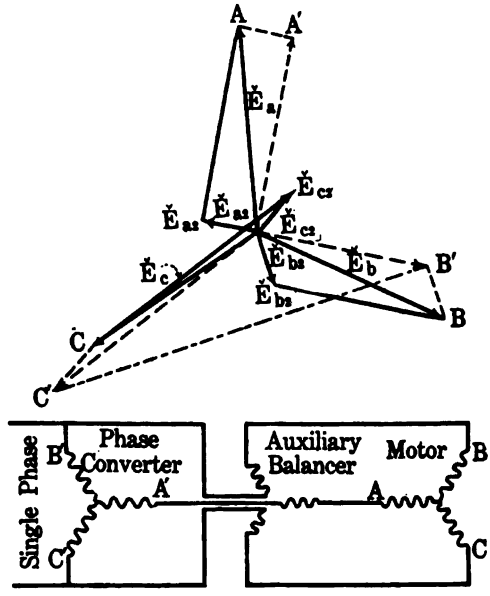

FIG. 16—PHASE CONVERTER WITH AUXILIARY BALANCER.

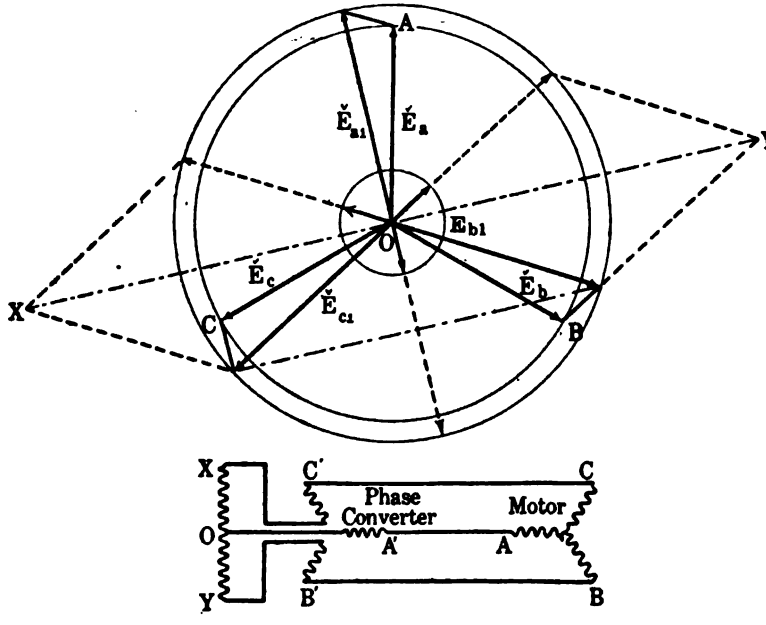

FIG. 17

SINGLE-PHASE IMPRESSED E.M.F. = *XY*

MOTOR E.M.F. = *ABC*

THERE IS A 2 TO 1 TRANSFORMATION OF E.M.F. FROM SINGLE-PHASE TO THREE-PHASE IN THIS CONNECTION

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With pitch less than π the curve will have a different form, the amplitude being greater on one side of the plane of the coils than on the other, the areas of each wave will remain the same and second harmonic terms will appear. Let $2\,m_0\,\pi$ be the new pitch then the average amplitude of the induction will be the same as before, namely B, and the value on one side of the coil will be $2\,(1-m_0)B$ and on the other side $2\,m_0\,B$ so that the total flux will be the same on either side. To obtain the values of the coefficients we have

$$2\,(1-m_0\,B\int_0^{m_0\pi}\cos n\,\theta\,d\,\theta+2\,m_0\,B\int_{m_0\pi}^{2\pi}\cos n\,\theta\,d\,\theta=\frac{\pi}{2}A_n$$

$$2\,(1-m_0)\,B\left[\frac{1}{n}\sin n\,\theta\right]_0^{m_0\pi}-2\,m_0\,B\left[\frac{1}{n}\sin n\,\theta\right]_{m_0\pi}^{2\pi}=\frac{\pi}{2}\,A_n$$

$$A_n=\frac{4\,B}{\pi}\left\{\frac{(1-m_0)+m_0}{n}\sin n\,m_0\,\pi\right\}$$

$$A_n=\frac{4\,B}{\pi}\left(\frac{1}{n}\sin n\,m_0\,\pi_1\right) \tag{2}$$

Let $2\,m_0\,\pi=\frac{2}{3}\,\pi$, then $(1-m_0)\,\pi=\frac{2}{3}\,\pi$ and

$$\mathfrak{B}=\frac{2\sqrt{3}\,B}{\pi}\left(\cos\theta+\frac{1}{2}\cos 2\,\theta-\frac{1}{4}\cos 4\,\theta-\frac{1}{5}\cos 5\,\theta\right.$$

$$\left.+\frac{1}{7}\cos 7\,\theta+\frac{1}{8}\cos 8\,\theta-\frac{1}{10}\cos 10\,\theta\;.\;.\;.\right) \tag{3}$$

A general expression for $\mathfrak{B}$ where B is the average of the positive and negative, maximum value for any pitch coil would be

$$\mathfrak{B}=\frac{4\,B}{\pi}\,\Sigma\left(\frac{1}{n}\sin n\,m_0\,\pi\cos n\,\theta\right) \tag{4}$$

and includes all possible coil pitches. If the number of teeth in a pole pitch be n_T; in addition to the average induction as indicated by (4), there will also be a tooth ripple of flux, the maximum value of which will depend upon the average value of the induction at each point. The value of m_0 must be a fraction having n_T as denominator and an integral numerator. The

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value of the integral numerator is therefore always $m_0 n_\tau$. The correct value for the max. induction will therefore be

$$\mathfrak{B}_m = \left\{ \frac{4B}{\pi} \Sigma \left(\frac{1}{n} \sin n\, m_0 \pi \cos n\, \theta \right) \right\} (1 - (-1)^{m_0 n_\tau} K_\tau \cos n_\tau \theta) \qquad (5)$$

where K_τ is the ratio of the average to the min. air gap. "m_0" must always be chosen so that $m_0 n_\tau$ is an integer.

If the length of the average effective air gap in centimeters be d the value of B is given by

$$B = \frac{4\pi}{10} \frac{IN}{2d} \text{ gauss}$$

where I is the maximum value of the current in the coil and N is the number of turns. If d is given in inches we may write

$$B = \frac{4\pi}{10} \frac{IN}{2d} \times 2.54 \text{ maxwells per square inch.}$$

If we integrate (5) between the limits $(\theta - m_0 \pi)$ and $(\theta + m_0 \pi)$ we shall have the total flux φ through the coil

$$\varphi = \frac{4Brl}{\pi} \int_{\theta - m_0 \pi}^{\theta + m_0 \pi} \Sigma \left(\frac{1}{n} \sin n\, m_0 \pi \cos n\, \theta \right) d\theta$$

$$- \frac{4Brl}{\pi} (-1)^{m_0 n_\tau} \int_{\theta - m_0 \pi}^{\theta + m_0 \pi} \Sigma \left(\frac{1}{n} \sin n\, m_0 \cos n\, \theta \right) K_\tau \cos n_\tau \theta\, d\theta$$

$$= \frac{4Bre}{\pi} \left[\frac{1}{n^2} \sin n\, m_0 \pi \sin n\, \theta \right]_{\theta - m_0 \pi}^{\theta + m_0 \pi}$$

$$- \frac{4Brl}{\pi} (-1)^{m_0 n_\tau} K_\tau \Sigma \frac{1}{n} \sin m_0 n \pi \left[\frac{\sin (n - n_\tau)\theta}{2(n - n_\tau)} + \frac{\sin (n + n_\tau \theta)}{2(n + n_\tau)} \right]_{\theta - m_0 \pi}^{\theta + m_0 \pi} \qquad (6)$$

The second expression is zero for all values of θ which are integral multiples of the tooth pitch angle, so long as $m_0 n$ is also an integer and therefore it is zero for all mutual inductive relations of similar coils on a symmetrical toothed core we therefore have:

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The induction through a coil displaced an angle θ from the axis of a similar coil carrying a current giving a mean induction B both coils being wound on the same symmetrical toothed core is

$$\varphi = \frac{8\,B\,r\,l}{\pi}\,\Sigma\left(\frac{1}{n^2}\,\sin^2 n\,m_0\,\pi\,\cos n\,\theta\right) \tag{7}$$

The second term in equation (6) also becomes zero when n_τ becomes infinite independent of the value of θ. We may therefore safely make use of an imaginary uniformly distributed winding when considering self and mutual impedances. It will also be shown later on, that with certain groupings of windings the second term may be reduced to zero for every value of θ.

If N_1 be the total number of complete loops in one complete pole pitch, we may take $\frac{N_1}{2\,\pi}$ as the density of winding per unit angle of the complete pole pitch. The mutual induction per turn in a coil angularly displaced an angle θ from another coil of winding density $\frac{N_1}{2\,\pi}$ with an effective total air gap $2\,d$ and with windings subtending an angle $2\,m_1\,\pi$ is given by

$$M_1 = \frac{8\,N_1\,r\,l}{10^9\,\pi\,d}\int_{-m_1\pi}^{+m_1\pi}\Sigma\left\{\frac{1}{n^2}\sin^2 n\,m_0\,\pi\cos n(\theta+\theta')\right\}d\,\theta' \text{ henrys} \tag{8}$$

$$= \frac{8\,N_1\,r\,l}{10^9\,\pi\,d}\,\Sigma\,\frac{1}{n^3}\,\sin^2 n\,m_0\,\pi\,[\sin n\,(\theta+\theta)']_{\theta'=-m_1\pi}^{\theta'=m_1\pi} \text{ henrys}$$

$$M_1 = \frac{16\,N_1\,r\,l}{10^9\,\pi\,d}\,\Sigma\left(\frac{1}{n^3}\,\sin^2 n\,m_0\,\pi\,\sin n\,m_1\,\pi\,\cos n\,\theta\right) \text{ henrys} \tag{9}$$

Next, if the loop of which M_1 is the mutual inductance is part of a winding having distribution density of winding $\frac{N_2}{2\,\pi}$ and subtending an angle $2\,m_2\,\pi$ its mutual inductance with the other winding will be

$$M_{12} = \frac{8\,N_1\,N_2\,r\,l}{10^9\,\pi^2\,d}\int_{-m_0\pi}^{m_2\pi}\Sigma\frac{1}{n^3}\,\sin^2 n\,m_0\,\pi\,\sin n\,m_1\,\pi\,\cos n\,(\theta+\theta')\,d\,\theta' \text{ henry} \tag{10}$$

$$= \frac{8\,N_1\,N_2\,r\,l}{10^9\,\pi^2\,d}\,\Sigma\,\frac{1}{n^4}\,\sin^2 n\,m_0\,\pi\,\sin n\,m_1\,\pi\,[\sin n\,(\theta+\theta')]_{\theta'=-m_2\pi}^{\theta'=m_2\pi} \text{ henrys}$$

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$$M_{12} = \frac{16\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, d}\, \Sigma \left(\frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$

$$\left. \sin n\, m_2\, \pi \cos n\, \theta \right) \text{ henrys} \qquad (11)$$

This is the general expression for the mutual inductance between two groups of connected coils of like form on the same cylindrical core. It should be noted how much the harmonics have been reduced due to grouping.

When the coils are not of like design as in the case of a rotor and stator and the pitch of the coils is different in one from the other, $\sin n\, m_0\, \pi$ will not appear twice in the equation but one of its values must be replaced by $\sin n\, m_x\, \pi$ where $2\, m_x\, \pi$ is the pitch of the new coil. Equation (**11**) then becomes

$$M_{1a} = \frac{16\, N_1\, N_a\, r\, e}{10^9\, \pi^2\, d}\, \Sigma \left(\frac{1}{n^4} \sin n\, m_0\, \pi \sin n\, m_x\, \pi \right.$$

$$\left. \sin n\, m_1\, \pi \sin n\, m_2\, \pi \cos n\, \theta \right) \text{ henrys} \qquad (12)$$

This formula is strictly correct when m_x is an integer and when θ is an integral multiple of the tooth pitch. It is true for all values of θ if either m_0 or m_x or both are unity.

By considering the axes of two similar groups of coils as coincident we obtain the value of $\Delta_1\, L_1$ which is part of the self inductance of the group, thus

$$\Delta_1\, L_1 = \frac{16\, N_1^2\, r\, e}{10^9\, \pi^2\, d}\, \Sigma \left(\frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin^2 n\, m_1\, \pi \right) \qquad (13)$$

The other factor that enters into the self inductance is the slot leakage inductance which depends upon the number of turns in a coil, the number of coils in a group and the width and depth of the slot and the length of the air gap. Since with the value of $\Delta_1\, L_1$ all the field which links the secondary winding has been included, only the portion of the slot leakage which does not link all the turns in the opposed secondary coil should be considered. No hard and fast rule can be made for determining this quantity since it depends upon the shape of the slots, there should be little trouble in making the calculation when the data are given. Denoting this quantity by $\Delta_2\, L_1$ we have

$$L_1 = \Delta_1\, L_1 + \Delta_2\, L_1 \qquad (14)$$

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Symmetrically Grouped Windings. The above formulas give the mutual impedance between groups of coils, each group of which may be unsymmetrical. Generally machines are designed so that, although the individual groups of coils due to fractional pitch may be unsymmetrical, the complete winding is symmetrical. When two coils are together in a slot this may be done by connecting one group of coils opposite the north pole in series with the corresponding group opposite the south pole; that is to say, the group displaced electrically by the angle π. If therefore we take equation (**11**) and consider the mutual induction as due to a group having axis at θ = zero and another having its axis at $\theta = \pi$ with a similarly arranged group of coils having its axis at θ, we find that (**11**) becomes

$$M_{12} = \frac{16\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, d}\ \Sigma \left\{ \frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$

$$\left. \sin n\, m_2\, \pi\, (1 - \cos n\, \pi)^2 \cos n\, \theta \right\} \text{henrys} \qquad (\mathbf{15})$$

Similarly

$$M_{1a} = \frac{16\, N_1\, N_a\, r\, l}{10^9\, \pi^2\, d}\ \Sigma \left\{ \frac{1}{n^4} \sin n\, m_1\, \pi \sin n\, m_z\, \pi \right.$$

$$\left. \sin n\, m_1\, \pi \sin n\, m_a\, \pi\, (1 - \cos n\, \pi)^2 \cos n\, \theta \right\} \text{henrys} \qquad (\mathbf{16})$$

Since $1-\cos n\pi$ is zero for all even values of n it is evident that (**15**) and (**16**) contain no even harmonics, moreover the above formulas give the mutual induction between two similarly connected groups of windings, but if $(1-\cos n\, \pi)$ is used only with the first power these formulas give the mutual impedance between one pair of such symmetrically grouped windings and another single group with axis inclined at an angle θ.

The value of self induction is

$$\Delta_1 L_1 = \frac{16\, N_1^2\, r\, l}{10^9\, \pi^2\, d}\ \Sigma \left\{ \frac{1}{n^4} \sin^2 n\, m_0\, \pi \sin^2 n\, m_1\, \pi \right.$$

$$\left. (1 - \cos n\, \pi)^2 \right\} \qquad (\mathbf{17})$$

$\Delta_2 L_1$ is found in the same manner as before

$$L_1 = \Delta_1 L_1 + \Delta_2 L_1 \qquad (\mathbf{18})$$

It is obvious from (**15**) and (**16**) that the effect of dissymmetry is to introduce more or less double frequency into the wave form of generated e. m. f.

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It will be seen from an examination of (**15**) and (**17**) that, for example, a winding of pitch $\frac{2\pi}{3}$ and subtending an angle $\frac{\pi}{3}$ when connected in a symmetrical group of two has the same field form and characteristics as a full pitch winding of the same number of turns subtending an angle $\frac{2\pi}{3}$.

There are many symmetrical forms of winding but all will be found to be covered by the formulas (**15**) and (**16**).

Unsymmetrical Windings. These may take many forms which may be classified:

(1) Dissymmetry of flux form due to even harmonics.

(2) Dissymmetry in axial position of polyphase groups.

(3) Dissymmetry in windings due to incorrect grouping of coils.

(4) Dissymmetry due to unsymmetrical magnetic characteristics of the iron.

Of these various forms of dissymmetry the most common is a combination of (**1**), (**2**) and (**3**). These forms of unsymmetrical windings may all be calculated by the formulas (**11**) to (**16**).

It is to be noted that the mutual inductance between a symmetrical and an unsymmetrical winding is harmonically symmetrical. Hence, if the field of a machine is harmonically symmetrical, the e. m. f. generated will be also harmonically symmetrical whatever may be the form of the windings.

The reciprocal nature of M is fully established by its form, for it is immaterial in obtaining (**16**) whether we start out with the winding whose pitch is m_x or with that whose pitch is m_0, the result will be the same. The effect of saturation will be to tend to alter the values of the coefficients of M but the general form will not vary appreciably. We shall now consider some standard windings of generators and motors.

Three-Phase Symmetrical Full Pitch. Here m_0, m_1 and m_2 are 0.5, 0.1666 and 0.1666 respectively. Using formula (**15**) all the even harmonics disappear and $(1 - \cos n\pi)^2$ is equal to 4 or zero.

$$M_{12} = \frac{16\, N_1\, N_2\, r\, l}{10^9\ \pi^2\, d} \left(\cos\theta + \frac{4}{81}\cos 3\theta + \frac{1}{625}\cos 5\theta + \frac{1}{2401}\cos 7\theta + \frac{4}{6561}\cos 9\theta + \ldots\right) \qquad (19)$$

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Theoretical Symmetrical Three-Phase Winding. Here $m_0 = 0.5, m_1 = m_2 = 0.333$. Using formula (**11**)

$$M_{12} = \frac{3}{4} \frac{16\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, d} \left(\cos\theta + \frac{1}{625}\cos 5\,\theta + \frac{1}{2401}\cos 7\,\theta + \frac{1}{14641}\cos 11\,\theta + \ldots\right) \tag{20}$$

Here the third group of harmonics is entirely eliminated.

Three-Phase Symmetrical $\frac{2\pi}{3}$ *Pitch Winding.* Here $m_0 = 0.333, m_1 = m_2 = 0.166$. Using formula (**15**)

$$M = \frac{3}{4} \frac{16\, N_1\, N_2\, r\, l}{10^9\, \pi^2\, a} \left(\cos\theta + \frac{1}{625}\cos 5\,\theta + \frac{1}{2461}\cos 7\,\theta + \frac{1}{14641}\cos 11\,\theta + \ldots\right) \tag{21}$$

which gives the same result as (**20**).

Formulas for Salient Pole Machines

The formulas given in the preceding discussion are appropriate for distributed winding and non-salient poles. Where salient poles are used the field form due to the poles with a given winding will be arbitrary so that with the polar axis as reference we shall have

$$\mathfrak{B} = \frac{2\,\pi\, N_a\, I_a}{d}\, \Sigma\,(A_n \cos n\,\theta) \tag{22}$$

Where $\mathfrak{B}$ is the induction through the armature or stator. When the poles are symmetrical $A_n \cos n\,\theta$ might be chosen at once for this condition and in this case we do not require coefficients of mutual induction between pole windings, since the value of $\mathfrak{B}$ is obtained by considering the mutual reaction between pole windings to be such as will produce symmetry. We may however assume $\mathfrak{B}$ to be perfectly general in form in which case the flux through a coil of pitch $2\, m_0\, \pi$ is

$$\varphi = \frac{4\,\pi\, N_a\, I_a\, r\, l}{10\, d}\, \Sigma\left(\frac{A_n}{n} \sin n\, m_0\, \pi \cos n\,\theta\right) \tag{23}$$

We have therefore for the mutual induction between one pole and a group of coils at an angle θ and subtending an angle $2\, m_1\, \pi$

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$$M_{a1} = \frac{4\, N_a\, N_1\, r\, l}{10^9\, d}\, \Sigma \left(\frac{A_n}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \cos n\, \theta \right) \quad (24)$$

and where there is symmetry due to grouping of windings, we have

$$M_{a1} = \frac{4\, N_a\, N_1\, r\, l}{10^9\, d}\, \Sigma \left\{ \frac{A_n}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$

$$\left. (1 - \cos n\, \pi)^2 \cos n\, \theta \right\} \quad (25)$$

where N_a is the number of turns for one pole and (**25**) applies to one pair of poles and the corresponding group of coils. When there are more than one pair of poles in series and the corresponding groups of winding are also in series, if it is desired to consider the mutual inductance of the complete winding, the result given above must be multiplied by the number of pairs of poles.

If in equation (**16**) we take

$$\left.\begin{aligned} &\frac{N_a}{2\,\pi}\, \frac{1}{n} \sin n\, m_a\, \pi = N_a \\ \text{and} \qquad &\frac{1}{\pi\, n} = B_n \end{aligned}\right\} \quad (26)$$

it becomes

$$M_{1a} = \frac{32\, N_1\, N_a\, r\, l}{10^9\, d}\, \Sigma \left\{ \frac{B_n}{n^2} \sin n\, m_s\, \pi \sin n\, m_0\, \pi \right.$$

$$\left. \sin n\, m_1\, \pi\, (1 - \cos n\, \pi)^2 \cos n\, \theta \right\} \quad (27)$$

which is the expression corresponding to (**25**) starting with the winding flux form. (**25**) and (**27**) must therefore be identical and we have

$$\frac{32\, N_1\, N_a\, r\, e}{10^9\, d}\, B_n \sin n\, m_s\, \pi - \frac{4\, N_a\, N_1\, r\, e}{10^9\, d}\, A_n$$

or

$$\mathfrak{B}_n = \frac{A_n}{8 \sin n\, m_s\, \pi} \quad (28)$$

and

$$\mathfrak{B}_1 = \frac{2\, \pi\, I_1}{10\, d}\, \Sigma\, (B_n \sin n\, m_0\, \pi \cos n\, \theta) \quad (29)$$

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and is the induction wave form for a single turn of the winding.

The expression for the mutual inductance between windings of the same core for salient poles is obtained in terms of the pole flux wave form by substituting in the formulas $\frac{A_n}{8 \sin n\, m_s\, \pi}$ for $\frac{1}{n\,\pi}$. We have therefore the following formulas for salient poles.

General expression considering only one pole and one group of coils.

$$\mathfrak{B}_a = \frac{2\,\pi\, N_a\, I_a}{10\, d}\, \Sigma\, (A_n \cos n\,\theta) \tag{a}$$

$$\mathfrak{B}_1 = \frac{\pi\, I_1}{20\, d}\, \Sigma \left(A_n\, \frac{\sin n\, m_0\, \pi}{\sin n\, m_a\, \pi} \cos n\,\theta\right) \tag{b}$$

$$M_{a1} = \frac{4\, N_a\, N_1\, r\, l}{10^9\, d}\, \Sigma \left(\frac{A_n}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \cos n\,\theta\right) \tag{c}$$

$$M_{12} = \frac{2\, N_1\, N_2\, r\, l}{10^9\, \pi\, d}\, \Sigma \left(\frac{A_n}{n^2}\, \frac{\sin n\, m_0\, \pi}{\sin n\, m_s\, \pi} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$
$$\left. \sin n\, m_2\, \pi \cos n\,\theta\right) \tag{d}$$

$$\Delta_1 L_a = \frac{4\, \pi\, N_a^2\, r\, l}{10^9\, d}\, \Sigma \left(\frac{A_n}{n} \sin n\, m_s\, \pi\right) \tag{e}$$

$$\Delta_1 L_1 = \frac{2\, N_1^2\, r\, l}{10^9\, \pi\, d}\, \Sigma \left(\frac{A_n}{n}\, \frac{\sin^2 n\, m_0\, \pi \sin^2 n\, m_1\, \pi}{\sin n\, m_s\, \pi}\right) \tag{f}$$

General expressions considering only poles to be symmetrical. Considered on the basis of two poles, N_a being turns on one pole.

$$\mathfrak{B}_a = \frac{2\, \pi\, N_a\, I_a}{10\, d}\, \Sigma\, \{A_n\, (1 - \cos n\,\pi) \cos n\,\theta\} \tag{a'}$$

$$\mathfrak{B}_1 = \frac{\pi\, I_1}{20\, d}\, \Sigma \left\{A_n\, \frac{\sin n\, m_0\, \pi}{\sin n\, m_s\, \pi}\, (1 - \cos n\,\pi) \cos n\,\theta\right\} \tag{b'}$$

$$M_{a1} = \frac{4\, N_a\, N_1\, r\, l}{10^9\, d}\, \Sigma \left\{\frac{A_n}{n^2} \sin n\, m_0\, \pi \sin n\, m_1\, \pi \right.$$
$$\left. (1 - \cos n\,\pi) \cos n\,\theta\right\} \tag{c'}$$

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$$M_{12} = \frac{2\,N_1\,N_2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left\{\frac{A_n}{n^2}\,\frac{\sin n\,m_0\,\pi}{\sin n\,m_x\,\pi}\,\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\right.$$

$$\left.\sin n\,m_2\,\pi\,\cos n\,\theta\right\}\text{ henrys} \qquad (\mathbf{d}')$$

$$\Delta_1 L_a = \frac{4\,\pi\,N_a^2\,r\,l}{10^9\,d}\,\Sigma\left\{\frac{A_n}{n}\,\sin n\,m_x\,\pi\,(1-\cos n\,\pi)\right\} \qquad (\mathbf{e}')$$

$$\Delta_1 L_1 = \frac{2\,N_1^2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left\{\frac{A_n}{n}\,\frac{\sin^2 n\,m_0\,\pi\,\sin^2 n\,m_1\,\pi}{\sin n\,m_x\,\pi}\right\} \qquad (\mathbf{f}')$$

General expression with both polar and winding symmetry.

$$\mathfrak{B}_a = \frac{2\,\pi\,N_a\,I_a}{10\,d}\,\Sigma\,\{A_n\,(1-\cos n\,\pi)\cos n\,\theta) \qquad (\mathbf{a}'')$$

$$\mathfrak{B}_1 = \frac{\pi\,I_1}{20\,d}\,\Sigma\left\{A_n\,\frac{\sin n\,m_0\,\pi}{\sin n\,m_x\,\pi}\,(1-\cos n\,\pi)\cos n\,\theta\right\} \qquad (\mathbf{b}'')$$

$$M_{a1} = \frac{4\,N_a\,N_1\,r\,l}{10^9\,d}\,\Sigma\left\{\frac{A_n}{n^2}\,\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\,(1-\cos n\,\pi)^2\right.$$

$$\left.\cos n\,\theta\right\} \qquad (\mathbf{c}'')$$

$$M_{12} = \frac{2\,N_1\,N_2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left\{\frac{A_n}{n^2}\,\frac{\sin n\,m_0\,\pi}{\sin n\,m_x\,\pi}\,\sin n\,m_0\,\pi\,\sin n\,m_1\,\pi\right.$$

$$\left.\sin n\,m_2\,\pi\,(1-\cos n\,\pi)^2\cos n\,\theta\right\} \qquad (\mathbf{d}'')$$

$$\Delta_1 L_a = \frac{4\,\pi\,N_a^2\,r\,l}{10^9\,d}\,\Sigma\left\{\frac{A_n}{n}\,\sin n\,m_x\,\pi\,(1-\cos n\,\pi)^2\right\} \qquad (\mathbf{e}'')$$

$$\Delta_1 L_1 = \frac{2\,N_1^2\,r\,l}{10^9\,\pi\,d}\,\Sigma\left\{\frac{A_n}{n}\,\frac{\sin^2 n\,m_0\,\pi\,\sin^2 n\,m_1\,\pi}{\sin n\,m_x\,\pi}\right.$$

$$\left.(1-\cos n\,\pi)^2\right\} \qquad (\mathbf{f}'')$$

In using any of the formulas given above for machines having more than two poles, it must be divided by the number of pairs of poles and likewise the expression for M or $\Delta_1 L$ must be multiplied by the number of pairs of poles, which leaves the formula for these quantities unchanged.

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Let us next consider the actual induction in the air gap with a distributed winding operating with three-phase currents. Let i_{m1} be the magnetizing current of the first phase i_{m2} and i_{m3} those of the other phases. The induction due to one group of coils of phase 1 is

$$\mathfrak{B}_1 = \frac{8\,N_1\,i_{m1}}{10\,\pi\,d}\,\Sigma\left\{\frac{1}{n^2}\sin n\,m_0\,\pi\sin n\,m_1\,\pi\cos n\,\theta\right\} \tag{30}$$

and if the phase displacement of 2 and 3 from 1 be φ_{12} and φ_{13}

$$\mathfrak{B}_2 = \frac{8\,N_2\,i_{m2}}{10\,\pi\,d}\,\Sigma\left\{\frac{1}{n^2}\sin n\,m_0\,\pi\sin n\,m_2\,\pi\cos(n\theta-\varphi_{12})\right\} \tag{31}$$

$$\mathfrak{B}_3 = \frac{8\,N_3\,i_{m3}}{10\,\pi\,d}\,\Sigma\left\{\frac{1}{n^2}\sin n\,m_0\,\pi\sin n\,m_2\,\pi\cos(n\,\theta-\varphi_{13})\right\} \tag{32}$$

For symmetrically grouped coils the formulas become

$$\mathfrak{B}_1 = \frac{8\,N_1\,i_{m1}}{10\,\pi\,d}\,\Sigma\left\{\frac{1}{n^2}\sin n\,m_0\,\pi\sin n\,m_1\,\pi\,(1-\cos n\,\pi)\cos n\,\theta\right\} \tag{33}$$

$$\mathfrak{B}_2 = \frac{8\,N_2\,i_{m2}}{10\,\pi\,d}\,\Sigma\left\{\frac{1}{n^2}\sin n\,m_0\,\pi\sin n\,m_2\,\pi(1-\cos n\,\pi)\cos m\,(\theta-\phi_{12})\right\} \tag{34}$$

$$\mathfrak{B}_3 = \frac{8\,N_3\,i_{m3}}{10\,\pi\,d}\,\Sigma\left\{\frac{1}{n^2}\sin n\,m_0\,\pi\sin n\,m_3\,\pi\,(1-\cos n\,\pi)\cos m\,(\theta-\varphi_{13})\right\} \tag{35}$$

For a symmetrical three-phase motor with full pitch coils $m_0 = 0.5$, $m_1 = m_2 = m_3 = 0.166$ (**33**), (**39**) and (**35**) become of the four

$$\mathfrak{B}_1 = \frac{8\,N_1\,i_{m1}}{10\,\pi\,d}\left\{\cos\theta-\frac{2}{9}\cos 3\,\theta+\frac{1}{25}\cos 5\,\theta+\frac{1}{49}\cos 7\,\theta-\frac{2}{81}\cos 9\,\theta+\frac{1}{121}\cos 11\,\theta+\frac{1}{169}\cos 13\,\theta+\right\} \tag{36}$$

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which is the field due to one group of coils alone. The wave is flattened by the third group of harmonics but all the other harmonics are peaking values. There is therefore a decided gain in such a wave form of flux since it permits of high fundamental flux density.

The maximum value of flux is approximately

$$B_{max} = 0.823 \cdot \frac{8\, N_1 i_m}{10\, \pi\, d} \text{ gaus} \tag{37}$$

where d is given in centimeters.

$$B_{max} = \frac{1.67\, N_1 i_m}{\pi\, d} \text{ maxwells per square inch,}$$

with d given in inches.

For the total winding the resultant induction will be the sum of B_1, B_2, and B. If we take the symmetrical winding with angles between planes of symmetry

$\varphi_{12} = \frac{2\pi}{3}$ and $\varphi_{13} = \frac{4\pi}{3}$, we have

$$\left.\begin{aligned} \cos n\,\theta &= \frac{\epsilon^{jn\theta}}{2} + \frac{\epsilon^{-jn\theta}}{2} \\ \cos n\left(\theta - \frac{2\pi}{3}\right) &= a^{-n}\,\frac{\epsilon^{jn\theta}}{2} + a^{n}\,\frac{\epsilon^{-jn\theta}}{2} \\ \cos n\left(\theta - \frac{4\pi}{3}\right) &= a^{n}\,\frac{\epsilon^{jn\theta}}{2} + a^{-n}\,\frac{\epsilon^{-jn\theta}}{2} \end{aligned}\right\} \tag{38}$$

If we multiply these three quantities successively by $\check{I}_{m1}$, $a^2\check{I}_{m1}$, $a\check{I}_{m1}$ and add, we have

$$\check{I}_{m1}\left\{\frac{\epsilon^{jn\theta}}{2}\,(1 + a^{-(n-2)} + a^{(n+1)})\left(+\frac{\epsilon^{-jn\theta}}{2}\right.\right.$$
$$\left.\left.\times\,(1 + a^{n+2} + a^{-(n-1)})\right)\right\} \tag{39}$$

and giving n successive odd values from 1 up, we find for (39) the following values

$n = 1$ (39) becomes $\frac{3}{2}\,\check{I}_{m1}\,\epsilon^{-j\theta}$

$n = 3$ " " 0

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$$n = 5 \quad " \quad " \quad \frac{3}{2} \check{I}_{m1} \epsilon^{j5\theta}$$

$$n = 7 \quad " \quad " \quad \frac{3}{2} \check{I}_{m1} \epsilon^{-j7\theta}$$

$$n = 7 \quad " \quad " \quad 0$$

$$n = 11 \quad " \quad " \quad \frac{3}{2} \check{I}_{m1} \epsilon^{j11\theta}$$

$$n = n \quad " \quad " \quad 2 \check{I}_{m1} \sin^2 \frac{2 n \pi}{3} \epsilon^{-j \frac{2}{\sqrt{3}} \sin \frac{2 n \pi}{3} n \theta}$$

We may therefore express $\mathfrak{B}$ by

$\mathfrak{B}$ = real part of

$$\frac{16 N_1 \check{I}_{m1}}{10 \pi d} \Sigma \left\{ \frac{1}{n^2} \sin n m_0 \pi \sin n m_1 \pi (1 - \cos n \pi) \right.$$

$$\left. \times \sin^2 \frac{2 n \pi}{3} \epsilon^{-j \frac{2}{\sqrt{3}} \sin \frac{2 n \pi}{3} n \theta} \right\} \tag{40}$$

It will be obvious that if we proceed around the cylinder in the negative direction of rotation at an angular speed w and $\check{I}_{m1}$ is equal to $I_{m1} \epsilon^{jwt}$, for $n = 1$ the value of B_1 will remain constant and real, hence B_1 must be a constant field rotating at angular velocity w in the negative direction. The value of B may be expressed in harmonic form, but in this form it does not illustrate the rotating field theory so aptly. The harmonic form is given below and is simpler in appearance than (40).

$$\mathfrak{B} = \frac{16 N_1 i_{m1}}{10 \pi . d} \Sigma \left(\frac{1}{n^2} \sin n m_0 \pi \sin n m_1 \pi (1 - \cos n \pi) \right.$$

$$\left. \sin^2 \frac{2 n \pi}{3} \cos n \theta \right) \tag{41}$$

For a symmetrical three-phase motor with full pitch coil ($m_0 = 0.5\ m_1 = 0.166$) $\mathfrak{B}$ becomes

$$\mathfrak{B} = \frac{12 N_1 i_{m1}}{10 \pi d} \Sigma \left\{ \cos \theta + \frac{1}{25} \cos 5 \theta + \frac{1}{49} \cos 7 \theta \right.$$

$$\left. + \frac{1}{121} \cos 11 \theta + \frac{1}{169} \cos 13 \theta + \ . \ . \right\} \tag{42}$$

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This gives for the maximum induction approximately

$$\mathfrak{B}_{max} = \frac{1.075 \times 12\, N_1\, i_{m1}}{10\, \pi\, d} = \frac{1.29\, N_1\, i_m}{\pi\, d} \text{ gauss} \tag{43}$$

where d is measured in centimeters.

$$\mathfrak{B}_{max} = \frac{3.28 \times N_1\, i_{m1}}{\pi\, d} \text{ maxwell per square inch} \tag{44}$$

where d is measured in inches and N is the total number of turns per pair of poles.

APPENDIX II

Graphical Construction for Obtaining Symmetrical Components

The graphica method for finding the symmetrical components of $S\,(\check{E}_a)$ given in the text serves as a geometrical interpretation

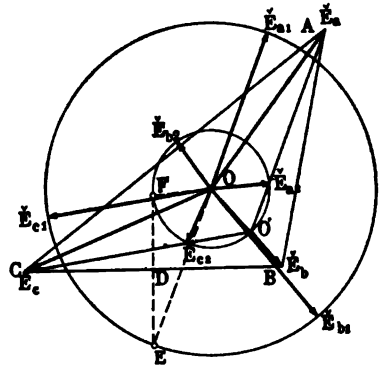

FIG. 18—GRAPHICAL METHOD FOR OBTAINING THE SYMMETRICAL COMPONENTS OF THE THREE-PHASE VECTORS $\check{E}_a, \check{E}_b, \check{E}_c$

of equation (5), but the graphical method shown in Fig. 18 is much simpler and more convenient, the construction is as follows. Find E and F the centroids of the two equilateral triangles with $B\,C$ as base: With 0 the centroid of triangle $A\,B\,C$ as centre describe the two circles passing through E and F then; $E\,O$ extended till it touches the circle through E at the opposite end of the diameter gives $\check{E}_{a1}$; $\check{E}_{b1}$ and $\check{E}_{c1}$ are obtained by laying of points on the circle 60 degrees from E. Similarly if $F\,O$ is extended to meet the circle through F at the opposite end of the diameter we obtain $\check{E}_{a2}$; and $\check{E}_{b2}$ and $\check{E}_{c2}$ are obtained by the same construction as before.

The proof of this construction is as follows: If G and H are

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the apexes (not shown in figure) of the equilateral triangles having $B\,C$ as base

$$O\,E = \frac{O\,B + O\,C + O\,G}{3} = \frac{-\,O\,A + O\,G}{3} = \frac{-\,\check{E}_a + O\,G}{3}$$

$$O\,F = \frac{O\,B + O\,C + O\,H}{3} = \frac{-\,O\,A + O\,H}{3} = \frac{-\,\check{E}_a + O\,H}{3}$$

$$-\,a^2\,.\,O\,C = -\,a^2\,(O\,D + D\,C)$$
$$-\,a\;.\,O\,B = -\,a\,(O\,D + D\,B) = -\,a\,(O\,D - D\,C)$$
$$-\,a^2\,.\,O\,C - a\,.\,O\,B = -\,(a + a^2)\,O\,D + (a - a^2)\,.\,D\,C$$
$$= O\,D + j\,\sqrt{3}\,D\,C$$
$$= O\,G$$

That is

$$O\,G = -\,(a\,\check{E}_b + a^2\,\check{E}_c)$$

Similarly

$$O\,H = -\,(a^2\,\check{E}_b + a\,\check{E}_c)$$

and therefor

$$O\,E = -\,\frac{\check{E}_a + a\,\check{E}_b + a^2\,\check{E}_c}{3} = -\,\check{E}_{a1}$$

and

$$O\,F = -\,\frac{\check{E}_a + a^2\,\check{E}_b + a\,\check{E}_c}{3} = -\,\check{E}_{a2}$$

The construction when $\check{E}_a + \check{E}_b + \check{E}_c$ is not zero is so obvious that it is not necessary to show it here.

If lines be drawn from E_a to $\check{E}_{a2}$, E_b to $\check{E}_{b2}$, E_c to $\check{E}_{c2}$ these lines will be parallel to $\check{E}_a'\,\check{E}_b'$ and $\check{E}_c'$ respectively and will meet at a point O'. The vectors $O'\,E_a\,O'\,E_b$ and $O'\,E_c$ give the values of the e. m. f. across each member of a star delta bank of transformers when operated on the three-phase circuit $S\,(\check{E}_a)$ with ratios changed so as to give a balanced secondary triangle of e. m. f's.

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DISCUSSION ON "METHOD OF SYMMETRICAL CO-ORDINATES APPLIED TO THE SOLUTION OF POLYPHASE NETWORKS" (FORTESCUE), ATLANTIC CITY, N. J., JUNE 28, 1918.

J. Slepian (by letter): During the past eighteen months it has been my very good fortune to have been in close contact with Mr. Fortescue and to have had many interesting discussions on the ideas embodied in this paper. Since I have had so long to think over and digest these ideas, I think I may be pardoned for going at great length here into the viewpoint I have reached. I feel all the greater need for a long discussion, because on reading the paper I see that the great wealth of material to be presented in a limited space has crowded out much detailed explanation and appeal to analogy, which in discussions with Mr. Fortescue have contributed so much to my clearer conception of the ideas presented here.

The method given here had its origin in considering the operation of balanced induction machines under unbalanced conditions. Using the "coordinates" proposed here, the theory of these machines may be given with beautiful simplicity. I think I am right in saying that the utility of the method is practically entirely limited to the case of rotating induction machines. Purely static apparatus treated in this way does not show any simplification. When one considers, however, that almost every practical alternating-current circuit contains at least one rotating machine, namely the generator, the broad field of application of the method becomes apparent.

The root of the ideas given here is old and was given early in treatments of single-phase motors. The simple constant rotating nature of the flux in a balanced polyphase motor was well known. It was discovered that the more complicated flux in a single-phase motor could be resolved into the sum of two such constant rotating fluxes of opposite rotations, with the magnitude of these fluxes not necessarily equal. It was observed also, that if the balanced polyphase states, which would give independently each of these rotating fields, were superimposed, the resultant state would give correctly the currents. voltages, torques, etc., of the single-phase motor. For a recent discussion of this, see Mr. Lamme's paper, page 627, Volume I, TRANSACTIONS 1918.

Mr. Fortescue has generalized this method of resolution of the flux in a polyphase machine under unbalanced conditions into the sum of fluxes, each corresponding to a balanced condition, to a similar resolution of an unbalanced system of any polyphase quantities whatever.

In my discussion here I shall, for simplicity, confine myself to the three-phase case. To illustrate the resolution of a set of unbalanced three-phase currents into symmetrical or balanced components, consider three arbitrary currents I_a I_b I_c flowing into the terminals of a three-phase, star-connected apparatus. (Fig. 1). Mr. Fortescue shows that this set of currents may be had by adding together the following three sets of currents.

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Method of Symmetrical Coordinates. First a current I_{a0} in each phase. These three currents then are all equal and all in phase. They combine to give a current in the neutral. Second: a current I_{a0} in phase a, a current $a^2 I_{a1}$, in phase b, and a current aI_{a1} in phase c, where

$$a = -\tfrac{1}{2} + j\tfrac{1}{2}\sqrt{3}$$

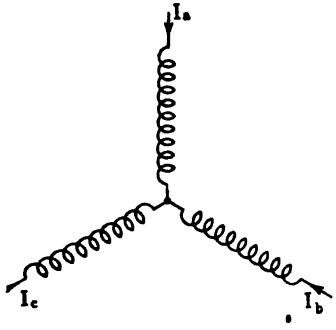

FIG. 1

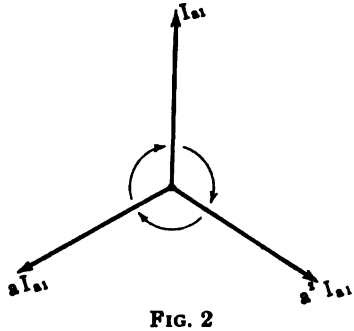

FIG. 2

is a cube root of unity. This set of currents is clearly a balanced polyphase set of what is called positive sequence. (Fig. 2). Third a current I_{a2} in phase a, a current $a\ I_{a2}$ in phase B and a current $a^2\ I_{a2}$ in phase c. (Fig. 3.). This set of currents is clearly a balanced polyphase set of what is called negative phase sequence. If these three sets of currents be made to flow simultaneously in the three-phase apparatus the resultant current in

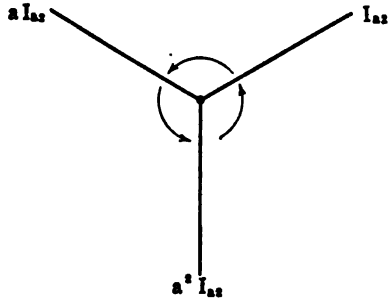

FIG. 3

each phase will be respectively I_a, I_b, I_c. The values of I_{a0}, I_{a1}, I_{a2} are obtained from equations (5). They are:

$$I_{a0} = 1/3\ (I_a + I_b + I_c).$$
$$I_{a1} = 1/3\ (I_a + aI_b + a^2\ I_c).$$
$$I_{a2} = 1/3\ (I_a + a^2 I_b + aI_c).$$

In a similar way, any set of three voltages to neutral acting on the three-phase apparatus can be resolved into the sum of

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three sets of balanced voltages of zero phase sequence, positive phase sequence, and negative phase sequence, respectively.

The sequence operators, S^0, S^1 and S^2, which Mr. Fortescue introduces, have a close analogy, it seems to me, with the j used in the usual treatment of alternating currents. In the elementary theory, it is found useful to resolve an alternating quantity into two components, one in phase with some reference alternating quantity, and the other component in phase quadrature. If i_1 and i_2 are the magnitudes respectively, of the in-phase and the quadrature components, their resultant I, is denoted by $i = i_1 + ji_2$. Here j may be looked upon as a unit alternating quantity in phase quadrature with a reference alternating quantity. Thus, the equation $i = li_1 + j\,i_2$ states that the quantity i may be obtained by adding together i_1 times a unit in phase quantity, and i_2 times a unit quadrature component.

In the same way, the symbols S^0, S^1, S^2 may be regarded as unit polyphase vectors* of zero, positive and negative phase sequence. Thus $S^0 = (1, 1, 1)$, if referring to currents, represents a unit current in each phase of a three-phase apparatus the three currents being in phase with some reference alternating quantity. Similarly, $S^1 = (1, a^2, a)$ represents a unit balanced, three-phase current, of positive phase sequence, in the three-phase apparatus, the current in the first phase being in phase with the reference alternating quantity. Likewise $S^2 = (1, a, a^2)$ represents a unit balanced, three-phase current of negative phase sequence. Thus the equation (**15**) $S\,(I_a) = S^0\,I_0 + S^1\,I_{a1} + S^2\,I_{a2}$ states that a system of three currents $S\,(I_a)$ is equal to the sum of three sets of currents, the first set being of the type S^0, that is of zero phase sequence, the second set of the type S^1, that is of positive phase sequence, and the third set of type S^2, that is of negative phase sequence. The three currents of any set are obtained by multiplying the three currents of the corresponding unit polyphase vector by the complex quantity indicated. Thus the currents in the second set are obtained by multiplying the three currents, 1, a_2, a, respectively, by I_{a1}.

The utility of resolving three-phase currents and voltages in this way, when applied to rotating balanced machines, lies in this fact, that a symmetrical set of voltages of any phase sequence applied to the machine will produce a symmetrical set of currents of the *same phase sequence*, and that a symmetrical set of currents of any phase sequence flowing into the machine will produce a symmetrical set of terminal voltages of the same phase sequence. This fact is well known in the theory usually given of balanced polyphase apparatus, although it is seldom explicitly stated. Other methods of dividing a set of unbalanced three-phase quantities acting on a balanced machine into components would

*This very fortunate term is due to Mr. C. T. Allcutt and expresses the idea of the paragraph most succinctly.

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not have this simplicity. For example: suppose the three actual currents in the three phases, and the three actual voltages of the terminals relative to neutral were taken as the components or coordinate of the currents and voltages. Then one component of current alone, that is a current in one phase only, would not produce only the corresponding component of voltage, but would produce voltages in all three phases.

Neglecting saturation, for a given rotor speed, the symmetrical currents produced in a balanced machine by symmetrical applied voltage, are proportional and in a definite phase relation to the symmetrical applied voltage. Thus the currents may be obtained by multiplying the individual members of the symmetrical set of applied voltages by some complex number. Similarly, a symmetrical set of currents flowing into a balanced machine produces a symmetrical set of terminal voltages which may be obtained by multiplying the individual members of the symmetrical set of currents by some complex number. The first complex number could be called the symmetrical admittance of the machine, and the second the symmetrical impedance.

This symmetrical impedance and admittance will be different, of course, for symmetrical components of different phase sequence. Thus for a star-connected, ungrounded neutral machine, the admittance for symmetrical voltage of zero phase sequence will be zero; if the neutral is grounded, the impedance to zero phase sequence current will be principally the leakage reactance between phases; if the rotor is running near synchronism in the sense of positive phase sequence, the impedance Z_1 to positive phase sequence current will be large, while the impedance Z_2 to negative phase sequence will be small. The complete expressions for these impedances Z_1, Z_2, are given in equations (**122**), (**123**). The relations between current and voltage components are given in equations (**120**) and (**121**). In these four equations is concentrated the whole theory of symmetrical machines operating under unbalanced conditions. But so simple are these equations and their physical meaning so clear, that once understood, they enable us to predict qualitatively, without computation, the behavior of rotating balanced machines under any unbalanced condition whatever.

Consider, for example, the simple picture of the action of a phase balancer which the above treatment gives. The balancer is merely a balanced machine across the polyphase line running near synchronism. It offers high impedance for the symmetrical voltage of normal or positive phase sequence, but offers a very low impedance for any negative phase sequence component of voltage. Thus the negative phase sequence voltage is "short-circuited out," and balance on the line is preserved.

The harmful effects of slight unbalance in the terminal voltage of a polyphase machine upon the machine's rating is also clearly shown. Since the negative sequence impedance is very low, a small negative phase sequence voltage will produce large negative

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phase sequence currents with their attendant heating. This suggests a better quantity to denote the degree of unbalance of a line than the one commonly used. It is the ratio of the negative phase sequence component of the line voltage to the positive phase sequence component.

Balanced stationary apparatus also enjoys the simplicity pointed out in balanced rotating machines, namely that symmetrical voltages of any phase sequence produce symmetrical currents of the same phase sequence, but here the impedances for positive and negative phase sequence are always the same. The zero phase sequence impedance is generally different.

When we pass to unbalanced apparatus, things become more complicated. Symmetrical currents of one phase sequence no longer produce e. m. fs. of that phase sequence alone, but the total e. m. fs. are unsymmetrical and contain components of other phase sequences. Let us consider in detail an unbalanced star impedance, with grounded neutral. Let the phase impedances be Z_a, Z_b, Z_c.

Let us first study the e. m. fs. to neutral produced by a zero phase sequence current. If I_{ao}, I_{ao}, I_{ao}, are the three currents, the three e. m. fs. are Z_aI_{ao}, Z_bI_{ao}, Z_aI_{ao}. Resolving these three e. m. fs. into symmetrical components, we find by equations (5) that the zeroth order component is:

$$\tfrac{1}{3}\,(Z_aI_{ao} + Z_bI_{ao} + Z_cI_{ao}) = \tfrac{1}{3}\,(Z_a + Z_b + Z_c)\,I_{ao}$$

The positive phase sequence components will be:

$$\tfrac{1}{3}\,(Z_aI_{ao} + aZ_bI_{ao} + a^2Z_cI_{ao}) = \tfrac{1}{3}\,(Z_a + aZ_b + a^2Z_c)\,I_{ao}$$

The negative phase sequence component will be:

$$\tfrac{1}{3}\,(Z_aI_{ao} + a^2Z_bI_{ao} + aZ_cI_{ao}) = \tfrac{1}{3}\,(Z_a + a^2Z_b + aZ_c)\,I_{ao}$$

Thus the total e. m. f. using the symbols $S^0S'\,S^2$ would be written:

$$S^0\,[\tfrac{1}{3}\,(Z_a + Z_b + Z_c)\,I_{ao}] + S^1\,[\tfrac{1}{3}(Z_a + aZ_b + a^2Z_c)I_{ao}] + S^2\,[\tfrac{1}{3}(Z_a + a^2Z_b + aZ_c]\,I_{ao} = S^0\,(Z_{ao}I_{ao}) + S'\,(Z_{a1}I_{ao}) + S^2\,(Z_{a2}I_{ao}).$$

Where $Z_{ao} = \tfrac{1}{3}\,(Z_a + Z_b + Z_c)$

$Z_{a1} = \tfrac{1}{3}\,(Z_a + aZ_b + a^2Z_c)$

$Z_{a2} = \tfrac{1}{3}\,(Z_a + a^2Z_b + aZ_c)$

Now consider the e. m. f's. given by the positive phase sequence currents, I_a, a^2I_{a1}, aI_{a1}. They will be Z_aI_{a1}, $Z_ba^2I_{a1}$ Z_caI_{a1}.

Resolve the system of em. f's. into symmetrical components. We find for the zero phase sequence component:

$$\tfrac{1}{3}\,(Z_aI_{a1} + Z_ba^2I_{a1} + Z_caI_{a1}) = \tfrac{1}{3}\,(Z_a + a^2Z_b + aZ_c)\,I_{a1} = Z_{a2}I_{a1}$$

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For the positive phase sequence component we find

$$\tfrac{1}{3}\,(Z_a I_{a1} + aZ_b a^2 I_{a1} + a^2 Z_c a I_{a1}) = \tfrac{1}{3}\,(Z_a + Z_b + Z_c)\, I_{a1} = Z_{a0} I_{a1}$$

For the negative phase sequence component we find

$$\tfrac{1}{3}\,(Z_a I_{a1} + a^2 Z_b a^2 I_{a1} + aZ_c a I_{a1}) = \tfrac{1}{3}\,(Z_a + aZ_b + a^2 Z_c)\, I_{a1} = Z_{a1} I_{a1}$$

Lastly, consider the e. m. f.'s produced by negative phase sequence currents, I_{a2}, $a\ I_{a2}$, $a^2 I_{a2}$. The zero phase sequence component will be

$$\tfrac{1}{3}\,(Z_a I_{a2} + Z_b a I_{a2} + Z_c a^2 I_{a2}) = \tfrac{1}{3}\,(Z_a + aZ_b + a^2 Z_c)\, I_{a2} = Z_{a1} I_{a2}.$$

The positive phase sequence e. m. f. component will be

$$\tfrac{1}{3}\,(ZaI_{a2} + aZ_b a I_{a2} + a^2 Z_c a^2 I_{a2}) = \tfrac{1}{3}\,(Z_a + a^2 Z_b + aZ_c)\, I_{a2} = Z_{a2} I_{a2}.$$

The negative phase sequence e. m. f. component will be

$$\tfrac{1}{3}\,(Z_a I_{a2} + a^2 Z_l a I_{a2} + aZ_c a^2 I_{a2}) = \tfrac{1}{3}\,(Z_a + Z_l + Z_c)\, I_{a2} = Z_{a0} I_{a2}$$

The components of voltage are then expressed easily in terms of the quantities Z_{a0}, Z_{a1}, Z_{a2}, defined in equations (8).

Let us bring these results together in tabular form where the relations between them can be observed.

Currents	Voltage Components		
	S^0 or zero phase sequence	S^1 or positive phase sequence	S^2 or negative phase sequence
Zero phase sequence, $S^0\ (I_{ao}) = (I_{ao}\ I_{ao}\ I_{ao})$	$Z_{ao} I_{ao}$	$Z_{a1} I_{ao}$	$Z_{a2} I_{ao}$
Positive phase sequence, $S^1\ (I_{a1}) = (I_{a1},\ a^2 I_{a1},\ aI_{a1})$	$Z_{a2} I_{a1}$	$Z_{ao} I_{a1}$	$Z_{a1} I_{a1}$
Negative phase sequence $S^2\ (I_{a2}) = (I_{a2},\ aI_{a2},\ a^2 I_{a2})$	$Z_{a1} I_{a2}$	$Z_{a2} I_{a2}$	$Z_{ao} I_{ao}$

Studying the table above, we see that each symmetrical component of current gives rise to symmetrical voltages of all three phase sequences. We notice first the e. m. f. component which is of the same phase sequence as the current by which it is produced, is obtained by multiplying the producing currents by Z_{ao}. That is the unbalanced apparatus may be said to have one component of impedance, Z_{ao}, which gives e. m. f. 's of the same phase sequence as the currents. This component of impedance does not change the exponent of the S symbol defining the phase sequence of the currents. Thus Z_{ao} alone corresponding to the currents $S^0\ (I_{ao})$ gives the e. m. fs. $S^0\ (Z_{a1} I_{ao})$; corresponding to the

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Currents S^1 (I_{a1}) gives the e. m. fs. S^1 $(Z_{ao}I_{a1})$; and corresponding to the

Currents S^2 (I_{a2}) gives the e. m. fs. S^2 $(Z_2\, I_{a2})$.

Now let us examine how Z_{a1} enters in the above table. Corresponding to the zero phase sequence.

Currents S^0 (I_{ao}) we find the e. m. fs. S^1 $(Z_{a1}I_{ao})$ of positive phase sequence. Corresponding to the positive phase sequence.

Currents S^1 (I_{a1}) we find the e. m. fs. S^2 $(Z_{a1}I_{a1})$ of negative phase sequence. Corresponding to the negative phase sequence.

Currents S^2 (I_{a2}) we find the e. m. fs. S^0 $(Z_{a1}I_{a2})$ of zero phase sequence.

If we agree now to the following definitions of the symbols S when affected with higher exponents:

$$S^3 = S^0;\ S^4 = S^1;\ S^5 = S^2;\ S^6 = S^0;\ S^7 = S^1;\ \text{etc.}$$

We see that the way in which Z_{a1} terms enter can be summarized in this way:

The unbalanced apparatus has a component of impedance, Z_{a1}, which increases the exponent of the phase sequence symbol by unity.

Lastly, let us see how Z_{a2} enters the above table. We find that:

Currents S^0 (I_{ao}) give e. m. fs. S^2 $(Z_{a2}I_{ao})$.
Currents S^1 (I_{a1}) give e. m. fs. S^0 $(Z_{a2}I_{a1})$.
Currents S^2 (I_{a2}) give e. m. fs. S^1 $(Z_{a2}I_{a2})$.

This may be summarized by saying that the unbalanced apparatus has a component of impedance Z_{a2} which increases the exponent of the sequence symbol by two.

All these results will be obtained automatically if we suppose that symbols S^0, S^1, S^2, are attached respectively to Z_{ao}, Z_{a1}, Z_{a2}, and when multiplying the current components, S^0 (I_{ao}), S^1 (I_{a1}), S^2 (I_{a2}), the exponential laws hold. Thus:

$$\begin{aligned}
S^0\,(Z_{ao})\,S^0\,(I_{ao}) &= S^0\,(Z_{ao}\,I_{ao});\ S^1\,(Z_{a1})\,S^0\,(I_{ao}) = S^1\,(Z_{a1}I_{ao}).\\
S^0\,(Z_{ao})\,S^1\,(I_{a1}) &= S^1\,(Z_{ao}\,I_{a1});\ S^1\,(Z_{a1})\,S^0\,(I_{a1}) = S^2\,(Z_{a1}\,I_{a1}).\\
S^0\,(Z_{ao})\,S^2\,(I_{a2}) &= S^2\,(Z_{ao}\,I_{a2});\ S^1\,(Z_{a1})\,S^2\,(I_{ao}) = S^3\,(Z_{a1}\,I_{a2}).\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad = S^0\,(Z_{a1}\,I_{a2}).
\end{aligned}$$

$$\begin{aligned}
S^2\,(Z_{a2})\,S^0(I_{ao}) &= S^2\,(Z_{a2}I_{ao})\\
S^2\,(Z_{a2})\,S^1(I_{a1}) &= S^3\,(Z_{a2}I_{a1}) = S^0(Z_{a2}I_{a1})\\
S^2\,(Z_{a2})\,S^2\,(I_{a2}) &= S^4\,(Z_{a2}I_{a2}) = S^1\,(Z_{a2}I_{a2})
\end{aligned}$$

The whole result can be expressed by writing the total impedance, $S(Z_a) = S^0\,Z_{ao} + S^1\,Z_{a1} + S^2\,Z_{a2}$.

To get the e. m. fs. corresponding to any currents, we multiply together the total impedance by the total current, following merely the rules of algebra, and interpreting higher powers of S as described above. Thus we get:

$$\begin{aligned}
S(E_a) &= S(Z_a)\,S(I_a)\\
&= (S^0Z_{ao} + S^1Z_{a1} + S^2Z_{a2})\,(S^0I_{ao} + S^1I_{a1} + S^2I_{a2})\\
&= S^0(Z_{ao}I_{ao} + Z_{a1}I_{a2} + Z_{a2}I_{a1}) + S^1(Z_{ao}\,I_{a1}\\
&\qquad + Z_{a1}I_{ao} + Z_{a2}I_{a2}) + S^2\,(Z_{ao}I_{a2} + Z_{a1}I_{a1} + Z_{a2}I_{ao})
\end{aligned}$$

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We get here the foundation of a complex three-phase algebra, which performs the same functions for three-phase systems as the usual complex quantity does for simple alternating current.

This leads to another interpretation or mode of viewing the sequence symbols, S^0, S^1, S^2, and here again we may profit by analogy with the j in the complex algebra of simple alternating quantities. Suppose i is a current in phase with some reference alternating quantity. If this current flows through a resistance r. the resulting e. m. f., ri is also in phase with reference alternating quantity. If the resistance is of unit value the resulting e. m. f. is i. Suppose, however, that the current flows through a reactance whose impedance is x ohms. The resulting e. m. f. will be $x\,I$ in magnitude, but it will be in phase quadrature with the reference alternating quantity. This is taken care of in the usual theory by affecting the expression for the reactance with a j, thus xj, and making j have the property that when multiplying a vector or alternating quantity it does not change the value of the vector but merely advances its phase by ninety degrees This is a different meaning from what was given before. j here no longer represents a unit vector or alternating quantity. It now represents an operator, or a symbol for advancing the phase of any vector which it multiplies. Where j by itself is referred to as a vector, it should be understood that ji is meant; where i is a unit vector in phase with the reference vector.

It is clear that operating twice successively by j upon a vector merely reverses the phase of that vector. Thus $j\,(j.i) = -1$, which leads to the rule of multiplication $J^2 = -1$. Lastly, we see that the e. m. f. induced by a current flowing through an impedance may be obtained correctly both in phase and magnitude by multiplying the current by a number of the form $r + jx$, which represents the complete impedance. Thus the complex algebra of simple alternating quantities is born.

We may attach significance as operators in a similar way to the symbols S^0, S^1, S^2, Now they shall no longer represent polyphase vectors, but shall merely be operators which when written next to a polyphase vector, change it into a polyphase vector of another phase sequence. The quantities which the S's affect shall be considered as polyphase vectors. Thus the symbol I shall stand for three currents, each equal to I. Operating on I with S^0 means multiplying each of these currents by unity. Thus S^0 is a unity operator and does not change the polyphase vector upon which it operates. A separate symbol for it might have been omitted.

$S^1\,I$ or S^1 operating on I, means that the three currents I, I, I, are to be multiplied by 1, a^2, a, respectively; or that the first current is to be unchanged, the second to be advanced in phase by 120 deg. and the third by 240 deg. S' by itself shall not mean anything unless it is understood to be followed by 1 in which case it stands for the unit polyphase vector of positive, rotation obtained by operating with S^1 on the three currents (1, 1, 1).

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Similarly, S^2I or S^2 operating on I, means that the three currents I, I, I, are to be multiplied by I, a, a^2, respectively; or that the first current is to be unchanged, the second advanced in phase by 240 deg. and the third by 120 deg. S^2 by itself shall not mean anything unless it is understood to be I, in which case it stands for the unit polyphase vector of negative phase rotation obtained by operating upon the polyphase quantity (I, I, I).

With the meaning now given to S^0, S^1, S^2, it at once follows that they satisfy the law of exponents. For example, S^1 (S^1I) means leave the first current I unchanged; advance the second current I, in phase by 120 deg., and then again by 120 deg; advance the third current, I, by 240 deg. and than again by 240 deg. It is clear that the final results are exactly the same as the results of operating with S^2 on I. Also it is clear that multiplication by an operators and a constant Z is commutative. Thus $ZS^1I = S^1Z_{a1}I$. This serves as the foundation of the complex three-phase algebra.

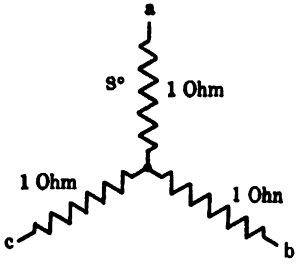

FIG. 4

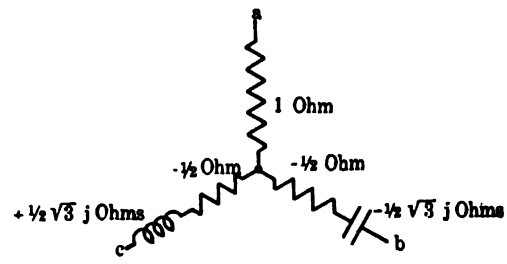

FIG. 5

In the complex algebra of simple alternating quantities, it is easy to give an illustration of the operator J as in impedance. In fact, a reactive impedance of one ohm gives for any current an e. m. f. which may be obtained from the current by multiplying by J. Can we similarly illustrate the three operators S^0, S^1, S^2.

S^0, of course, is easy. Take a grounded star of resistances, each of one ohm. Then we find for the currents S^0 (I) = (I, I, I) the voltages, $(I, I, I) = S^0 I$; for the currents S^2 (I) = (I, aI, a^2I) the voltages $(I, aI, a^2I) = S^2I$.

Now for S^1. Take a grounded star in which the impedance of phase a is 1 ohm, of phase b, $(-\frac{1}{2} - \frac{1}{2}\sqrt{3J})$ ohms $= a^2$ ohms, and of phase c, $(-\frac{1}{2} + \frac{1}{2}\sqrt{3J})$ ohms $= a$ ohms. Now we find for the currents S^0 (I) = (I, I, I), the e. m. fs. (I, a^2I, aI) = $S^1I = S^1$ (S^0I) for the currents S^1 (I) = (I, a^2I, aI), the e. m. fs. $(I, aI, a^2I) = S^2I = S'$ $(S'I)$; for the currents S^2 (I) = (I, aI, a^2I), the e. m. fs. $(I, I, I) = S^0I = S^1$ (S^2I). Fig.5.

Lastly, S^2. Take a grounded star in which the impedance of phase a is 1 ohm, of phase b, $(-\frac{1}{2} + \frac{1}{2}\sqrt{3J})$ ohms, and of

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phase c, $(-\frac{1}{2} - \frac{1}{2}\sqrt{3}j)$ ohms. Then we find for the currents $S^0\ (I) = (I,\ I,\ I)$, the e. m. fs. $(I,\ aI,\ a^2I) = S^2I = S^2S^0I$; for the currents $S'\ (I) = (I,\ a^2I,\ aI)$ the e. m. fs. $(I,\ I,\ I) = S^0I = S^2S'I$; for the currents $S^2\ (I) = (I,\ aI,\ a^2I)$ the e. m. fs. $(I,\ a^2I,\ aI) = S^1I = S^2S^2I$. Fig. 6.

In these examples, negative resistances appear. This is not surprising, as it is clear that unbalanced apparatus, when current of one phase sequence flows, may feed energy into impressed

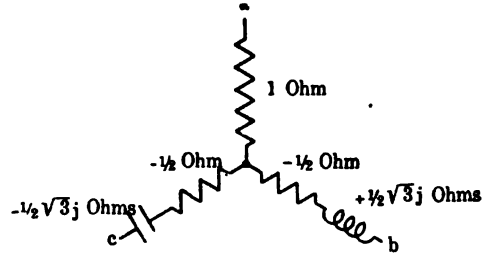

FIG. 6

e. m. fs. of another phase sequence. The negative resistance may be represented physically by a series commutator a-c. motor, driven at constant speed in the opposite of its motoring direction of rotation.

The resolution of an arbitrary three-phase star impedance into its three-phase components is merely the problem of finding three three-phase impedances of the S^0, S^1 and S^2 type, respectively, which put in series as in Fig. 7 will reproduce the given three-phase star.

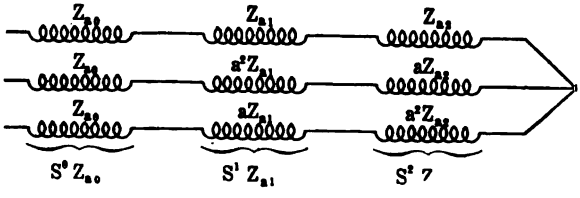

FIG. 7

It is clear that admittances may be treated in the same way as impedances above. Thus the general three-phase admittance may be written in terms of its components, thus:

$$S\ (Y_a) = S^0Y_{a0} + S^1Y_{a1} + S^2Y_{a2}$$

The currents produced by a general three-phase voltage $S\ (E_a) = S^0E_{a0} + S^1E_{a1} + S^2E_{a2}$ will be:

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$$S(I_a) = (S^0 Y_{ao} + S^1 Y_{a1} + S^2 Y_{a2}) \quad (S^0 E_{ao} + S^1 E_{a1} + S^2 E_{a2}).$$
$$= S^0 (Y E_{ao} + Y_{a1} E_{a2} + Y_{a2} E_{a1})$$
$$+ S^1 (Y_{ao} E_{a1} + Y_{a1} E_{ao} + Y_{a2} E_{a2})$$
$$+ S^2 (Y_{ao} E_{a2} + Y_{a1} E_{a1} + Y_{a2} E_{ao}).$$

In the complex algebra of simple alternating currents we pass formally from an impedance $R + jx$ to the corresponding admittance by taking the reciprocal, $\frac{l}{r + jx}$ and multiplying numerator and denominator by $r - jx$ to reduce it to the standard form.

$$\text{Thus } \frac{l}{r + Jx} \times \frac{r - jx}{r - jx} = \frac{r}{r^2 + x^2} - j \frac{x}{r^2 + x^2}$$

Similarly in this three-phase complex algebra we obtain the admittance corresponding to an impedance $S^0 Z_{ao} + S^1 Z_{a1} + S Z_{a2}$ by taking the reciprocal $\frac{1}{S^0 Z_{ao} + S^1 Z_{a1} + S Z_{a2}}$. But now to reduce to standard form we must multiply numerator and denominator by

$$S^0 (Z_{ao} + a S^1 Z_{a1} + a^2 S^2 Z_{a2}) (S^0 Z_{ao} + a^2 S^1 Z_{a1} + a S^2 Z_{a2}).$$

Thus:

$$\frac{1}{S^0 Z_{ao} + S^1 Z_{a1} + S^2 Z_{a2}} \cdot \frac{S^0 Z_{ao} + a S^1 Z_{a1} + a^2 S^2 Z_{a2}}{S^0 Z_{ao} + a S^1 Z_{a1} + a^2 S^2 Z_{a2}} \cdot \frac{S^0 Z_{ao} + a^2 S^1 Z_{a1} + a S^2 Z_{a2}}{S^0 Z_{ao} + a^2 S^1 Z_{a1} + a S^2 Z_{a2}}$$

$$= \frac{S^0 (Z_{ao}^2 + a^2 Z_{a1} Z_{a2} + a Z_{a1} Z_{a2}) + S^1 (a^2 Z_{ao} Z_{a1} + a Z_{ao} Z_{a1} + Z^2_{a2}) + S^2 (a^2 Z_{ao} Z_{a2} + Z^2_{a1} + a Z_{ao} Z_{a2})}{D}$$

$$= \frac{S^0 (Z^2_{ao} - Z_{a1} Z_{a2}) + S^1 (Z^2_{a2} - Z_{ao} Z_{a1}) + S^2 (Z^2_{a1} - Z_{ao} Z_{a2})}{S^0 (Z^3_{ao} + Z^3_{a1} + Z^3_{a2} - 3 Z_{ao} Z_{a1} Z_{a2})}$$

Remembering that S^0 is an operator which does not change the quantity upon which it acts, and therefore that a symbol for it might have been omitted, we may leave off the S^0 in the denominator and get for the final admittance:

$$S(Y_a) = S^0 \frac{Z^2_{ao} - Z_{a1} Z_{a2}}{Z^3_{ao} + Z^3_{a1} + Z^3_{a2} - 3 Z_{ao} Z_{a1} Z_{a2}}$$
$$+ S^1 \frac{Z^2_{a2} - Z_{ao} Z_{a1}}{Z^3_{ao} + Z^3_{a1} + Z^3_{a2} - 3 Z_{ao} Z_{a1} Z_{a2}}$$
$$+ S^2 \frac{Z^2_{a1} - Z_{ao} Z_{a2}}{Z^3_{ao} + Z^3_{a1} + Z^3_{a2} - 3 Z_{ao} Z_{a1} Z_{a2}}$$

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The principles just enunciated enable us to put together three-phase apparatus and calculate the resulting three-phase components just as is done in single-phase apparatus. For apparatus in parallel, we add admittances; for apparatus in series we add impedances.

So far, in the static apparatus considered, the mutual inductance between "legs" of the three-phase apparatus has been assumed zero. When these mutual inductances are not zero, things become more complicated. A three-phase apparatus with grounded neutral is, of course, a four terminal network, and as is well known, in the most general case would require $\frac{4 \times 3}{2} = 6$ complex constants to characterize it under steady state conditions. It is clear that a complex three-phase expression $S^0 Z_{a0} + S^1 Z_{a1} + S^2 Z_{a2}$ depends on only three complex constants, and hence cannot represent the most general three-phase apparatus.

The relations between the symmetrical components of voltage and current on the general static three-phase network are given in equations (**25**). One way of summarizing these equations would be to say that the three-phase apparatus has three different impedances for the three phase-sequence currents. Thus the impedance to zero phase sequence current as given by equations (**25**) is:

$$S^0 (Z_{aa0} + 2 Z_{bc0}) + S^1 (Z_{aa1} - Z_{bc1}) + S^2 (Z_{aa2} - Z_{bc2});$$

the impedance to positive phase sequence currents is

$$S^0 (Z_{aa0} - Z_{bc0}) + S^1 (Z_{aa1} + 2 Z_{bc1}) + S^2 (Z_{aa2} - Z_{bc2});$$

and the impedance to negative phase sequence currents is

$$S^0 (Z_{aa0} - Z_{bc0}) + S^1 (Z_{aa1} - Z_{bc1}) + S^2 (Z_{aa2} + Z_{bc2}),$$

Another way of summarizing the equations (**25**) would be to say that the actual three-phase apparatus is equivalent to a three-phase network having the three-phase impedance

$$S^0(Z_{aa0} - Z_{bc0}) + S^1 (Z_{aa1} - Z_{bc1}) + S^2 (Z_{aa2} - Z_{bc2}),$$

in series with a network whose impedance to zero, positive and negative phase sequence current are respectively, $S^0 3 Z_{bc0}$; $S^1 3 Z_{bc1}$; $S^2 3 Z_{bc2}$.

With the ideas developed in this paper, solutions of problems on symmetrical rotating machines with unbalanced static apparatus may be worked with comparative ease. As an example I shall consider the case of a generator, with ungrounded neutral, acting on a given star load, of leg impedances, Z_a, Z_b, Z_c. Suppose the machine is symmetrical, *i. e.*, non-salient poles, and a damping squirrel cage on the field. In this case the generator has an impedance Z_1 to positive phase sequence current, and a

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small impedance Z_2 to negative phase sequence currents. The positive phase sequence impedance Z_1 is what has been called the synchronous impedance of the machine such that if E is the no load voltage, $E - Z_1 I$ is the voltage on a positive sequence balanced load I. The problem is to find the currents and voltages on the load, and the voltage between load neutral and generator neutral.

We have given that $I_{ao} = O$.

We calculate the impedance drops of positive phase sequence and put their sum equal to the generated positive sequence voltage. Thus:

$$Z_1 I_{a1} + Z_{ao} I_{a1} + Z_{a2} I_{a2} = E.$$

Similarly, since the generated negative phase sequence voltage is zero,

$$Z_2 I_{a2} + Z_{a1} I_{a1} + Z_{ao} I_{a2} = 0.$$

Solving for I_{a1} and I_{a2}

$$I_{a1} = E \frac{Z_2 + Z_{ao}}{Z_1 Z_2 + (Z_1 + Z_2) Z_{ao} + Z^2_{ao} - Z_{a1} Z_{a2}}$$

$$I_{a2} = E \frac{- Z_{a1}}{Z_1 Z_2 + (Z_1 + Z_2) Z_{ao} + Z^2_{ao} - Z_{a1} Z_{a2}}$$

The terminal voltages will be:

$$E_{a1} = E - Z_1 I_{a1} = Z_{ao} I_{a1} + Z_{a2} I_{a2} = E \cdot \frac{Z^2_{ao} - Z_{a1} Z_{a2} + Z_2 Z_{ao}}{0}$$

$$E_{a2} = - Z_2 I_{a2} = Z_{a1} I_{a1} + Z_{ao} I_{a2} = E \frac{Z_2 Z_{a1}}{0}$$

The voltage between load neutral and generator neutral will be:

$$E_{ao} = Z_{a1} I_{a2} + Z_{a2} I_{a1} = E \cdot \frac{- Z^2_{a1} + Z_{ao} Z_{a2} + Z_2 Z_{a2}}{D}$$

A special case of interest is where $Z_b - Z_c$ are zero, that is, a single-phase short circuit. We then have:

$$Z_{ao} = \tfrac{1}{3} Z_a \qquad Z_{a1} = \tfrac{1}{3} Z_{a1} \qquad Z_{a2} = \tfrac{1}{3} Z_a$$

$$I_{a1} = E \frac{(Z_a + 3 Z_2)}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

$$I_{a2} = E \frac{-Za}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

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$$E_{a1} = E \cdot \frac{Z_2 Z_a}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

$$E_{a2} = E \cdot \frac{Z_2 Z_a}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

$$E_{ao} = E \frac{Z_2 Z_a}{3 Z_1 Z_2 + (Z_1 + Z_2) Z_a}$$

If just previous to the single-phase short circuit the generator was unloaded, Z_a becomes infinite and our final results are:

$$I_{a1} = - I_{a2} = \frac{E}{Z_1 + Z_2}$$

$$E_{a1} = E_{a2} = E_{ao} = E \frac{Z_2}{Z_1 + Z_2}$$

If the generator above did not have a damper winding on the field, it would be an unsymmetrical machine having a single-phase rotor. As we know, the theory of unsymmetrical rotating machines is of considerable complication. Mr. Fortescue has promised a treatment by these methods of the general unsymmetrical rotating machine in a future paper, which I look forward to with great interest.

C. P. Steinmetz: In dealing with calculations or investigations of polyphase systems, or, as usually the case, three-phase systems, the difficulties which we meet are not so much mathematical difficulties, but are what I may call mechanical difficulties. The equations, while mathematically not complicated, lead to expressions which are so complicated and extensive in form as to make any such calculations extremely difficult.

In dealing with the balanced polyphase system, this difficulty has been overcome by the introduction of the equivalent single-phase system, by considering the polyphase system as resolved into a number of single-phase systems, each comprising the circuit from one of the phase wires to the neutral point of the system. This method however, fails in the unbalanced polyphase system—and naturally practically all existing commercial polyphase systems are more or less unbalanced—and the theory of the vector method given to us today in a more extensive description by Mr. Fortescue, gives the solution by showing us in the case of the general three-phase system that it can be resolved into two balanced three-phase systems of opposite phase rotation. We can apply the same plan to other polyphase systems.

V. Karapetoff: Mr. Fortescue deserves the gratitude of the profession for bringing out a new method for numerical compu-

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tations in unsymmetrical polyphase systems, and also for applying the method to a number of practical cases.

Expressions (4) represent the result of solution of certain equations, which equations unfortunately are not given in the paper. The expressions quoted are unnecessarily involved, and it is difficult to see their method of derivation. Moreover, it may be shown that they do not represent the most general case of resolution of an unsymmetrical system of vectors. The following procedure would seem to be preferable.

Definition. A multiple-angle symmetrical polyphase system of vectors is defined as one in which the vectors are numbered not consecutively, but skipping a certain number of vectors. Thus, Fig. 8 represents a triple-angle seven-phase clockwise system, because three angles are comprised between phase 1 and phase 2, and also between phase 2 and 3, etc. Such a generalization of the concept of polyphase systems is useful when the number of phases and the number of angles skipped are prime numbers, so that all the phases may be numbered consecutively, without omitting or repeating any. If the total number of phases n is a prime number, then the total number of possible combinations or multiple systems is $(n-1)$, the angles between the consecutive phases being $\frac{2\pi}{n}$, $2 \times \frac{2\pi}{n}$, . . . $(n-1)\frac{2\pi}{n}$. It is assumed that the numbering in *all* the systems is either clockwise or counter-clockwise, so that there is no confusion between a positive and a negative phase sequence.

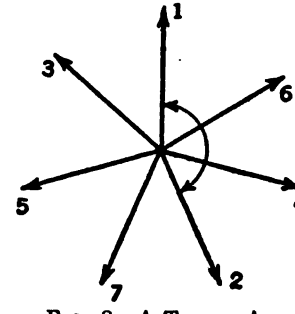

FIG. 8—A TRIPLE-ANGLE SEVEN-PHASE SYMMETRICAL SYSTEM

Theorem 1. An arbitrary system of n unequal and unsymmetrical vectors, *without residue*, may be represented by $(n-1)$ symmetrical multiple n-phase systems. A system without residue is defined as one in which the sum of the vectors is equal to zero. Let the given vectors be $E_1, E_2, \ldots\ldots E_n$, and let the $(n-1)$ unknown systems be denoted by $A, B, \ldots M$, where A is a single-angle system, B is a double-angle system, etc. It is required to prove that the following equations are consistent and may be solved for $A, B, \ldots\ldots M$:

$$\left.\begin{array}{l} E_1 = A_1 + B_1 + \ldots + M_1 \\ E_2 = A_2 + B_2 + \ldots + M_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ E_n = A_n + B_n + \ldots + M_n \end{array}\right\} \qquad (1)$$

These equations correspond to eqs. (4) in Mr. Fortescue's paper when $\Sigma E = 0$. In order to preserve the same conventions as in

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the original paper, the component systems are assumed to be numbered clockwise (though the vectors are rotating counter-clockwise). Then if a is an operator which rotates a vector by an angle $2\pi/n$ counter-clockwise, the preceding equations become

$$\left.\begin{array}{l} E_1 = A_1 + B_1 + \ldots + M_1 \\ E_2 = a^{-1} A_1 + a^{-2} B_1 + \ldots + a^{-(n-1)} M_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ E_n = a^{-(n-1)} A_1 + a^{-2(n-1)} B_1 + \ldots a^{-(n-1)(n-1)} M_1 \end{array}\right\} \quad (2)$$

These n equations contain $(n-1)$ unknown vectors $A_1, B_1, \ldots M_1$, but only $(n-1)$ equations are independent of each other, because of the condition $\Sigma E = 0$. To solve for A_1 multiply the second equation by a, the third by a^2, etc. and add them together. The result is

$$n A_1 = E_1 + a E_2 + \ldots + a^{n-1} E_n \quad (3)$$

This checks with the second line of Mr. Fortescue's equations (4) By analogy we obtain

$$n B_1 = E_1 + a^2 E_2 + \ldots\ldots + a^{2(n-1)} E_n \ldots\ldots \quad (4)$$

and

$$n M_1 = E_1 + a^{n-1} E_2 + \ldots\ldots a^{(n-1)(n-1)} E_n \quad (5)$$

The theorem is thus proved because equation (2) can be solved for the $(n-1)$ unknown vectors in terms of the given E's.

Theorem 2. Referring to theorem 1, if the given system of vectors has a *residue*, say, equal to a vector R, this residue may be split into n arbitrary vectors, $\rho_1, \rho_2, \ldots \rho_n$, in phase or out of phase with one another, and equations (1) become

$$\left.\begin{array}{l} E_1 = \rho_1 + A_1 + B_1 + \ldots\ldots + M_1 \\ E_2 = \rho_2 + A_2 + B_2 + \ldots\ldots + M_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ E_n = \rho_n + A_n + B_n + \ldots\ldots + M^2 \end{array}\right\} \quad (6)$$

This proposition follows at once from the fact that the vectors $(E_1 - \rho_1)$, $(E_2 - \rho_2)$, etc. form a system without residue, so that theorem 1 applies to them. Mr. Fortescue considers only a particular case when

$$\begin{aligned} \rho_1 = \rho_2 = \ldots\ldots = \rho_n &= R/n \\ &= (E_1 + E_2 + \ldots E_n)\ /n \end{aligned} \quad (7)$$

and writes this expression as the first term on the right-hand side in all the formulas of his equations (4). This is perfectly justifiable in so far as he is after a symmetrical solution, but there may be practical cases in which a more general representation according to equations (6) is preferable. In this case the expressions $(E_1 - \rho_1)$, $(E_2 - \rho_2)$, etc. must be used in equations (3) to (5), instead of E_1, E_2, etc.

The net result is that only a system without residue may be resolved in a perfectly definite manner, while a system with a residue first must be converted into one without residue by

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subtracting arbitrary parts of the whole residue from each vector.

With a three-phase system the double-angle component simply becomes a three-phase system of an opposite phase rotation, so that it is not necessary to introduce the question of multiple-angle systems. From a practical point of view the paper would gain much if the author would begin directly with the three-phase system on the basis of his equations (**6**), and leave the general consideration of n-phase systems for the end of the paper. For reference purposes one would not have to study the general theory and perhaps get discouraged before obtaining the required bit of specific information.

The introduction of the sequence operator S does not seem to be necessary, at least for the three-phase system. A proper cyclic notation should accomplish the same purpose with much less theory and much less writing. As a matter of fact, the index of S may be obtained correctly from the subscripts in the expression to which it refers. For example, in Mr. Fortescue's formula (**22**) on the first line the sum of the subscripts of $Z\,I$ is $1 + 2 = 3$, and the order of S is 3 or, which is the same, 0. On the second line the sum of the subscripts of $Z\,I$ is $1 + 0 = 1$, and the order of S is 1. The same is true for all the fundamental formulas on pp. 1038 to 1042 as well as in applications. Therefore, it would seem that the operator S ought to be dropped, and the formulas so rewritten that they would apply to any phase of the system in cyclic rotation.

The fundamental formula (**21**) will then simply become

$$E = Z\,I + W_+\,I_+ + W_-\,I_- \tag{8}$$

Here E, Z, and I apply to any of the three phases, I_+ is the current in the next consecutive phase, and I_- is the current in the preceding phase. For example, if E, Z, and I refer to phase b, I_+ refers to phase c and I_- refers to phase a. At the same time the equation is so written that any of the three phases may be taken as the phase under consideration. W_+ and W_- are the corresponding mutual reactances.

When the mutual inductance between a primary and a secondary polyphase circuit must be considered, Fortescue's equation (**28**) applies, and in the simplified cyclic notation it becomes

$$E = X\,J + X_+\,J_+ + X_-\,J_- \tag{9}$$

where J, J_+ and J_- are the secondary currents in the three phases. The letter X is used instead of W to indicate that the mutual inductance is between the phases of the primary and the secondary circuit. X without subscript covers the combinations of the "corresponding" phases $a - u$, $b - v$, and $c - w$, while X_+ refers to the combinations $a - v$, $b - w$, $c - u$ of the phases "tied one forward", and X_- refers to the remaining combination of the phases "tied two forward", or "tied one backward".

Each of the quantities in equations (**8**) and (**9**) may be rep-

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resented by means of symmetrical coordinates in a general manner, without specifying phases a, b and c. Thus,

$$\left.\begin{aligned} I &= I_0 + I_p + I_n \\ I_+ &= I_0 + a\,I_p + a^{-1}\,I_n \\ I_- &= I_0 + a^{-1}\,I_p + a\,I_n \end{aligned}\right\} \qquad (10)$$

Similarly,

$$W_+ = W_0 + a\,W_p + a^{-1}\,W_n \qquad (11)$$

The first equation (**10**) refers to any phase, and states that the current in that phase is a sum of the residue current I_0, plus the vectors I_p and I_n of the symmetrical coordinates *of the same phase*. The subscripts p and n refer to the systems of positive and negative phase rotation respectively. In an n-phase system subscripts 1, 2, 3, etc. should be used in place of $_p$ and $_n$, to indicate single-angle, double-angle, triple-angle, etc. coordinates.

Substituting expressions such as (**10**) and (**11**) into equations (**8**) and (**9**) one obtains comparatively simple polynomials with three groups of terms: those without a, those multiplied by a, and those multiplied by a^{-1}. These final expressions give the coordinates of E, and are the fundamental equations upon which all applications should be based, without any need for the sequence operator S.

Incidentally, I believe that the expression "symmetrical components" is a more correct and descriptive term of the method than the term "symmetrical coordinates", and I should like to see it so changed while it is not too late.

I also suggest that the term "mutual reactance" be used throughout the paper in place of the "mutual impedance". The latter term implies a combined resistance and inductance action, while the formulas in the paper seem to refer to the magnetic inductive action exclusively. If the resistance component is intended to be taken into account, a mathematical definition of the mutual impedance becomes necessary, because two coils are presupposed to have different resistances, and it is not clear how these resistances enter into the expression for mutual impedance.

It may also be mentioned here that there are cases of unsymmetrical polyphase connections which can be solved more readily without resolving the given electrical quantities into their symmetrical components. Several such cases are considered in the present writer's work entitled "Ueber Mehrphasige Stromsysteme bei Ungleichmaessiger Belastung (Enke, Stuttgart, 1900). For example, in the solution of any arbitrary n-phase star-connected system with given voltages E and phase admittances Y, it is convenient to take the voltage e between the two neutral points as the independent variable. Then, for each phase the current $I = Y(E - e)$, and according to the first Kirchhoff law

$$\Sigma\, I = \Sigma\, Y(E - e) = 0,$$

so that

$$e = (\Sigma\, Y E) / \Sigma\, Y.$$

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Knowing e, the currents in the individual phases are computed from the first equation. In the most general case of a complicated network with a-c. voltages inserted at different places, the Kirchhoff equations of two kinds, written for the vectors of currents and voltages, furnish a complete solution, and one has to consider in individual applications whether or not the resolution into symmetrical components is desirable or not.

A. M. Dudley: Attention has been called to the practical applications of this solution, and one or two may be mentioned here. It has been long known that in the ordinary induction motor in addition to the main rotating working field there may exist several other fields of different amplitude and frequency and phase rotation and that the existence of such fields is probably responsible for the operating freaks sometimes noticed in motors. For example, a motor is sometimes found which will not reverse its mechanical direction of rotation when running light even though the leads be reversed so as to give the opposite direction of phase rotation; or, a two-phase motor will be found which when running light takes power from the line on one phase and returns power to the line on the other, or a, squirrel-cage motor will start from rest with considerably more torque than it is capable of accelerating up to full speed so that it apparently has a so-called sub-synchronous speed. These are actual practical operating conditions often mentioned but never determined quantitatively. Mr. Fortescue's analysis offers the means of such quantitative study.

As a further practical instance may be mentioned the two cases of single-phase connections referred to in the analysis of equations (**141**), (**142**) and (**150**), one of which is equivalent to two coupled polyphase motors connected in parallel and the other to two coupled polyphase motors connected in series. This was fully discussed from another view point in Mr. Lamme's April paper before the Institute and the identity of the two conclusions is a check on Mr. Fortescue's method.

To those who have expressed the idea that the paper is hard to read I would commend Dr. Slepian's discussion also printed herewith as illuminating both to the method and the results. He makes the suggestion that the method as outlined may constitute the basis of a new complex polyphase algebra which will greatly simplify the study of polyphase network problems of all kinds.

Dr. Steinmetz and Prof. Karapetoff have told us that as a mathematical achievement and as a demonstration of theory this paper is a masterpiece. I should like to venture the prediction that as a practical working tool it will eventually come into the greatest usefulness.

Let us then pay our respects to Mr. Fortescue both as a great mathematician and as a scientist and also as the developer of a practical tool which shall make easier the daily task of the engineer in the ranks.

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Charles F. Scott: Mathematics and engineering have their varying conditions, and they have their supporters and their critics. To a certain class of engineers the mathematician is an abstract man who starts somewhere and finds something that can lead to consecutive conclusions. On the other hand, a practical man, using that term in the extreme, is apt to rely wholly on experience and to have no use for the mathematics he does not understand. We all know that the really valuable position is the mean between these two, in which the theoretical and the practical do go together.

The reference to papers of a mathematical and explanatory character, and their relations to engineering reminded me somehow of the first Institute paper I came in contact with. It was through the pages of the *Electrical World*. I had been experimenting with alternating currents and was associated with men who knew only slightly more about it than I did. Many things seemed mysterious. Then the paper by Mr. William Stanley on "Alternating-Current Phenomena," appeared, and that mysterious thing which was the stumbling block of about everybody who came up against alternating-current, the simple single-phase phenomenon of current and voltage out of phase, was cleared up. Mr. Stanley drew some triangles and explained a few things. That put me on a new basis. I could see how the mathematical explanation, the physical phenomena, were related, and it gave me a real understanding of the alternating current. A little later came the polyphase, and it was a step from that to the work on the unbalanced polyphase system.

As we advance then, getting into more and more complicated problems, these papers of solution and explanation come to guide the engineer in his work.

What is this paper? When you look at a paper of this sort it looks as if a mathematician had produced it, leading on and on to interesting formulas, one after another. Now, to bring out my point, I want to emphasize and say I do not think this is a product of the mathematician, but of the engineer. Mr. Fortescue did not work these things out from the theoretical, mathematical standpoint, but as an engineer who was confronted with problems in which unbalanced polyphase systems were concerned. It may have been transmission or some kind of particular machinery which involved problems which he did not have the proper tools to solve.

Therefore, in studying the problems and seeing the inadequacy of the mathematical tools at hand, he went to work and constructed tools to perform that work. Mr. Fortescue has done just as some other engineers do, who work first with experiments, then go to mathematics, and then back to the experiments. That combination of first theory and then practise, back and forth, constitutes the work of the engineer who can be constructive and do pioneer work. I think I am right in saying that this is not a paper of a theoretical mathematician, but a

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paper of a practical engineer, who is developing a new tool for himself and offering it to others.

C. O. Mailloux: This is the second great paper which Mr. Fortescue has presented before this Institute. Much that I had to say in the way of friendly criticism has already been anticipated by Prof. Karapetoff. I sympathize with Prof. Karapetoff's views, and I share them, in regard to the possibilities of simplification of this paper. I thank Mr. Fortescue and many other members of the Institute will also thank him, for having given an introduction to the paper, because that introduction is really the mathematical specifications in accordance with which the paper has been prepared; and to me it was more valuable, as a criterion of the value of the paper, than the paper itself, because the paper itself cannot be read offhand. The bewildering exhibit of subscripts to be found in it is something that will, well, make one pause; but the introduction, to any one who is at all familiar with mathematics, tells exactly the basis on which the paper is written, what it aims to do, and substantially the method by which it accomplished its aim. I am very thankful, indeed, personally, that that was done.

Part I is also very useful, but if it had followed the method of presentation suggested by Prof. Karapetoff, it would have been much simplified. The paper is so valuable, that it would be well worth while to rewrite it for the purpose of making it more easily digestible and more easily accessible to the great body of practical men. It appears to have been written too much from the point of view of the professed mathematician, who seeks first to make a generalization, and then proceeds to experimentation to find practical and special cases. It would have been a better arrangement if that generalization had been put in an appendix, and if the statement had been made that from the general case is derived the particular case which interests us, namely, the case of the three-phase current. I think that to discuss *n-phases* is of academic interest only at present, even though it is possible that there may be some day when it will be of practical importance. Until that seemingly remote day comes, a generalization which includes *n*-phases might as well lie in an appendix, out of harm's way, and especially where it would not encumber the rest of the discussion. If Mr. Fortescue had written the paper by putting into the appendix, the generalization, the thing which makes it complete and comprehensive, he would have simplified it and made it more useful.

It is because I think so highly of the paper, I would like to see it made clearer. It is not a difficult paper, the mathematics are simple, but they look complex, because one is bewildered by a maze of subscripts and by many "operators." The fundamental idea of the operator is a splendid one, but it is just as well to put that explanation in an appendix, in all its generality, and then use it in the simple form that Prof. Karapetoff suggests. Those who deal with the matter practically, would not then

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have so many equations to analyze and not so many symbols to deal with.

The analysis of the actions and reactions of the various forces present in electrical circuits, and the interpretation of the equations devised for expressing these actions and reactions, under different conditions, whether they occur in balanced or in unbalanced systems, may be facilitated by remembering that all physical forces and all forms of energy are subject to the same general principles of conservation and equilibrium, and, especially, that Newton's principle of dynamic equilibrium,—namely, that under all conditions and at all times, action and reaction are opposed and equal to each other, exactly—, applies to electrical, as well as to mechanical, forces, powers, and energies.

Analogy requires that, in any electrical circuit, at any instant of time whatever, the forces of action and those of reaction should always be opposing and balancing each other exactly, just as they do in mechanical systems; and it is known that they always do balance exactly. Analogy also requires that, for electrical circuits, when dealing with instantaneous values, the equations of electrodynamic equilibrium should have the same general characteristics as the general equation of dynamic equilibrium or balance for mechanical forces. The latter, it is well known, reduces to an algebraical sum of four terms, representing four distinct kinds or amounts of force, or power, or energy, according to the case, whose resultant, when a state of equilibrium occurs, is equal to zero.

The four kinds or amounts of force, power, or energy, in terms of which every dynamic reaction whatever, in mechanics, can be completely expressed, find exact parallels in electrodynamics. Here, also, we find four kinds of force, power, or energy; we have reversible forces of two kinds, corresponding to those which, in mechanics, produce or result from changes in kinetic energy, or changes in potential energy; we have the dissipated force, which is expended in overcoming ohmic resistance, corresponding to the force lost in overcoming friction in mechanics, and we have, for the fourth kind, the force, power, or energy representing the balancing actions or reactions which are necessary to maintain the equilibrium that must exist at every instant of time between action and reaction. The first two kinds represent energy which is not immediately dissipated but is displaced and stored in the system; the third kind represents energy which is immediately consumed in the system; and the fourth kind represents energy which is put into the system from an outside source, or else which is taken out of the system by overflow into another system or circuit, as may be required to maintain equilibrium.

These considerations show that even in an unbalanced electrical system, so-called, there must still be dynamic balance or equilibrium. The balancing force, power, or energy, according to the case, appears in the fourth term, where it represents the compensating force, power, or energy by which equilibrium is maintained.

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Charles L. Fortescue: I maintain that the paper is quite simple. The mathematical portion is not difficult to understand, anybody can follow it if he takes the pains. I admit that the appearance of the equations is cumbersome, but that is almost impossible to overcome. The nature of the subject makes the equations cumbersome.

Prof. Karapetoff in his discussion used the word "stimulation". I wish to say that necessarily in a paper of this kind there are many sources of stimulation, as I pointed out in my introduction, many of the ideas are not new. The idea of symmetrical component three-phase systems is being used more or less by others, but the theory has never been presented systematically, and I think the idea of symmetrical operators is new.

Dr. Steinmetz's and Prof. Karapetoff's discussions are what I may term conjugate discussions. Dr. Steinmetz discusses the paper purely from the practical point of view. He thinks that the system is capable of practical application, and will be of great use for that purpose. Dr. Karapetoff points out the possibility, from a theoretical point of view, of the n-phase system. I wish to say nothing would have given me greater pleasure than to go into that purely theoretical matter much more extensively. It is very fascinating and has great promise, but I felt that there was needed, in this paper, some practical justification for presenting it. The theoretical part was so long drawn out, that I felt it was necessary to show by practical illustrations why I presented it.

I felt that the presentation of mathematical solutions did not alone afford a justification for the paper, but that there must also be a good practical reason for it, and I felt if I went into all the theoretical ramifications of this very interesting subject people would ask: "What is he about? What does he mean by driving us through all this painful stuff without giving us a good reason?" So I thought that the presentation of the theory should be as concise and short as possible, and for that reason I left out a good deal of explanation that some people think ought to be there.

Prof. Karapetoff has apparently lost sight of the fact that unless his arbitrary vectors are all equal to $\dfrac{\check{E}_1 + \check{E}_2 + \ldots \check{E}_n}{n}$ they in turn may be resolved into $(n - 1)$ symmetrical systems and the "residue" will eventually reduce to the same value as that given by me, namely n equal vectors of value

$$\frac{\check{E}_1 + \check{E}_2 + \ldots \check{E}_n}{n}.$$

The form in which the theorem is presented does not preclude the adoption of any artifice such as the subtraction of n arbitrary vectors to make the vector sum equal to zero, when any material advantage is gained thereby. Such an artifice, in fact, is gener-

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ally adopted in the case of Y-connected circuits where the vector sum is rarely zero on account of the presence of the third harmonic. Furthermore, in any symmetrical system there is a definite impedance to each component symmetrical current including the "residue" as Prof. Karapetoff has termed the constant vector, this is not true if an arbitrary "residue" is adopted.

Prof. Karapetoff brings up another point, the use of sequence symbols. In three-phase systems, once a person has become familiar with the use of this method, it becomes quite a natural system. He thinks quite naturally in terms of symmetrical components, and he usually omits the use of the symbols. In other words, the symbols are not absolutely necessary, but they simply afford a sure method of keeping track of the mechanical part of the mathematical work. If one makes a mistake in carrying out the mathematical work, one can always go back to the sequence symbols and check it up. If a quantity looks unsymmetrical, and a mechanical error is suspected, one can always follow it out with the sequence symbols and be sure of getting the correct result. Furthermore, in dealing with a system of more than three phases, the sequence operator becomes important, because there are so many cross mutual inductances that the formulas become very much involved, and it is necessary to have some reliable mechanical device that will keep one in the right path; the sequence operators are such guides. They are almost indispensable when dealing with power both in three-phase and n-phase systems. Those are the principal reasons for their use.

Prof. Karapetoff brings up the question—What is the "mutual impedance?" I have not used the term perhaps quite correctly as I have applied it to cases in which the dissipative forces are not strictly proportional to the velocities or currents. Mutual impedance may be defined in this manner: If we have two terminals of a circuit carrying a given current, and two other terminals of another circuit, the electromotive force produced across the second pair of terminals, given a sine-wave current of a given frequency is, the product of the mutual impedance between the two circuits and the current in the first pair of terminals. In some cases the current follows paths common to both circuits, so that the mutual impedance may have a real component; in other cases the energy component may be due to eddy currents or losses in subsidiary circuits common to both.

Mr. Dudley has pointed out that the practical engineer must always keep track of the theoretical side of his work. The object of all mathematical investigation and theory should be to carry our knowledge of operating conditions further, and to investigate such obscure phenomena as arise from time to time. Very often we find that a new tool has to be devised in order to enable us to carry out our theoretical investigations without becoming involved in too great complication.

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I feel that a paper presented before a scientific body that cannot be read by every member is hardly justified. I believe that all can follow this paper if they sincerely try to do so, it may take a little time, but the mathematics is not difficultt.

V. Karapetoff (by letter): In my Theorem 2, ρ_2, ρ_2 etc. are arbitrary vectors such that $\Sigma\, \rho = R$, where R is the total residue of the given system of vectors. Each ρ may be resolved geometrically into a vector R/n, in phase with total R, and another vector, say ρ, so that $\rho = \rho + (R/n)$. Hence $\Sigma\, \rho + (R/n) = R$, or $\Sigma\, \rho' = 0$. Thus, the system of vectors is without residue and consequently may be resolved into $(n - 1)$ symmetrical systems. The system of n vectors each equal to R/n, in phase with total R, is an *irreducible residue.*

When it is desired to sift out all the symmetrical components from the given vectors, the form R/n for the residue should be used. However, there may be conceivable cases in which there might be some advantage in splitting the residue into certain unequal or unsymmetrical parts, without attempting to extract all of its symmetrical components. For this reason, Theorem 2 is stated in the most general form. I am glad to have my attention called to the fact that only in the case where each ρ is equal to R/n, we have the *true* or the *irreducible* residue.

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SUSTAINED SHORT-CIRCUIT PHENOMENA AND FLUX DISTRIBUTION OF SALIENT-POLE ALTERNATORS

BY N. S. DIAMANT

ABSTRACT OF PAPER

It is shown in Section IV that with the ordinary field forms met with in practise, the resultant flux wave under s. s. c. (sustained short circuits) will be extremely distorted, see Figs. 16, (4th wave) 23, 27, 27A, etc., for the simple reason that with the very low voltages obtained under such conditions, the fundamental of the *B*-curve of the field is reduced so much by the armature reaction that the higher harmonics assume a very predominant rôle and become several hundred per cent of the s.s.c. fundamental. See Fig 26 and Tables XI, XII, and XIII.

As a corollary to the foregoing it is found that the *B*-curve under load will not differ radically from the no-load field form since the fundamental will remain large enough to hold its own. See Fig. 22 and also footnote (4). The cross magnetizing effect of the armature reaction is, of course, to make the *B*-curve unsymmetrical with respect to the mid-pole axis. Compare Figs. 7 and 22.

The magnetic oscillations are studied not only by means of full-pitch stator coils but also by means of rotor coils No. 7 and 8, Fig. 3, and stator coils No. 9, 10, 11, 12, 15, and 16, Figs. 3 and 3A.

Attention is called to the following facts for which explanations and theoretical proofs are offered.

(1) *The ripples at the crest of the e. m. f.* waves of the stator coils are due not so much to the flux pulsations as to (a), the to-and-fro flux swing across the pole, and (b) the cutting of a *B*-curve similar to wave (b), Fig. 6. See equations 22, and 23; also Table I and Note A, at the end. Compare Figs. 7 and 2nd wave Fig. 4.

(2) *The most important oscillations set up by the armature reaction* are 4 and 6 times machine frequency for two- and three-phase machines respectively. See supplementary Notes B and C; also Figs 8A, 8B wave lengths marked *B* and *D*, and Fig. 5, wave length marked *B*, and Table I.

(3) *Eccentric rotor sets up pulsations* once in every revolution, under load or at no-load, but none under s.s.c. See Fig. 4, 2nd wave, Fig. 5 wave length marked *A*, and Figs. 8A, 8B, and 8C. Also see Table I, column 4. For a tentative explanation see p 1183.

(4) *The e. m. fs. and fluxes* for coils No. 10 at the top of a tooth are very much larger than the e. m. fs. and fluxes for coil No. 9, at the top of a wedge. See Tables II and III; also Figs. 10, 11, 12.

(5) *The spacing of the ripples of the no-load e. m. fs.* is unequal, contrary to theory. See Fig. 13 and Table IV. Also equations (22) and (23) p. 1195.

(6) *The e. m. fs. and fluxes* for search coil No. 5, at the bottom of the slot, are larger than the e. m. fs. and fluxes for the coil No. 1 at the top of the slot. See Fig. 7 and Tables V, VI, and XV.

(7) *The third harmonic e. m. fs.* of the stator search coils increase with the increasing excitation. See Table IX.

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(8) *Under s.s.c. there is present some cross-magnetizing armature reaction* which (a), makes crests 1 and 3 Fig. 27, unequal. See Table XV. (b), it shifts the axis of the flux towards the larger crest, Figs. 23, 27, etc., and thus makes angle *B*, Fig. 27, less than (1/2) *A*. See Table XIV.

(9) *The agreement between the armature reaction* obtained by subtracting e_r from e_{fo}, (see Figs. 24, 25, 31 and Figs. 16, 27, etc.) and that obtained by subtracting the equation of the s. s. c. waves as given in Tables XI and XII, from corresponding open-circuit e. m. f. equations as given in Tables VII and VIII, is fairly satisfactory. See last column of Table XIII.

(10) *The direct component of the armature reaction of an imperfect two-phase machine* consists of (a) a regular demagnetizing component, plus (b), a transverse or cross magnetizing component in quadrature to (a). See equations 56, 59, etc.

(11) *The analysis of the no-load field form* gives the 3rd, 5th, and 7th harmonics as 5 to 15 per cent but the $(2q \pm 1)$ ths seem to be small.

(12) *The armature current under s.s.c.* is very nearly sinusoidal. See Figs. 15 and 16. Also Table X. The *H*-curve of armature reaction set up by such a current will be sinusoidal, see equations 32, 33, etc. and succeeding two paragraphs. The *B*-curve, however, will not be sinusoidal for lack of constancy and uniformity of the permeability at different points of the magnetic circuit, and on account of saturation. See sections III and IV.

I. Introduction

IN A paper dealing with the sudden short circuits of alternators, presented at the Panama-Pacific Convention, in passing, the author called attention to the following[1]: First, that the voltages induced in full-pitch stator exploring coils under sudden short circuits "are not of as simple a form as theory would indicate." Second, that the flux distribution within about one or two cycles, after the short circuit takes the general shape which it has under steady or sustained short circuit. The e.m.fs. induced in exploring coils placed at the top, middle and bottom of different slots, all, very shortly after the sudden short circuit, assume a very distorted wave form which persists for the whole length of the film, *i.e.*, about 10 or 15 cycles. The above are clearly seen from Figs. 1 and 2[2] which represent the two extreme cases of symmetrical and totally unsymmetrical armature currents.

The ordinary theories of armature reaction or of short-circuit impedance tests, would hardly lead one to expect results which these oscillograms seem to indicate. A number of valuable investigations on the flux distribution[3] at various loads and power

1. Trans., A. I. E. E., 1915. Vol. 34, part II, p. 2259.
2. See Table III, p. 2255.
3. *Journal, I. E. E.*, Vol. 37, p. 148. G. W. Worrall and F. T. Wall. See Figs. 2 to 7 inclusive.

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factors have given results quite consistent with those deduced from the usual theory. The flux distribution was in no case sinusoidal, to be sure, but it never deviated greatly from the no-load field form usually met with in practise. This is clearly

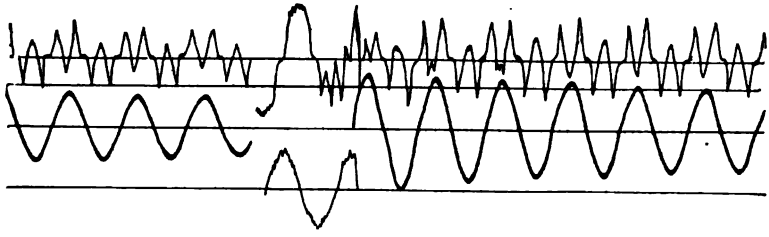

FIG. 1

Oscillogram No. 101. Table III, footnote (2). Machine series Y-connected and suddenly short-circuited at its terminals. Field excitation = 5.1 amperes. Speed = 1200 rev. per min. First wave: E_{1T} = e.m.f. induced in coil No. 1, at top of slot. Second wave: armature current. Third wave: phase voltage. Second straight line: calibration for second wave = 293 amperes.

brought out in a series of interesting oscillograms given by W. J. Foster.[4]

In view of the above and the entire lack of reliable data, as to the detailed analysis of sustained short-circuit (s.s.c.) phenomena, the writer could make no more definite statement in the paper referred to, than, that "it would seem that a good way

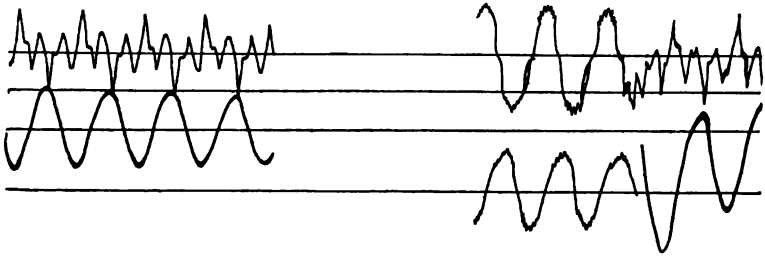

FIG. 2

Oscillogram No. 102. Table III, footnote (2). Machine series Y connected and suddenly short-circuited at its terminals. Field excitation = 5.05 amperes. Normal speed. First wave: E_{3M} = e.m.f. induced in full-pitch coil No. 3 at middle of slot. Second wave: armature current. Third wave: phase voltage. Second straight line: calibration for second wave = 297 amperes.

to study sudden short circuits was to begin by studying s.s.c. in detail. Although a great deal of work has been done on s.s.c. it is believed that so far little or no attention has been given to the question of flux distribution under such conditions."

4. TRANS., A. I. E. E., Part I, 1913, p. 749. See Curves 31, 34, 37, 38, 40, 48, etc.

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In the following pages an attempt is made to show that all the complicated phenomena of s.s.c. can be explained by taking proper account of certain details and secondary effects. At the end a critical resumé and discussion of results is given which will enable the reader to obtain some of the principal results of the investigation without studying the paper in detail.

II. Data Relating to the Machine, Exploring Coils, etc.

The machine used for the tests is a standard three-phase, six-pole, revolving field alternator rated at 45 kv-a., 480 volts.

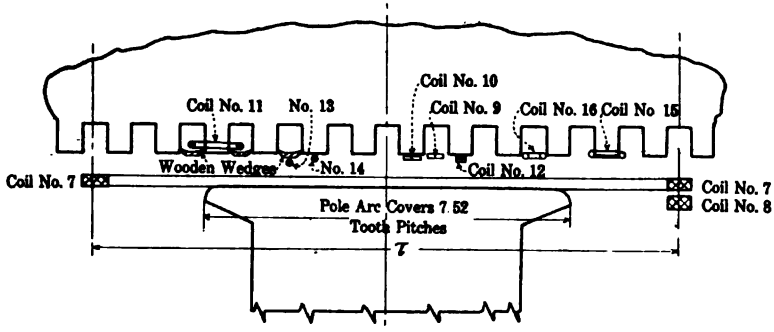

Fig. 3.

Coil No. 7: 40 Full pitch concentrated turns.
Coil No. 8: 20 Concentrated turns.
Coil No. 9: 5 Concentrated turns.
Coil No. 10: 5 Concentrated turns.
Coil No. 11: 5 Concentrated turns.
Coil No. 12: 2 Concentrated turns, one on top of other thus having effective pitch of single turn.
Coil No. 13: Single conductor on top of wedge.
Coil No. 14: Single conductor on top of tooth.
Coil No. 15: Single turn around tooth.
Coil No. 16: Single turn around slot.

(278 across phase). It has an amortisseur winding and is equipped with a set of search coils as follows:

Coil No. 1. Full-pitch, 5 concentrated turns at top of slot, right under wooden wedge.

Coil No. 2. Same as No. 1, except that it is $\frac{2}{3}$ pitch.

Coil No. 3. Similar to No. 1, but placed at the middle of the slot.

Coil No. 4. Same as No. 3, except that it is $\frac{2}{3}$ pitch.

Coil No. 5. Similar to No. 1 but placed at the bottom of the slot.

Coil No. 6. Same as No. 5 except that it is $\frac{2}{3}$ pitch.

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These coils were placed in different slots. The other search coils are schematically shown and described in Figs. 3 and 3A. The armature winding is of the ordinary double-layer type with a $\frac{2}{3}$ pitch, and it has two circuits which may be connected either in parallel or in series.

In connection with the coils most commonly employed the following notation will be used:

$*E_{1T}$ = e.m.f. induced in coil No. 1. (Subscripts: No. 1 top of slot.)

$*E_{3M}$ = e.m.f. induced in coil No. 3. (Subscripts: No. 3 middle of slot.)

$*E_{5B}$ = e.m.f. induced in coil No. 5. (Subscripts: No. 5 bottom of slot.)

$*E_{RF}$ = e.m.f. induced in coil No. 7, (Subscripts: Rotor full-pitch coil.)

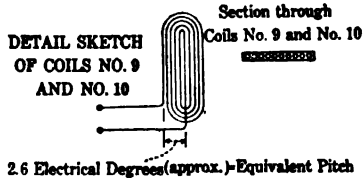

FIG. 3A

$*E_{RC}$ = e.m.f. induced in coil No. 8. (Subscripts: Rotor concentrated coil.)

$*E_{TW}$ = e.m.f. induced in coil No. 9. (Subscripts: Coil at top of wedge.)

$*E_{TT}$ = e.m.f. induced in coil No. 10. (Subscripts: Coil at top of tooth.)

$*E_{AT}$ = e.m.f. induced in coil No. 11. (Subscripts: Coil around tooth.)

$*E_{TT2}$ = e.m.f. induced in coil No. 12. (Subscripts: 2 turns on top of tooth.)

$*E_{CTS}$ = e.m.f. induced in coil No. 13. (Subscripts: Conductor on top of slot.)

$*E_{CTT}$ = e.m.f. induced in coil No. 14. (Subscripts: Conductor on top of tooth.)

B = Flux density as further specified by its subscript.

f = (actual) machine-frequency in cycles per second = r.p.s. x p.

*All under conditions to be specified.

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k = Order of space harmonic. See p. 1164 for explanation.
p = Number of pairs of poles.
p_k = Number of pairs of poles of kth space harmonic $= k\,p$
q = Number of slots per pole.
s.s.c. = sustained short circuit.
τ = Pole pitch in centimeters or inches.
τ_k = Pole pitch of kth space harmonic $= \tau/k$.
$e_{rot.st.}$ = Instantaneous value of rotational e.m.f. induced in stator coil.
$e_{pul.st.}$ = Instantaneous value of e.m.f. induced in stator coil due to pulsations of flux. (In magnitude only.)
$e_{sw.st.}$ = Instantaneous value of e.m.f. induced in stator coil due to to-and-fro swing of flux. (Without change in magnitude.)
$e_{pul.r.}$ = Same as $e_{pul.st.}$ except that it refers to coil on rotor.
$e_{sw.r.}$ = Same as $e_{sw.st.}$ except that it refers to coil on rotor.

III. Flux Distribution at Various Loads and Power Factors

The oscillographic study of flux distribution in alternators and other apparatus by means of exploring coils would have been extremely simple, were it not for the fact that the quantity recorded on the film is not the flux wave in which we are interested but a quantity proportional to its *time rate of change.*

A conductor moving across a magnetic field, constant both in time and space, *i.e.*, a field constant in magnitude and stationary with regard to the pole axis, will have an e.m.f. induced in it which may be designated as motional or rotational e.m.f.; this will be maximum when an element $d\,l$, of the conductor under consideration, its velocity v, and the flux density B_m, all three, are mutually perpendicular. However, without any relative motion between flux and conductor, if the flux enclosed by the circuit varies for any cause and in any manner whatsoever, there will be an e.m.f. induced proportional to the rate of change of flux. This may be called static or variational e.m.f.

Thus in the most general case,[5] which is the one met with in practise, the e.m.f. induced in a conductor or coil is:

$$e_{\text{total}} = e_{\text{motional}} + e_{\text{variational}},$$

where e_{total} is the quantity recorded on the film.

5. See, in this connection, A. Blondel, *Sur l'énoncé le plus général des lois de l' induction.* Compt. Rend. 1914, Vol. 160, Nos. 20 and 22.

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The application of the above to the analysis and interpretation of the complex waves obtained under s.s.c. or even the simpler phenomena of open circuit is difficult. Therefore, it will be well to call attention at first, to some general facts which though not entirely new do not seem to be generally known.

It is clear that the e.m.f. induced in a conductor or a number of concentrated turns (on the armature) will be a true reproduction of the B-curve under the pole, provided the variational e.m.f. were zero—which is seldom the case. The variational or static e.m.f. will depend upon the manner in which B varies with respect to time only. As a result of some valuable researches[6] it has been found that B may vary with respect to time in two ways: First, due to what may be termed the inductor alternator action of the teeth, there may be a *variation in the amount of reluctance* of the magnetic circuit of the machine. This will cause pulsations in the intensity of B and thus an e.m.f. will be induced in the armature coils as well as the pole pieces, yoke, etc. The latter will give rise to parasitic eddy currents which will tend to wipe out the pulsations. Thus, as indicated by theory and as shown by test, the variational e.m.f. due to this cause will be very small (compared to the rotational e.m.f.). Secondly, it has been found that the flux density at any point under the pole may vary due to a *to-and-fro swing* of the flux-wave with respect to the (geometrical) pole-axis. It has been shown by Worrall[6] by trimming the pole-shoes and varying the number of slot-pitches covered by it, that the pulsation is maximum when the pole-arc covers a whole number of slot-pitches, while the flux-swing is maximum when the pole-arc covers (an integer + ½) slot-pitches. This neglects fringing and in the experiments referred to, it seems to hold remarkably well. However, it seems to the writer that fringing should not be neglected since it is not the geometrical tooth or slot but the manner in which the flux is distributed in these that is responsible for the static e.m.f. Thus, his conclusions seem to be correct, except that it would be more rational to consider the effective teeth and slots (so far as flux is concerned).

The study of the experimental results as well as the quantitative expressions developed in the supplementary notes show that the e.m.f. waves of coils No. 7 and No. 8 give chiefly the

6. G. W. Worrall, *Jour., Inst. E. E.*, 1907, Vol. 39, p. 206. Also 1908, Vol. 40, p. 413. Guery, *L' Eclairage Electrique*, 1903, Vol. 36, p. 51. Arnold and La Cour, *Samn. Elek. Vort.*, 1901, Vol. 3, p. 58. K. Simon, *E. T. Z.*, 1908, Vol. 27, p. 631. *Electrician*, Vol. 57, p. 581.

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e.m.f. due to flux pulsations. See second wave Fig. 4 and Fig. 5. The e.m.f. due to the to-and-fro motion of the flux is given by: (for $x_1 = 0$ see Fig. 6 and note A)

$$e_{sw.r.} = 2\,N\,l\,q\,\omega\,\epsilon^2\,B_1 \sin(4\,q\,\omega\,t) \qquad (22)$$

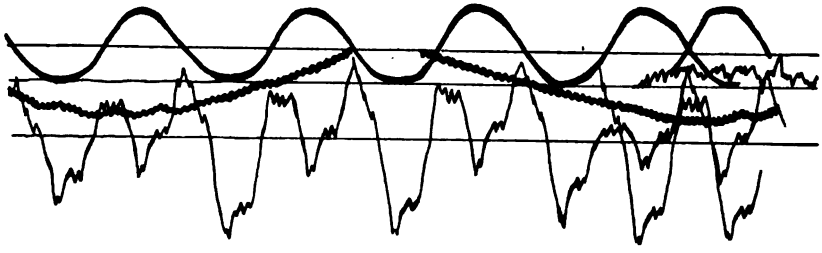

FIG. 4

Oscillogram No. O A 84. Test No. 196. First wave: armature current under s.s.c. at 4.96 amperes field excitation, normal speed and machine connected 2-phase. Third wave: E_{RF} = e.m.f. in full-pitch 40-turn rotor coil No. 7 under same conditions. Second wave: E_{RF} at same excitation and speed at no-load.

and that due to the pulsations of flux by:

$$e_{pul.r.} = 4\,N\,l\,q\,\omega\,\eta\,B_1 \sin(2\,q\,\omega\,t) \qquad (23)$$

where η and ϵ are small fractions denoting the amplitude of the variations of flux, in each case. The former would be expected to be small as it involves ϵ^2 and oscillograms give no indication of the existence of ripples of ($4\,q \times$ machine-frequency), as required by (22). As seen from Fig. 4, second wave, and Fig. 5, the little kinks in the E_{RF} wave (due to the pulsations

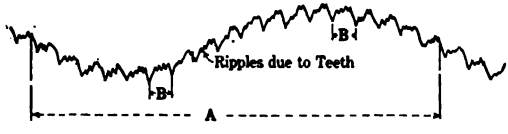

FIG. 5

Oscillogram No. O A 21. Test No. 91. Table I. Machine parallel Y-connected and run as synchronous motor. Field current = 4.4 amperes. Line current = 15.5 amperes. Line voltage = 220. E_{RF} = e.m.f. induced in rotor coil No. 7. Note: A = 1080 deg. B = 60 deg.

of flux) under synchronous motor and no-load operation, are fairly small and have a frequency of $2\,q\,f$, *i.e.*, show $2\,q = 24$ ripples per cycle. See Table I, column 6. This is what would be expected from (23).

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While it is clear that the kinks in the E_{RF} wave, Figs. 4 and 5, are due to the teeth, the long flat superimposed wave having a period A, is believed to be due to the rotor being eccentric with respect to the stator. This explanation seems the more plausible when it is considered that A, Fig. 5, is found to be 1080 electrical degrees, as indicated in Table I column 4;

TABLE I.

Data relating to the e.m.fs. induced in rotor exploring coils under conditions given below.

No. of oscill. / No. of test	E_{RF} / E_{RC}	Field current in amps.	A in degrees see Fig. 5	B in degrees see Fig. 8	No. of ripples per cycle	Freq. in cycles per sec.	Amplitude in max. volts see Figs. 8a, b, c pos.	neg.	Line current in amps.	Approx. power factor
1	2	3	4	5	6	7	8	9	10	11
Synchronous Motor Operation										
O A 21/91	E_{RF}	4.4	1080	54.8	24	59.5	0.69	0.82	15.5	1
O A 22/92	E_{AP}*	4.4	1080	60		59.5	0.66		15.2	1
O A 39/115	E_{AP}*	1.0	1080		24	59.5	1.12	mean	79	†0.27 lg.
O A 40/116	E_{RC}	8.3	1080	61.3		59.5	1.12	mean	76	0.28 ld.
O A 36/118	E_{RC}			61	24	59.5	0.037	0.053	85.5	0.25 lg.
O A 37/109	E_{RC}	8		60.6	24	59.5	0.037	0.053	71.2	0.24 ld.
Sustained Short Circuit. Series Y Connection										
−/164	E_{RF}	5			24	54	0.97 mean			
−/165	E_{RC}	5			24	50	0.03 mean			
−/166	E_{RC}	3.16			24	52	0.027 mean			
−/167	E_{RF}	3.16			24	52	0.71 mean			
226/−	E_{RF}	5		59.2	24	60				
Open-Circuit Operation										
−/163	E_{RF}	5				60	2.46			
−/162	E_{RF}	3.16				60	1.94			
−/161	E_{RF}	5				54	2.31			
−/160	E_{RF}	3.16				54	1.98			
O A 24/94	E_{RF}	5	1080	60	24					

*E_{AP} = e.m.f. induced in search coil around pole, right under pole-shoe; this gave practically same results as full-pitch coil No. 7 and therefore it was disconnected from slip-ring and not used.

†lg. = lagging. ld. = leading.

i.e., the superimposed wave is found to have under load or at no-load a frequency $(\frac{1}{3})\, f = (1/p)\, f =$ one cycle per revolution.

For a conductor placed at the middle of the pole-arc, *i.e.*, at $x_1 = \tau/2$, Fig. 6, $e_{sw.r.}$ would be maximum and $e_{pul.r.}$[7] would be zero; *i.e.*, such a conductor would give ripples due to the

7. See equations (21) and (20) respectively in supplementary notes.

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flux-swing only. Attempts were made to take advantage of this but owing to the high centrifugal force it was found difficult to fasten down securely wires at the mid-pole position.

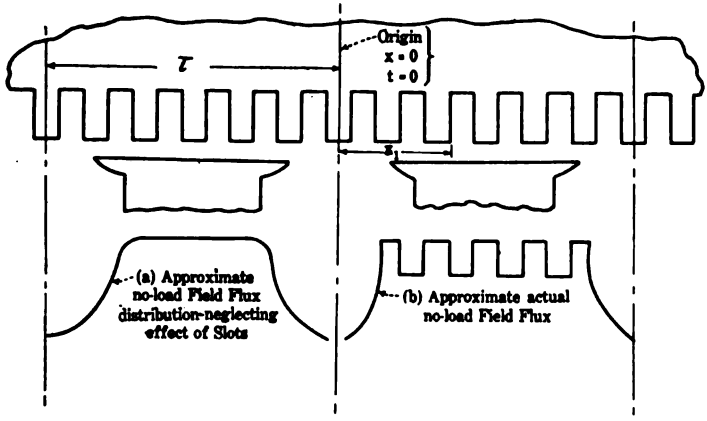

FIG. 6

However, a very good idea of the relative magnitude of the e.m.f. due to flux-swing, can be obtained by studying the e.m.f. waves induced in the stator coils. These will consist of (e_{rot}

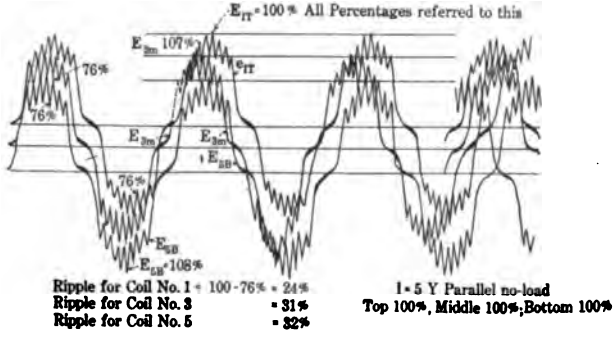

FIG. 7

Oscillogram No. O A 38. Test No. 110. Tables IV, V and VI. Machine run on open-circuit. First wave: E_{1T} = e.m.f. induced in exploring coil No. 1 at top of slot. Second wave: E_{3M} = e.m.f. induced in exploring coil No. 3, at middle of slot. E_{5B} = e.m.f. induced in exploring coil No. 5, at bottom of slot. Speed normal, *i.e.*, corresponding to 60. Excitation = 5 amperes. Same vibrator and resistance in vibrator circuit used for all waves. Compare to Fig. 22.

$+ e_{pul} + e_{sw}$). As indicated by equations (7), (13) and (18) Note A, e_{pul} will be negligible for a rotor coil placed at the middle of the pole-shoe, *i.e.*, at $x = \tau/2$, or for a stator coil

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whenever $x = \tau/2$, see Fig. 6. In other words, e_{pul} will be negligibly small *whenever* $e_{rot.st.}$ is maximum. Thus the ripples at the crest of the e.m.f. wave ($x = \tau/2$) of the stator coils 1, 3 and 5 are due mainly to flux-swing and a comparison of the ripples in them, Fig. 7, with the kinks of the E_{RF} wave, second wave Fig. 4 and Fig. 5, and E_{RC}, Fig. 8a, will show at once that the kinks (due to pulsation) are quite negligible compared to the ripples (due to flux-swing)—especially when it is remembered that coils No. 8 and 7 have many more turns than the stator coils and the volts per centimeter for the calibration of the former is about ten per cent of that for the latter.

The subject of tooth ripples is of great practical importance,

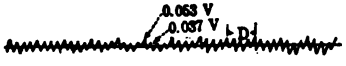

FIG. 8A

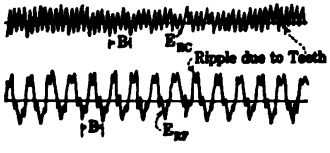

FIG. 8B

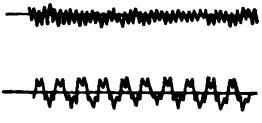

FIG. 8C

Fig. 8a Oscillogram No. O A 36. Table I. Machine run as synchronous motor. Line current 85.5 amperes lag. Line voltage = 200. E_{RC} = e.m.f. induced in coil No. 8. Exactly same wave shape and amplitude obtained in oscillogram No. O A 37 with 71 amperes of leading line current and 238 line voltage.

Fig. 8b Oscillogram No. O A 49. First wave E_{RC} = e.m.f. induced in coil No. 8. B = 60 deg. Second wave: E_{RF} = e.m.f. induced in coil No. 7. B = 60 deg. Machine series Y-connected and short-circuited at its terminals. Field current = 5 amperes. Frequency = 54 cycles per second. Resistance in vibrator circuit = 0 ohms and gold-leaf fuses short-circuited.

Fig. 8c Oscillogram No. O A 50. Similar to O A 49 in every respect, except field excitation = 3.16 amperes. Same vibrators used.

not only in case of accurate tests and measurements but also in connection with the operation of large alternators, induction motors, etc. Though effective methods have been developed for their elimination there seems to be lack of general concensus of opinion[8] as to their origin and detailed analysis. In this connection it seems pertinent to ask, whether the ripples are entirely variational in their origin or are they partly motional. The actual flux density in the air-gap has the general form indicated by curve (*b*) rather than curve (*a*) Fig. 6. Since the motional

8. S. P. Smith and R. H. Boulding, *Jour.*, *I. E. E.*, 1915, Vol. 53, p. 205. Discussion p. 238. Also see correspondence *Electrician*, June, 1914. Further, see references given under foot-note (6).

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e.m.f. will be duplicate of the B-curve, it is clear that the tooth ripples will partly have motional origin, *i.e.*, will be partly due to the cutting of a B-curve like that shown in (*b*) Fig. 6.

A general idea of the relative importance of the ***tooth kinks in the B-curve*** may be gained from Fig. 9 which is self-explanatory. The flux measurements were obtained by pulling out of the air gap an exploring coil fastened on a piece of cardboard and having a length approximately equal to the axial thickness of the armature core, and a width about 30 per cent of that of a tooth. In the test recorded in Fig. 9, the machine was stationary; in the next one, the rotor was excited and slowly rotated and the following maximum throws of the Grassot fluxmeter were observed.

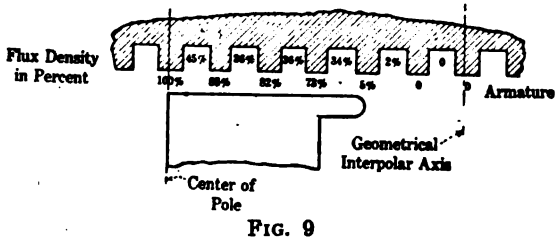

FIG. 9

Flux distribution with the machine stationary as measured by Grassot fluxmeter.

Coil No. 10—coil at top of tooth—maximum deflection = 100 per cent.

Coil No. 9—coil at top of wedge—maximum deflection = 41 per cent.

Coil No. 8—concentrated coil—maximum deflection = 8 per cent

The actual flux density under running conditions was further investigated by means of the exploring coils No. 9, 10 and 11, Figs. 3 and 3A, by taking oscillographic records of the e.m.fs. induced in them under no-load, s.s.c. and synchronous motor operation. The data obtained in this series of tests are brought together in Table II. As seen from Figs. 10, 11 and 12 the E_{AT}, E_{T1} and E_{Tw} waves have two prominent crests above, and two below, the zero line, per cycle. These are quite characteristic and are designated as 1st and 2nd positive and 1st and 2nd negative. See Table II, columns 8 and 11. The angular distance between successive + or − crests is designated by α and that between successive positive and negative ones by β. See Fig. 10 and Table II columns 6 and 7.

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TABLE II.—Data relating to the e.m.fs. induced in Exploring Coils No. 9, No. 10 and No. 11 (Fig. 3) under open-, sustained short-circuit and synchronous motor operation.

No of oscillograms / No. of test	$E_{A.T.}$ $E_{T.T.}$ $E_{T.W.}$	No load, s.s.c. or synch. mot. / field amperes	Line current in amperes	Approximate power factor leading or lagging	β (in degrees, See Figs. 10, 12, etc.)	α (in degrees, See Figs. 10, 12, etc.)	Consecutive crest values in max. volts per turn: Positive crests 1st	Positive crests 2nd	Negative crests 1st	Negative crests 2nd
1	2	3	4	5	6	7	8	9	10	11
O A 25/95	$E_{A.T.}$	N. L./5	0		118°	61°	1.47	1.39	1.49	1.27
O A 61/172	$E_{T.T.}$	N. L./4.95	0		120.5°	59.5°	0.508	0.422	0.466	0.416
O A 20/90	$E_{A.T.}$	S. M./4.49	15.5	0.95 lg.	117°	59.9°	1.18	1.24	1.02	1.23
O A 27/97	$E_{T.T.}$	S. M./4.24	15.5	1.0	116.5°	62.3°	0.33	0.33	0.38	0.34
O A 27/97	$T_{T.W.}$	S. M./4.24	15.5	1.0	119°	59.4°	0.15	0.16	0.15	0.15 +
O A 23/93	$E_{A.T.}$	S. M./3	18.5	0.81 lg.	113°	65.1°	1.19	1.07	1.05	1.19
O A 28/98	$E_{T.T.}$	S. M./2.9	39.5	0.56 lg.	119°	62.2°	0.24	0.28	0.24	0.29
O A 28/98	$E_{T.W.}$	S. M./2.9	39.5	0.56 lg.	115°	60.9°	0.12	0.14	0.12 +	0.14 +
O A 26/96	$E_{A.T.}$	S. M./	40.7	0.4 ld.	124.8°	58.5°	1.41		1.44	1.47
O A 29/99	$E_{T.T.}$	S. M./5.4	39.5	0.45 ld.	118.4°	62.15°	0.47	0.48	0.48 +	0.48 −
O A 29/99	$E_{T.W.}$	S. M./6.4	39.5	0.45 ld.	119.5°	61.7°	0.19	0.196	0.194	0.194
O A 37/109	$E_{A.T.}$	S. M./8	71.2	0.24 ld.	121.5°	57.3°	1.47	1.47	1.51	1.51
O A 36/108	$E_{T.T.}$	S. M./−	85.5	0.25 lg.	117.5°	63.3°	0.71 +	0.94	0.71 +	1.0 −
O A 41/117	E_{TT2}	N. L./ −	0		117°	62.4°	0.017	0.014		
O A 41/117	E_{TT2}	S. M./ −	24.7	0.93 lg.	116°	61.4°	0.041	0.034		
O A 40/116	E_{TT2}	S. M./−	76	0.26 ld.	120°	61.5°	0.045	0.047	0.045	0.047
O A 39/115	E_{TT2}	S. M./−	79	0.27 lg.	111.5°	62.6°	0.022	0.033	0.022	0 033
		Field amperes	Kind of s.s.c.	Frequency						
O A 61/172	E_{TT}	4.95	Y series	60	118.4°	61.6°	0.23	0.187	0.25	0.187
O A 59/180	T_{TW}	5.0	Y series	60	119°	60.1°	0.18	0.12	0.16	0.12

NOTE: α and β are in terms of machine-frequency. Thus if one cycle at machine-frequency occupies m cms. on the film and α is n cms., then $\alpha = \frac{n}{m} \times 360$ deg.

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The peculiar shape of these curves may be explained as follows: As a concentrated coil like No. 9 or No. 10, moves past a (stationary) field pole, the amount of change in the enclosed flux will depend upon the tooth kinks of the B-curve, (*b*) Fig. 6. If the B-curve were flat topped, see (*a*) Fig. 6, there would be no change of flux through coils No. 9 or 10 while under the pole-arc. Thus the small ripples in the portion of the B-curve marked β, Fig. 10, are due partly to the bunching of the flux at the teeth and partly due to the flux variations (= pulsations in magnitude, and to-and-fro swing), as already explained. As the coil emerges from under the horn of, say, a north pole and begins to enter the

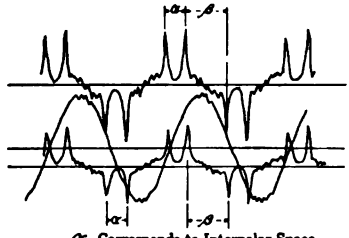

FIG. 10

Oscillogram No. O A 28. Test No. 98, Table II. First wave: E_{AT} = e.m.f. induced in coil No. 11 around tooth. Second wave: volts across the terminals of the machine. Third wave: E_{TW} = e.m.f. induced in coil No. 9 on top of wedge. Machine parallel Y-connected and operated as a synchronous motor. Line current = 39.5 amperes. Line voltage = 212 volts. Power factor = 56 deg. lag. α corresponds to interpolar space. β corresponds to space under pole-arc.

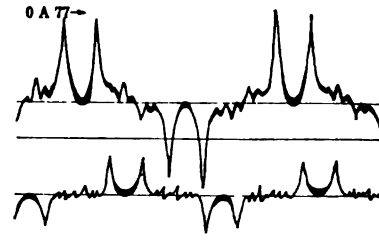

FIG. 11

Oscillogram No. O A 77. Test No. 189. First wave: E_{TT} = e.m.f. induced in search coil No. 10 at top of tooth. Second wave: E_{TW} = e.m.f. induced in exploring coil No. 9 at top of wedge. Machine run at open-circuit and same vibrator and same resistance in vibrator circuit for both waves.

interpolar region and then the tip of the south pole, there will be, two, comparatively large changes in the flux enclosed; these flux changes will give rise to two peaks whose angular spacing α, as seen from Table II is about 60 elec. deg. *i.e.*, α corresponds to the angular distance between two consecutive pole-horns. Evidently these crests should be on the same side of the zero line since one is due to a decreasing flux issuing from a north pole, while the other is due to an increasing flux of opposite polarity.[9]

9. See curve 32 given by W. J. Foster, TRANS., A. I. E. E., 1913, Vol. 32, p. 754.

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At first sight it might be expected that the e.m.fs. induced in coils No. 9 and 10 should be similar in shape and equal in amount since the only difference between them is that one is placed on top of a tooth, while the other is on top of a wedge. However, as seen from Table II this is by no means the case; further as

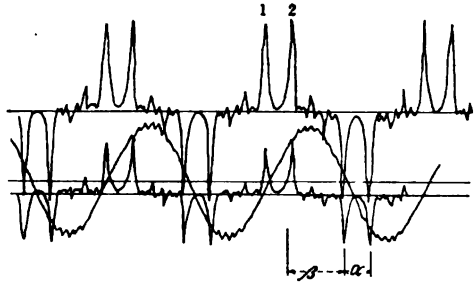

FIG. 12

Oscillogram No. O A 29. Test No. 99. Table II. Machine Y parallel connected and run as a synchronous motor. Field excitation, 6.4 amperes. Line voltage = 237 volts Line current = 39.5. Approximate power factor = 0.45 leading. First wave: E_{TT} = e.m.f. induced in coil No. 10 at top of tooth. Second wave: Line voltage. Third wave: E_{TW} = e.m.f. induced in coil No. 9 at top of wedge.

seen from Table III the area enclosed by E_{TT} for the coil at the top of a tooth, is very much larger than E_{TW} for the coil at the top of a wedge. This can be explained on the assumption that a given peak is due to the change of the enclosed flux to nearly zero, from the value it had just before the coil left the

TABLE III.

Data relating to synchronous motor operation of machine and giving the area per one-half cycle per turn enclosed by the e.m.f. waves induced in coils No. 9 and No. 10.

No. of oscill.	Field amps.	Line amps.	Line volts	Power factor	Maxwells or volts per turn per ½ cycle
O A 29	6.4	39.5	237	0.45 ld.	45,000, coil No. 10
O A 29	6.4	39.5	237	0.45 ld.	12,036, coil No. 9
O A 27	4.24	15.5	218	1.00	59,400, coil No. 10
O A 27	4.24	15.5	218	1.00	29,800, coil No. 9

horn of the pole. This change will be much larger for a tooth than for a slot. The above, in connection with Tables II and III, give a fairly good idea of the flux distribution and flux pulsations under actual running conditions, in contrast to the phenomena which obtain with the machine stationary and which are easily investigated, as indicated in Fig. 9.

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Study of Table II and the oscillograms given in connection therewith reveals several interesting facts. A comparison of Figs. 10 and 12 will show that the kinks between a + crest and a − crest, *i.e.*, under the pole-arc, are larger when the armature current is leading or demagnetizing than when it is lagging or magnetizing. Both E_{TT} and E_{TW} are larger for a given armature current when it is leading than when it is lagging; for example at 39.5 amperes of line current (under synchronous motor operation) $E_{TT} = 0.24$ and $E_{TW} = 0.12$ volts per turn. At 39.5 amperes of condensive current the volts per turn increase to 0.47 and 0.19 respectively. The field excitation in the former case was 2.9 amperes and the terminal voltage 212 (effective volts per phase); the corresponding quantities for the latter case being 6.4 amperes and 237 volts. In the light of the above, (see Table II) the explanation seems to be that in spite of the

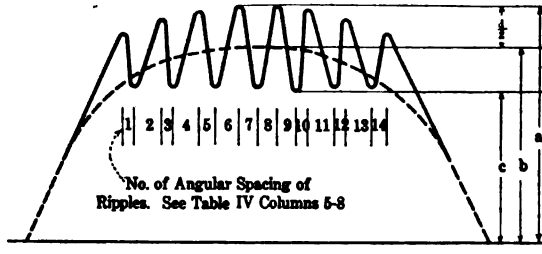

FIG. 13

Exaggerated figure showing e.m.f. induced in exploring coils at no-load and unequal spacing of tooth ripples.

demagnetizing action of the leading currents the flux density in the gap increases on account of the higher excitation and voltage.

Under s.s.c. the crest voltages E_{TT} and E_{TW} do not differ as widely which is no doubt due to the very peculiar and distorted flux distribution as will be explained in the next section.

In connection with the subject of tooth ripples, attention may be called to several other peculiarities. As seen from Table IV and indicated in Fig. 13, the angular spacing of the tooth ripples from crest to crest is not uniform as ordinary theory would indicate. (See equations **13** and **14**). In the first place, the angular spacing (1 + 2), (3 + 4), etc. Fig. 13, is slightly less than π/q, or in this case 15 electrical deg. Secondly the angular spaces 2, 4, 6, 11, and 13 are larger than 1, 3, 5, etc.; this is probably due to the hysteresis effect in the teeth. Thirdly, the amplitude d, Fig. 13 is distinctly smaller for E_{1T} than for E_{5B}

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this is easily seen from Fig. 7 in which the waves were taken under the same operating conditions, using the same vibrator

TABLE IV.

Angular spacing in degrees (electrical) of the maximum and minimum crests of the tooth ripples of the no-load search coil e.m.fs.

No. of oscill.	Search coil	Field amperes	Mean angular spacing of (see Fig. 13)			
			1, 3, 5 & 7	2, 4 & 6	(1 + 2), (3 + 4), etc.	(2 + 3), (4 + 5), etc.
1	2	3	4	5	6	7
272	No. 1, top of slot	8.8	6	8.5		
O A 35	No 1, top of slot		7.15	7.35	14.62	13.29
O A 35	No.2, midd. of slot		6.46	7.81	13.9	14
O A 35	No. 5, bott. of slot		6.94	7.21	14	14.2
O A 38	No. 1, top of slot	5	6.64	7.45	14.18	14.4
O A 38	No.3, midd. of slot	5	6.64	7.08	13.77	14
O A 38	No. 5, bott. of slot	5	6.84	8.03		
260	No. 1, top of slot	5	6.41	9.13		
263	No. 3, midd. of slot	4	6.03	8.43		
268	No. 5, bott. of slot	5	6.37	8		
Mean:			6.55	7.89	14.09	13.18

and the same vibrator resistance. A quantitative idea of the variations may be gained from Table V.

In Table VI are given the areas enclosed per ½ cycle by E_{1T},

TABLE V.

No. of oscillograms	E.m.f. of search coil	$\frac{d}{2} \times \frac{1}{b} \times 100$ See Fig. 13	Field amperes	Frequency cycles per second
272	E_{1T}	7.5%	8.8 appr.	50
272	E_{5B}	8.3%	8.8	50
O A 38	E_{1T}	7.2%	5	60
O A 38	E_{3M}	7.9%	5	60
O A 38	E_{5B}	9.2%	5	60

E_{3M} and E_{5B}, and it is seen that for the bottom coil No. 5 the flux is 107.5 per cent of that enclosed by the top coil. Under synchronous motor operation the percentage is smaller, about

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103 per cent for leading, *i.e.* demagnetizing, and larger, about 113 per cent for lagging currents.

These phenomena may be explained on the basis of the following assumptions. Consider two surfaces, one passing through the top and the other through the bottom of the armature slots.

TABLE VI.

Mean areas of consecutive positive and negative half-waves of e.m.f. induced in full-pitch, 5-turn exploring coils at top, middle, and bottom of slots = E_{1T}, E_{3M} and E_{5B} respectively.

No. of oscill. test No.	E_{1T} E_{3M} or E_{5B}	Frequency in cycles per sec.	Synchr. motor, short-circuit or no load operation	Approx. power factor	Line current V	$(\phi_1-\phi_2)$ in mega-maxwells or volts per turn per cycle	$(\phi_1-\phi_2)$ in per cent	Field amps.
1	2	3	4	5	6	7 8	9	10
O A 32/104	E_{3M}	59.5	S. M.	0.42 ld.	45.2 ld.	2.31	100	
O A 32/104	E_{5B}	59.5	S. M.	0.42 ld	45.2 ld.	2.40	104	
O A 34/106	E_{1T}	59.5	S. M.	0.41 lg.	52.6 lg.	1.95	100	
O A 34/106	E_{3M}	59.5	S. M.	0.41 lg.	52.6 lg.	2.05	105	
O A 35/107	E_{1T}	59.5	S. M.	0.37 ld.	52.6 ld.	2.22	100	
O A 35/107	E_{3M}	59.5	S. M.	0.37 ld.	52.6 ld.	2.235	101.5	
O A 35/107	E_{5B}	59.5	S. M.	0.37 ld.	52.6 ld.	2.281	103	
O A 33/105	E_{1T}	59.5	S. M.	lag.	60 appro	1.93	100	
O A 35/105	E_{3M}	59.5	S. M.	lag.	60 appro	2.0	103.7	
O A 35/105	E_{5B}	59.5	S. M.	lag.	60 appro	2.18	113	
O A 38/110	E_{1T}	60	no load			2.316	100	5
O A 38/110	E_{3M}	60	no load			2.38	103.0	5
O A 44	E_{1T}	54	s.s.c. connection Y series			0.415	100	5
O A 72	E_{5B}	54	s.s.c. connection Y series			0.282	70.3	5
O A 51	E_{1T}	52	s.s.c. connection Y series			0.3095	100	3.16
O A 54	E_{3M}	52	s.s.c. connection Y series			0.245	79.2	3.16
O A 67	E_{5B}	52	s.s.c. connection Y series			0.0969	34.4	3.16
O A 81	E_{1T}	60	s.s.c. connection 2-phase			0.1348	100	4.96
O A 82	E_{5B}	60	s.s.c. connection 2-phase			0.1056	78.3	4.96

Assume that the flux density in the slots is smaller and in the teeth is larger and more nearly perpendicular[10] to the direction of rotation of the armature conductors, for the second surface than for the first. Further, if the armature were revolving, the

10. Compare equations (4) and (4a) in supplementary notes.

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TABLE VII.

Order of harmonics	Wave analysis No.: (I − 2) No. of oscillogram: 206 Field current, amperes: 6.5 Frequency, cycles per sec.: 60 Exploring coil: No. 1, top of slot			(I − 4) 201 6 60 No. 1, top of slot			(I − 5) 206 5 60 No. 1, top of slot		
	A, volts B, volts	C, volts (C/C_1) 100	$\frac{A}{C_1} \times 100$ $\frac{B}{C_1} \times 100$	A, volts B, volts	C, volts $\left(\frac{C}{C_1} \times 100\right)$	$\frac{A}{C_1} \times 100$ $\frac{B}{C_1} \times 100$	A, volts B, volts	C, volts $\left(\frac{C}{C_1} \times 100\right)$	$\frac{A}{C_1} \times 100$ $\frac{B}{C_1} \times 100$
1	28.82	28.9	99.7%	27.93	27.99	99.85%	25.3	25.3	100%
	− 0.62	100%	− 2.14%	1.39	100%	4.97%	0.17	100%	0.66%
3	− 0.73	1.08	− 2.53%	− 1.03	1.05	− 3.68%	− 0.88	0.95	− 3.44%
	0.805	3.77%	2.78%	− 0.21	3.75%	− 0.75%	0.35	3.75%	1.36%
5	− 2.22	2.24	− 7.65%	− 2.41	2.42	− 8.61%	− 2.18	2.19	− 8.63%
	0.33	7.76%	1.12%	− 0.24	8.65%	− 0.86%	− 0.14	8.65%	0.56%
7	− 2.28	2.28	− 7.89%	− 2.09	2.55	− 0.75%	− 2.02	2.05	− 7.9%
	0.12	7.89%	0.403%	− 1.47	0.91%	− 5.26%	− 0.33	8.09%	− 1.3%
9	− 0.81	0.94	− 2.79%	0.02	0.09	0.07%	2.68	2.7	10.64%
	− 0.48	3.25%	− 1.67%	− 0.09	0.03%	− 0.03%	− 0.00	10.65%	− 0.02%
11	0.35	0.39	1.22%	0.14	0.64	0.05%	0.62	0.64	2.42%
	− 0.17	1.35%	− 0.58%	0.63	2.29%	2.25%	0.15	2.5%	0.59%
13	1.15	1.16	3.98%	0.44	0.72	1.57%	0.34	0.38	1.31%
	− 0.15	4.02%	− 0.52%	0.58	2.57%	2.07%	0.20	1.5%	0.78%
15	0.30	0.56	1.04%				0.13	0.26	0.50%
	0.47	1.93%	1.63%				0.22	1.02%	0.89%
17	− 0.26	0.26	− 0.88%				0.40	0.52	1.60%
	− 0.03	0.9%	− 0.12%				0.32	2.04%	1.26%
23	1.14	1.17	3.95%	− 0.29	0.87	− 1.04%	0.79	1.12	3.14%
	− 0.27	4.05%	− 0.9%	0.81	3.1%	2.93%	0.78	4.43%	3.09%
25	− 0.87	0.88	− 3%	0.49	1.09	1.75%	− 0.65	0.77	− 2.57%
	0.14	3.04%	4.95%	− 0.97	3.89%	1.39%	− 0.42	3.04%	− 1.66%

NOTE: Wave analysis, I − etc. refers to open-circuit waves. First column gives the coefficients of the series: $A_1 \sin \theta + A_3 \sin 3\theta + \text{etc.} + B_1 \cos \theta + B_3 \cos 3\theta + \text{etc.}$ The second column gives the coefficients of the same wave when expressed as: $C_1 \sin \overline{\theta + \varphi_1} + C_3 \sin \overline{3\theta + 3\varphi_3} + \text{etc.}$ A, B and C are also given in per cent of C_1 in second and third columns.

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TABLE VIII.

Wave analysis No.: (I − 10) No. of oscillogram: 206 Field amperes: 2.75 Frequency: 60 Exploring coil, No. 1, top of slot			(I − 13) 263 4 60 No. 3, middle of slot		(I − 15) 263 3 60 No. 3, middle of slot		(I − 21) 201 6 60 No. 5, bottom of slot		(I − 25) 228 3 60 No. 5, bottom of slot	
Order of harmonics	A, volts B, volts	C, volts (C/C_1) 100	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$	A, volts B, volts	C, volts $\frac{C}{C_1} \times 100$
1	16.75	16.8	22.2	2.22	17.2	17.2	28.9	28.9	17.4	17.43
	0.97	100%	−0.15	100%	0.05	100%	0.72	100%	0.002	100%
3	−0.66	0.74	−1.0	1.09	−0.9	−0.97	−1.89	−1.89	−0.62	0.79
	0.35	4.4%	0.28	4.91%	0.36	5.64%	0.05	6.54%	0.5	4.55%
5	−1.58	1.78	−2.4	2.41	−2.12	2.12	−3.66	3.75	−0.66	0.83
	−0.8	10.59%	−0.11	10.85%	−0.14	1.23%	−0.8	12.96%	−0.5	4.76%
7	−0.89	1.25	−1.08	1.12	−1.27	1.29	−1.67	1.67	−1.93	2.83
	−0.88	7.44%	−0.28	5.04%	−0.26	7.50%	−0.35	5.78%	−2.07	16.23%
9	0.05	0.09	1.01	1.01	0.06	0.15	0.39	0.68	−1.31	1.43
	−0.08	0.56%	0.03	4.55%	−0.13	8.55%	−0.56	2.35%	−0.58	8.20%
11	0.11	0.58	0.96	0.80	−0.68	0.68	0.92	1.08	0.14	1.65
	0.57	3.45%	0.26	3.60%	0.097	3.95%	0.57	3.74%	1.64	9.47%
13	−0.1	0.48					0.35	0.5		
	0.47	2.85%					0.36	1.73%		
15	0.75	0.17					−0.24	0.26		
	−0.16	1.01%					0.1	0.9%		
17	−0.44	1.13					−0.36	3.98		
	1.04	6.71%					−3.96	13.76%		
23	−0.34	−0.34					0.67	1.02		
	−0.00	2.02					1.77	3.53%		
25	0.47	0.47					−0.72	1.25	0.48	0.55
	0.067	2.8					−1.02	4.33%	0.26%	3.17%

NOTE: Wave analysis I − etc. refers to open-circuit waves. First column gives the coefficients of the series: $A_1 \sin \theta + A_3 \sin 3\theta$ etc. $+ B_1 \cos \theta + B_3 \cos 3\theta$ etc. Second column gives coefficients of the same wave when transformed into: $C_1 \sin \overline{\theta + \varphi_1} + C_3 \sin \overline{3\theta + \varphi_3}$ etc. Coefficients C in per cent of fundamentals C_1 are also given in column (2).

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peripheral velocity would have been larger for the second surface. These assumptions will partly at least explain the above and they seem reasonable as may be seen by examining the approximate flux distribution[11] in slotted armatures under different load conditions.

Tables VII and VIII give wave analyses of the e.m.fs. induced at no-load in different exploring coils. It will be seen that for a given field current, for example 6 amperes, wave analysis

TABLE IX.

Showing increase of 3rd harmonic of e.m.f. induced in stator exploring coils, with increasing excitation.

Excitation in amps. = I_f	Third harmonic e.m.fs. induced in stator full pitch exploring coils			Per cent values of columns				No. of oscillogram from which columns were calculated		
								No. 2	No. 3	No. 4
	No. 1 E_{1T}	No. 3 E_{3M}	No. 5 E_{5B}	No. 1 I_f	No. 2 E_{1T}	No. 3 E_{3M}	No. 4 E_{5B}			
1	2	3	4	5	6	7	8	9	10	11
No-load operation										
8.8	2.21		3.94	320	298		532	272		272
6.5	1.08			236	146			206		
6.0	1.05		1.89	218	142		255	201		201
5.0	0.95	1.67	1.28	182	128	225	173	206	261	266
4.0		1.09		145.5		147			263	
3.0	0.75	0.97	0.79	109	101	131	107	228	263	228
2.75	0.74			100	100			206		
Y series sustained short circuit										
5	3.99		7.52	182	532		1000	202		286
4			5.12	145.5			684			286
3	2.46		4.04	109	328		538	229		286

NOTE: The sustained s.c. percentages are in terms of 0.75 volts, induced in top search coil No. 1 at no-load and 3 amperes excitation.

$I - 4$, the amplitudes in maximum volts for the 1st, 3rd and 5th harmonics are respectively, 27.99, 1.05 and 2.42 for coil No. 1. The corresponding values from wave analysis $I - 21$, Table VIII, for coil No. 5 at the bottom of the slot, are: 28.9, 1.89

11. See for example, Figs. 110 and 111 p. 282 and p. 283, Principles of Electrical Design, by A. Still. *Jour., I. E. E.*, Magnetic flux distribution in toothed core armatures. H. S. Hele-Shaw, A. Hay and P. H. Powell. Figs. 183, 223 etc. pp. 165 and 220, in S. P. Thompson's Polyphase Electric Currents, Second Edition.

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and 3.75. As the third harmonic is of special interest in three-phase work Table IX has also been prepared; inspection of this table shows that taking the e.m.f. induced in coil No. 1, at the top of the slot, at 2.75 amperes of field excitation, as 100 per cent, the third harmonic of e.m.f. increases for coil No. 1 with increasing excitation until it reaches 298 per cent at 8.8 amperes of field current. Under s.s.c. we have a similar condition; the third harmonic for coil No. 5 reaches 1000 per cent while the corresponding value for coil No. 2 is 532 per cent at 5 amperes.[12]

In Fig. 14 the e.m.fs. E_{CTT} and E_{CTS} induced in a single

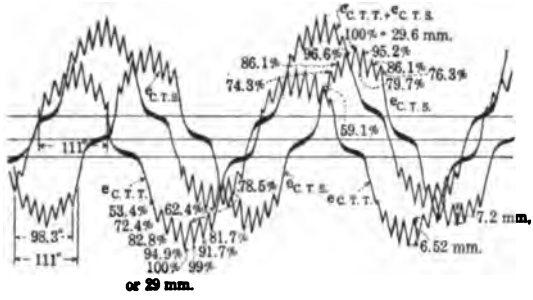

FIG. 14

Oscillogram No. O A 75. Test No. 187. Open-circuit test at 5 amperes field excitation and normal speed. First wave: (e.m.f. induced in single conductor No. 14 at top of tooth) + (e.m.f. induced in single conductor No. 13 at top of immediately succeeding wedge) = E_{CCT} + E_{CTS}. Second wave: E_{CTS}. Third wave: E_{CTT}. Same vibrator used for all exposures. 30 ohms resistance in vibrator for E_{CTT} + E_{CTS}; 10 ohms for other two waves.

conductor placed on top of a tooth and on top of a slot respectively (see coils No. 13 and 14, Fig. 3) show certain peculiar differences in that E_{CTT} has nine distinct kinks while E_{CTS} has eight. This is easily explained by following step by step the variational e.m.f. induced in a conductor on top of a tooth and one on top of a wedge, as the armature slots move with respect to the field poles.

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12. For a tentative explanation of the fact observed, namely that the third harmonic of e.m.f. for a given exploring coil will increase with excitation see, P. Janet, *Lecons d' électrotechnique générale*, T. II, p. 191. However, note further that not only the 3rd harmonic increases for a given coil with increase of field current but it seems to increase faster for a search coil at the bottom of slot than one at the top of a slot.

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IV. Experimental Investigation of the Direct Component of Armature Reaction and the Resultant Flux Distribution Under Sustained Short-Circuit Conditions

The armature current under s.s.c. has been found to be very closely sinusoidal, as can be seen by inspecting first two waves in Fig. 15 and fifth wave in Fig. 16, which are for Y and delta

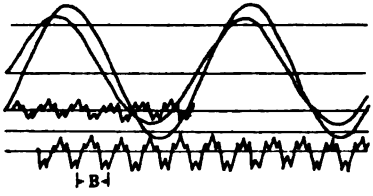

Fig. 15

Oscillogram No. 281. Table X. Sustained short circuit with machine series delta connected. Second straight line; zero line for first two waves representing armature phase current at 3.27 and 4.1 amperes field excitation respectively. Third and fourth waves give e.m.f. induced in 29-turn full-pitch rotor exploring coil No. 7, under same conditions.

connections. A quantitative idea of the approach of these waves to a sine wave may be gained from Table X.

Throughout this paper the armature current under s.s.c. *will be assumed* to be sinusoidal and if the armature winding were

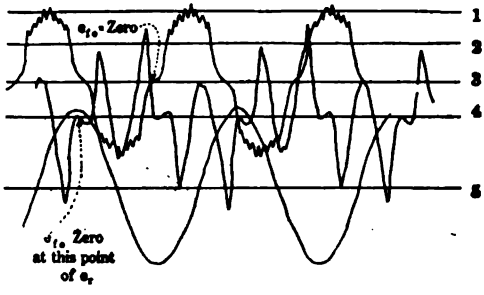

Fig. 16

Oscillogram No. O A 44. Test No. 148, Table XV. *Exposure No.* 1. Open-circuit test at 5 amperes field excitation and 54 cycles per second. First wave: contactor. Third wave: E_{1T} = e.m.f. induced in coil No. 1. *Exposure No.* 2. Sustained short-circuit test with machine series Y connected. Same excitation and speed as before. Fourth wave: E_{1T}. Fifth wave: line current. Second wave: contactor.

sinusoidally distributed, see Fig. 17, the armature m.m.f. would be sinusoidal. However, it can be shown (see supplementary notes, equation 30) than an ordinary armature winding with N conductors per slot and one slot per pole per phase, see Fig.

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18, supplied with a sinusoidal current will be exactly equivalent to: (1) a winding sinusoidally distributed (in space) having a pitch of τ cm. or inches. (2) a winding sinusoidally distributed having a pitch of $\tau/3$ and ⅓ as many turns as the first, etc., etc. In other words, imagine a set of sinusoidally distributed windings, of pitch τ/k, where k is the order of the space harmonic,

TABLE X.

Per cent amplitude of armature current, i_a, at every 30 deg., under s.s.c. and the machine Y-series and delta-series connected.

θ in degrees	0	30	60	90	120	150	180
sin $\theta \times 100$	0	50	86.6	100	86.6	50	0
i_a, Y conn. (oscill. O A 44)	0	51.1	84.5	100	85.4	50	0
i_a, Δ conn. (oscill.281)	0	48.7	81.2	100	85.1	48.7	0

all placed in series and carrying the same sinusoidal current (with respect to time): These will produce the same H-curve as the actual non-sinusoidal winding carrying the sine-wave current of fundamental (with respect to time) frequency.

Without going further into the analysis of this, suffice to

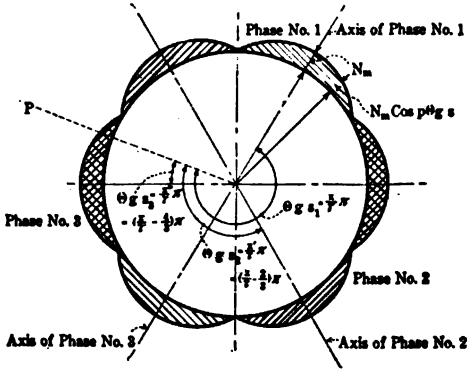

FIG. 17

Schematic representation of three-phase two-pole armature winding which is sinusoidally distributed (in space).

state[13] that for a polyphase alternator, with a sine wave (with respect to time) armature current and any ordinary type (non-

13. See A. Potier, *Journal de Physique*, 1897, 3me serie, T. VI, p.341 and 483. A. Blondel, *L' Eclairage Electrique*, 1895, T. IV, pp. 241, 308 et 358. T. V., p. 268. See also P. Janet, *Lecons d' Electrotechnique générale*, T. III, Chp. V. Alex. Russell, Altern. Currents Vol. II, Chp. XIV.

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sinusoidally distributed, in space) winding, the resultant m.m.f. curve can be considered as consisting of a series of H-curves, all of fundamental frequency (in time) but having a pole pitch τ/k and an amplitude equal to:

$$H_e = \frac{N_{ph}}{2} \cdot \frac{H_m}{k} S \cdot D_f \cdot S_{p.f.} \tag{31}$$

where N_{ph} = number of phases.

H_m = $(4/\pi)\ Y_m$. See Fig. 18.

S = slots per pole per phase.

D_f = distribution factor = $\dfrac{\sin \frac{1}{2} S \frac{\lambda}{\tau} \pi}{S \sin \frac{1}{2} \cdot \frac{\lambda}{\tau} \pi}$

λ = slot pitch.

$S_{p.f.}$ = short-pitch factor = $\cos \epsilon\, k\, \pi$, where $(1 - \epsilon)\tau$ = winding pitch.

For a three-phase alternator the first, seventh, thirteenth etc., or $(6\,n + 1)\ t\,h$ space harmonics, ($n = 0, 1, 2, 3$ etc.), will rotate in the same direction at a speed of $2\,\pi f/k$ or at an angular velocity $\omega/p\,k$. The $(6\,n - 1)\ t\,h$ harmonics will rotate in the opposite direction with the same velocity. In mathematical short-hand the above may be expressed as follows:

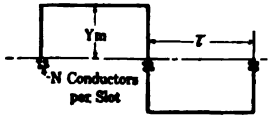

FIG. 18

$$H_1 = H_e \cos\left(\omega\, t - \frac{x}{\tau}\, \pi\right) \tag{32}$$

$$H_3 = 0 \tag{33}$$

$$H_5 = H_e \cos\left(\omega\, t + \frac{5\,x}{\tau}\, \pi\right) \tag{34}$$

$$H_7 = H_e \cos\left(\omega\, t - \frac{7\,x}{\tau}\, \pi\right), \text{ etc.} \tag{35}$$

where H_e is given by (31).

In case of two-phase machines the first, fifth etc., or $(4\,n + 1)\ t\,h$ space harmonics will travel in one direction while the third, seventh etc. or $(4\,n - 1)\ t\,h$ space harmonics will travel in the opposite direction. The expressions for these will be similar to the ones given above and their amplitude will be given by (31)

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The m.m.f. per centimeter at any point x of the stator periphery will be $(4\,\pi.\ 10^{-1}\,H)$ where,

$H = H_1 + H_5 + H_7$ etc.

= sum of series of sinusoidal H-waves, all of fundamental frequency and having phase angles, $(-\,6\,x_1\,\pi/\tau)$, $(+\,6\,x_1\,\pi/\tau)$, $(-\,12\,x_1\,\pi/\tau)$, etc., with respect to the fundamental whose phase angle $(-\,x_1\,\pi/\tau)$ is assumed to be zero. See Notes B and C.

This leads to the following important proposition: With polyphase balanced alternating currents varying according to a sine law (in time) and flowing through ordinary non-sinusoidally distributed windings, (in space) the H-curve will be sinusoidal (*i.e.*, will be the sum of a series of sine waves). Thus, under load

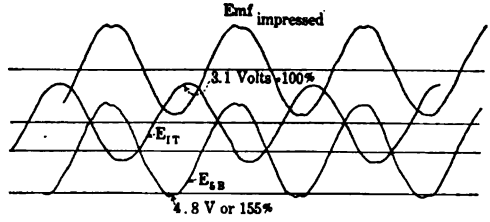

FIG. 19

Oscillogram No. O A 13. Test No. 32. Rotor removed and three-phase 85.8 amperes (158 per cent normal current) introduced into the stator from outside source. First wave: e.m.f. impressed on stator. Second wave: E_{1T} = induced in full pitch search coil No. 1 Third wave: E_{5B} = e.m.f. induced in full pitch exploring coil No. 5. Second and fou.th lines: calibrations for E_{1T} and E_{5B} respectively. Note E_{1T} and E_{5B} are due to the rotating field set up by three-phase currents.

or s.s.c., in case of induction motors or alternators, the H-curve due to sinusoidal (in time) armature currents will be sinusoidal. Consequently if the permeability be constant, and uniform across the pole-face, the B-curve due to armature reaction will also be sinusoidal. However, if we find the e.m.fs. induced in exploring coils due to armature reaction to be distorted this must be charged to the permeability μ, of the circuit. The importance of this result will be appreciated when we attempt to explain certain phenomena to be considered presently.

Fig. 19 gives E_{1T} and E_{5B} when the stator was supplied with 3-phase currents from an outside source and the rotor removed. It is seen that the impressed e.m.f. does not deviate greatly from a sine wave. But the e.m.f. E_{5B}, induced in the coil at the bottom of the slot is peaked and over 150 per cent of E_{1T} = e.m.f. induced in coil on top of slot. The waves given in Fig. 20 were taken with the rotor in place and running without

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excitation at a speed corresponding exactly to that of the gliding field produced by the three-phase stator currents. In this case the current supplied was only 28 per cent of full-load current. As in Fig. 19, E_{5B} is more peaked than E_{1T}; $E_{5B} = 1.17\ E_{1T}$.

Assuming the m.m.f. per centimeter to be the same at the top and bottom of the slot, it is clear that the inequality between E_{1T} and E_{5B} must be charged, at least partly, to the difference in permeance between the points considered. Generalizing the above it can be stated that although the H-curve due to armature currents may be sinusoidal the B-curve will differ from the former for two reasons: First, the lack of constancy and uniformity in μ across the pole face; further the B-curve at the bottom of the slot may differ from that at the top owing to the difference in the permeance per unit area at these two points.

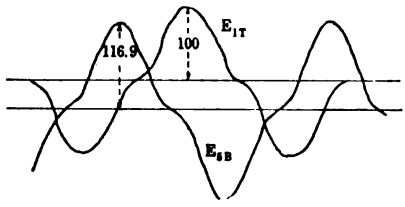

FIG. 20

Oscillogram O A 66. Test 177. First wave: E_{1T} = e.m.f. induced in full-pitch search coil at top of slot. Second wave: E_{5B} = e.m.f. induced in full-pitch search coil at bottom of slot. A-c. supplied from outside source = 15.1 amperes. Two exposures taken under identical conditions. Rotor not excited and run at synchronous speed. Same vibrator used for both exposures.

Second, according to the degree of saturation the B-curve will ' e more or less flat topped; this of course is well known especially in connection with induction motor design.

It is possible for the B-curve due to armature reaction alone, to undergo certain oscillations, similar to those described in Section III. However, a little study of Table I, columns 8 and 9 and of Figs. 19, 20 and of others to be given later will show that these secondary effects may safely be neglected.

Summarizing then we can state that the experimental and theoretical studies given herein and by others show that the no-load flux may and does deviate from a sine wave to the extent of containing the 3rd, 5th and 7th harmonics to the amount of 10 per cent or thereabouts. The field-form of most modern machines, at no load or full load approaches reasonably well the general shape of a sine wave[14]

14. W. J. Foster, TRANS., A. I. E. E., 1913, Vol. 32, p. 749. L. A. Herdt, TRANS., A. I. E. E., 1902, Vol. 19, p. 1093.

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and is symmetrical with respect to the $\pi/2$ or the (geometrical) pole-axis except in case of loads having considerable cross-magnetizing effect.

As explained before the H-curve due to armature reaction will be sinusoidal while the B-curve will show no *extreme* deviations from a sine wave. This, rather theoretical result, is

TABLE XI

Wave analysis No.:	*II* − 1		*II* − 2		*II* − 3		*II* − 4	
No. of oscillogram:	202		229		286		286	
Field excit. amperes:	4.95		3		5.02		3	
Expl. coil, No. 1, top of slot			No. 1, top of slot		No. 5, bottom of slot		No. 5, bottom of slot	
	A, volts	*C*, volts	*A*, volts	*C*, volts	*A*, volts	*C*, volts	*A*, volts	*C*, volts
Order of harmonics	*B*, volts	$\frac{C}{C_1} \times 100$	*B*, volts	$\left(\frac{C}{C_1}\right) 100$	*B*, volts	$\left(\frac{C}{C_1}\right) 100$	*B*, volts	$\left(\frac{C}{C_1}\right) 100$
1	1.24	1.24	1.16	1.16	− 1.05	1.39	0.85	0.86
	− 0.004	100%	− 0.03	100%	− 0.92	100%	0.13	100%
3	1.96	3.99	0.98	2.46	− 3.64	7.42	1.97	4.24
	3.32	323%	2.26	213%	− 6.47	532%	3.74	490.5%
5	1.06	2.14	0.17	1.77	1.77	2.24	− 1.15	1.49
	− 1.86	172.5%	− 1.16	152.5%	− 1.37	160.5%	− 0.96	173%
7	0.91	1.45	0.67	1.1	− 1.05	2.7	1.25	2.04
	− 1.12	116.8%	− 0.78	95%	− 2.47	193%	− 1.61	236%
9	0.45	0.45	0.03	0.03	− 1.18	1.38	1.51	1.75
	0.027	36.5%	− 0.001	2.8%	0.72	98.9%	0.87	203%
11	− 0.31	0.32	− 0.24	0.03	− 0.11	0.11	− 0.18	0.38
	− 0.097	25.86%	− 0.01	2.24%	− 0.02	81%	− 0.34	44.4%
13	− 0.193	0.39	− 0.01	0.27	− 0.88	1.29	− 0.33	0.71
	− 0.34	31.7%	− 0.02	2.29%	0.95	92%	0.63	82.5%
15	0.128	0.15	0.004	0.004				
	0.086	12.5%	0.003	0.4%				
17	− 0.056	0.12						
	0.10	9.5%						
23	0.005	0.12						
	0.12	9.5%						
25	0.004	0.006						
	0.004	0.5%						

NOTE: Wave analysis, II-etc. refers to analysis of waves obtained under s.c.c. with the machine series Y connected. See also note, Table VIII, as to meaning of *A*, *B*, *C*.

further corroborated by the oscillograms given in Figs. 19 and 20. According to these, either with the rotor removed or in place and running at synchronous speed, the e.m.fs. induced in stator search coils, due to stator currents alone, are fairly smooth. Furthermore the flux pulsations as indicated by the rotor coil e.m.fs., E_{RF}, and E_{RC} (under s.s.c.) were found to be extremely small.

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The resultant of two or more sine waves is, of course, a sine wave, and therefore it might be expected, at first sight, that the resultant of the B-curve due to the field and that due to the armature reaction will not deviate greatly from a sine wave, since neither of them is very distorted to begin with. However, turning to oscillograms obtained under s.s.c. we find indications of an extremely distorted flux distribution. A glance at Figs. 23, (fourth wave) 16, (fourth wave) etc. and Table XI which gives the coefficients of the harmonics of the waves obtained under s.s.c., show that the 3rd harmonic of e.m.f. induced in the stator search coils is several hundred per cent of the fundamental and that a number of other harmonics are quite comparable to it.

In order to avoid any false deductions and detect any peculiar defects in the machine under test to which the distorted s.s.c. waves might be due it was considered advisable to investigate

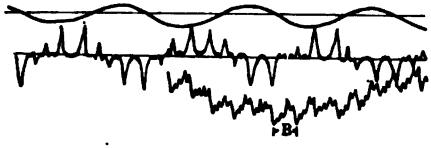

FIG. 21

Oscillogram No. O A 40. Test No. 116. Tables I and II. First wave: line current Second wave: E_{TT_2} = e.m.f. induced in coil No. 12. Third wave: E_{RF} = e.m.f induced in full-pitch rotor coil No. 7. Synchronous motor operation. Line current = 76 amperes lead. Line voltage = 241 volts. Power factor = 0.28 ld.

briefly the B-curves obtained under different loads. For this purpose the machine was run as a synchronous motor and a number of oscillograms taken, a few of which have been included in the paper.

As seen from Figs. 21, 22 etc., the waves E_{1T}, E_{3M}, E_{RF}, E_{RC} etc. are not unlike those obtained at no-load with small variations which might have been expected. It will be observed by comparing Fig. 22 and Fig. 7 that the tooth ripples are smaller under synchronous motor operation. One of the important results of this series of tests was that even with heavy magnetizing or demagnetizing armature reactions, (no field excitation, or 150 per cent or more excitation) the flux distribution did not in any way approach the B-curves obtained under s.s.c.

Other points of interest have already been considered in the previous section and as the subject is not new it will not be

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considered any further. However, attention may be called to the e.m.f. induced in the rotor search coils No. 7 and 8. The e.m.f. waves of these coils show $2\,q = 24$ small indentations per cycle which are due to the teeth. The e.m.f. of full-pitch coil No. 7, shows a large superimposed oscillation which recurs every B deg., see Fig. 21, 3rd wave.

As shown clearly in the supplementary notes this is due to the fifth and seventh space harmonics of armature reaction and as indicated in Table I column 5, $B = 60$ elec. deg., Fig. 21, *i.e.*, the periodicity of the superimposed wave is $360/60 = 6$ times machine-frequency, as we would expect. The long flat oscillations which repeat themselves every $A = 1080$ elec. deg., see Fig. 5, and Table I column 4, are due to eccentric rotor, as explained before.

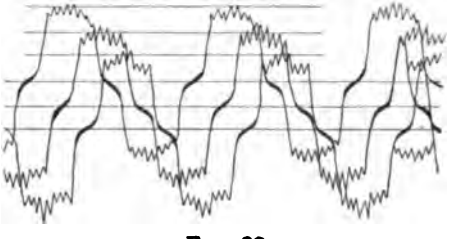

FIG. 22

Oscillogram No. O A 35. Test No. 106. Table VI. Machine Y parallel connected and run as synchronous motor. First wave: E_{1T} = e.m.f. induced in exploring coil No. 1 at top of slot. Second wave: E_{3M} = e.m.f. induced in exploring coil No. 3 at middle of slot. Third wave: E_{5B} = e.m.f. induced in exploring coil No. 5 at bottom of slot. Line current = 52.6 lead. Power factor = 0.37 approx. Compare to Fig. 7.

To determine the armature reaction under s.s.c., *experimentally*, in terms of the voltages induced in the stator coils the writer resorted to the following method. A contactor[15] having a fixed space relation with respect to the poles was placed on the machine. This closed a direct-current circuit of very little inductance so that the rise of current was quite abrupt and the instant of make well defined. See Fig. 23. An exposure was first made giving the contactor wave and the open circuit e.m.f. induced in one of the search coils. The zero lines of both waves were then moved, in the same direction, and a second exposure was made giving the contactor and search coil e.m.f. under the same operating conditions except that the machine was short-circuited at its terminals.

15. The contactor used is similar to the one described in TRANS., A. I. E. E., Vol. 34, p. 2254, except for slight changes in the design of the brush rigging.

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Obviously it is thus possible to determine accurately the point on the s.s.c. wave corresponding to any point on the open-circuit wave. If we designate by e_{fo} the e.m.f. induced in one of the stator coils No. 1, 3, or 5 at no-load and given field current

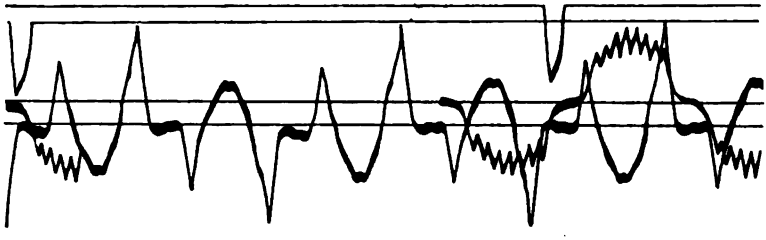

FIG. 23

Oscillogram No. O A 72. Test No. 184. Table XV. *Exposure No.* 1. Open-circuit run at 5 amperes field current and 54 cycles per second. First wave: contactor. Third wave: E_{5B} = e.m.f. induced in exploring coil No. 5, at bottom of slot. *Exposure No.* 2 Sustained short circuit; series Y connection and under same conditions. Second wave Contactor. Fourth wave: E_{5B}.

and speed, and by e_r the e.m.f. induced in the same coil under the same conditions of speed and excitation but with the machine short-circuited, we can, as a fair approximation assume that e_{fo}

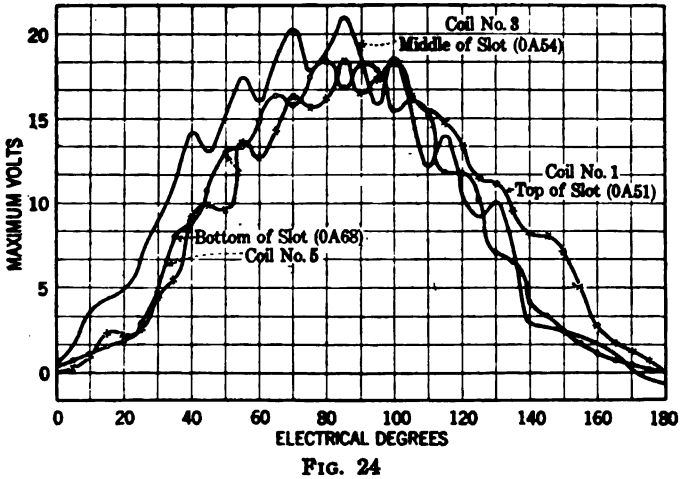

FIG. 24

E_a curves = difference between no-load and s.s.c. e.m.fs. induced in top, bottom and middle of slot full-pitch exploring coils, Nos. 1, 3 and 5 = $(E_{fo} - E_r)$—field current = 3.16 amperes — periodicity = 52 cycles per second — Y series connection.

plus a certain e.m.f. e_a, which we can charge to armature reaction, give us the resultant e.m.f. e_r.

Fig. 24 shows $e_a = (e_r - e_{fo})$ for the three full-pitch stator exploring coils taken at a field excitation of 3.16 amperes and a

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TABLE XII.

Wave analysis: VI – 1 No. of oscillogram: O A 44 Field excitation: 5 Frequency: 54 Exploring coil: No. 1, top of slot				(VI – 2) O A 51 3.16 52 No. 1, top of slot			(VI – 3) O A 68 3.1 52 No. 5, bottom of slot		
Order of harmonics	A, volts B, volts	(A/C_1) 100 (B/C_1) 100	C, volts (C/C_1) 100	A, volts B, volts	(A/C_1) 100 (B/C_1) 100	C, volts (C/C_1) 100	A, volts B, volts	(A/C_1) 100 (B/C_1) 100	C, volts (C/C_1) 100
1	– 23.02	– 99.8%	23.07	– 15.22	– 99.4%	15.31	14.75	100%	14.75
	1.16	5.03%	100%	1.64	10.73%	100%	0.2	1.35%	100%
3	4.87	21.0%	5.06	2.09	13.62%	2.14	0.85	5.76%	0.92
	– 1.38	– 6.01%	21.05%	– 0.65	– 4.27%	14%	0.35	2.37%	6.24%
5	– 0.87	– 3.77%	0.9	0.73			– 0.6	– 4.1%	0.62
	– 0.25	– 1.10%	3.93%	– 0.34			– 0.12	– 0.8%	4.18%
7	0.56	2.44%	0.68	– 0.08			0.22	1.47%	0.35
	0.38	1.67%	2.94%	– 0.52			0.27	1.8%	2.35%

NOTE: Wave analysis VI – etc. refers to C_a curves of armature reaction See Figs. 25 and 2_4. First column gives coefficients of series: $A_1 \sin \theta + A_3 \sin 3\theta$ etc. $+ B_3 \cos \theta + B \cos 3\theta$ etc. Second column gives A and B in per cent of C_1. Third column gives coefficients of series $C_1 \sin \overline{\theta + \varphi_1} + C_3 \sin \overline{3\theta + \varphi_3}$ etc. where $C^2 = A^2 + B^2$; also it gives C in per cent of C_1

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frequency of 52 cycles per second; 3.16 amperes being the field excitation necessary to send full-load current under s.s.c. Fig. 25 gives e_a for a frequency of 54 cycles per second and a field current of five amperes.

Table XII gives the wave analysis of e_a, Fig. 25 coil No. 1, and Fig. 24 coils No. 1 and 5. It is seen that the 3rd harmonic is 21 per cent for five amperes field excitation and drops to about 5 and 6 per cent at 3.1 amperes of excitation. A general idea of the shape of these e_a curves of armature reaction may be obtained from Fig. 24A which is an average "smoothed out" curve representing the waves given in Fig. 24 for coils No. 1 and 5.

The last two columns of Table XIII give the wave analysis of

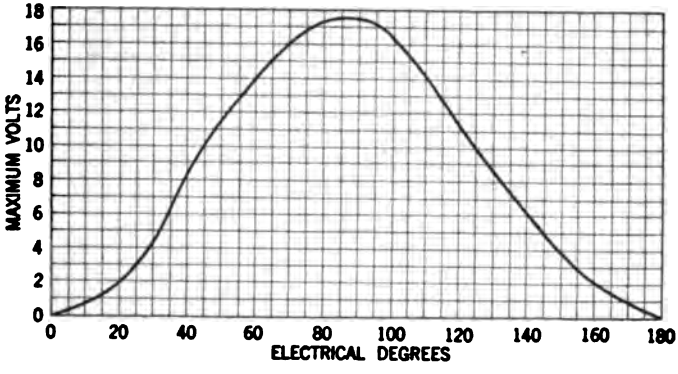

FIG. 24A

Average "Smoothed Out" curve representing e.m.f., E_S induced in coils No. 1 and No. 5 Fig. 24.

e_a as obtained by subtracting (1 − 5) Table IV, from (II − 1) Table XI. (I − 5) gives the wave analysis of the e.m.f. induced in coil No. 1 at no-load and 5 amperes of excitation, while (II − 1) gives the analysis of the corresponding wave of the same coil with the machine Y connected and short-circuited. Comparing (II − 1) − (I − 5) with the wave analysis of e_a, Table XII, it will be seen that the agreement is satisfactory.

The important result derived from the above is that the s.s.c. flux distribution is badly distorted primarily on account of the fact that the fundamental of the resultant flux is so greatly reduced that the higher harmonics become quite predominant. For example, from Table VII, wave analysis (I−5), the 3rd is seen to be 0.95 volts which is 3.75 per cent of the no-load fundamental, which is 25.3 volts; but it is (0.95/1.24) × 100 = 67 per cent

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TABLE XIII.

Wave analysis No. III – 2 No. of oscillogram: 225 Field excitation amperes: 5.05 Exploring coil, No. 1, top of slot			(III – 4) 227 3 No. 1, top of slot		(V – 4) O A 81 5 No. 1, top of slot		(V – 5) O A 82 5 No. 5, bottom of slot		(II – 1) – (I – 5) 5 No. 1, top of slot	
Order of harmonic	*A*, volts *B*, volts	*C*, volts (C/C_1) 100	*A*, volts *B*, volts	*C* volts (C/C_1) 100	*A*, volts *B*, volts	*C*, volts (C/C_1) 100	*A*, volts *B*, volts	*C*, volts (C/C_1) 100	*A*, volts *B*, volts	$(A/25.3)$ 100 $(B/25.3)$ 100
1	– 0.75	1.21	0.525	1.00	– 2.65	2.65	– 2.19	2.64	– 24.06	95.2
	0.96	100%	0.86	100%	0.05	100%	1.48	100%	– 0.18	0.71
3	1.74	3.54	1.17	2.13	– 3.77	3.78	– 4.9	4.96	2.84	– 11.21
	3.09	291.5%	1.78	213%	0.28	144%	0.82	188	2.97	– 11.74
5	– 1.4	1.76	– 0.94	1.0	1.93	1.99	1.6	2.15	3.24	– 12.8
	– 1.07	144.8%	0.38	100%	– 0.47	75%	– 1.44	81.5	– 1.72	6.8
7	0.97	0.99	0.38	0.64	1.22	1.375	1.29	1.9	2.93	– 11.58
	– 0.20	81.6%	0.51	63.7%	– 0.63	51.8	– 1.4	71.9	– 0.79	3.12
9	0.89	0.94	0.19	0.39					– 2.23	8.82
	– 0.28	77.1%	– 0.35	39.3%					0.03	
11	– 0.47	0.94	0.048	0.99					– 0.93	3.7
	0.82	77.4%	– 0.08	9.57%					– 0.25	0.99
13	– 0.25	0.40	0.15	0.32					– 0.53	2
	– 0.31	33.1%	– 0.28	0.32%					– 0.54	2.13
15	0.25	0.29							0	
	0.01	23.8%							– 0.31	1.23
17			0.07	0.25					– 0.46	1.82
			– 0.24	25.1%					– 0.22	0.87

NOTE: Wave analysis III etc. refers to analysis of waves obtained under s.a.c. with the machine series Y connected and neutral in. V etc. refers to waves obtained under s.a.c. with the machine connected two-phase. See Fig. 31. For II – 1 see Table XI; for I – 5 see Table VII. As to meaning of *A*, *B* and *C* see Table XII.

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of the s.s.c. fundamental. From wave analysis ($II - 1$) Table XI, the third harmonic under s.s.c. is 3.99 volts *i.e.*, (3.99/25.3) $\times$ 100 = 16 per cent of the no-load fundamental, but (3.94/1.24) $\times$ 100 = 323 per cent of the s.s.c. fundamental. The increase from 0.95 volts to 3.99, on the other hand, must be charged mainly to the B-curve of armature reaction which will be distorted, unlike the H-curve producing it, which will be sinusoidal

Turning to Table XII, consider wave analysis No. ($V\,I - 1$) which gives the coefficients for E_{rr} at five amperes excitation and 54 cycles per second. The fundamental is seen to be 23

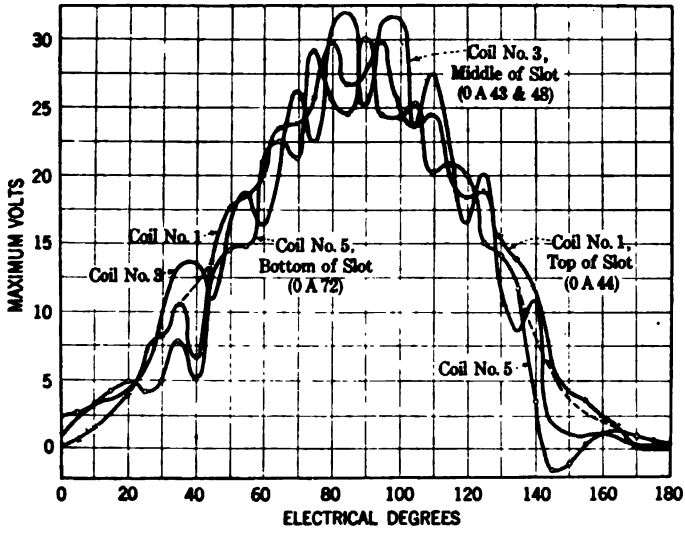

FIG. 25

E_s curves = $E_{fo} - E_r$ = difference between open-circuit and s.s.c. e.m.fs. induced in full-pitch exploring coils Nos. 1, 3 and 5 — field current = 5 amperes—periodicity = 54 cycles per second—Y series connection.

volts; the 3rd is 21 per cent of e_s but (60/54) (5.05/1.24) $\times$100 equals 450 per cent of the s.s.c. fundamental.

In general, then, if both the flux distribution at no-load and the flux wave due to armature reaction are distorted to the extent of containing 10 to 20 per cent 3rd and 5th harmonics, the resultant s.s.c. flux distortion will be badly distorted, because the above harmonics will amount to several hundred per cent of the fundamental, of the s.s.c. wave. Assume, for example, that e_{fo} and e_s consist of:

	e_{fo}	e_s
1st harmonic	100 per cent	− 95 per cent
3rd "	10 " "	10 " "
5th "	5 " "	5 " "

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These values are quite conservative as inspection of Tables VII, VIII, and XII will show, and neither of the waves can be said to be very distorted. The resultant s.s.c. flux distribution will be equal to $e_r = (e_{fo} + e_a)$ and the amplitudes of its 1st, 3rd, and 5th harmonics will be 5, 20 and 10 or 100, 400 and 200 per cent respectively, in terms of the s.s.c. fundamental. These

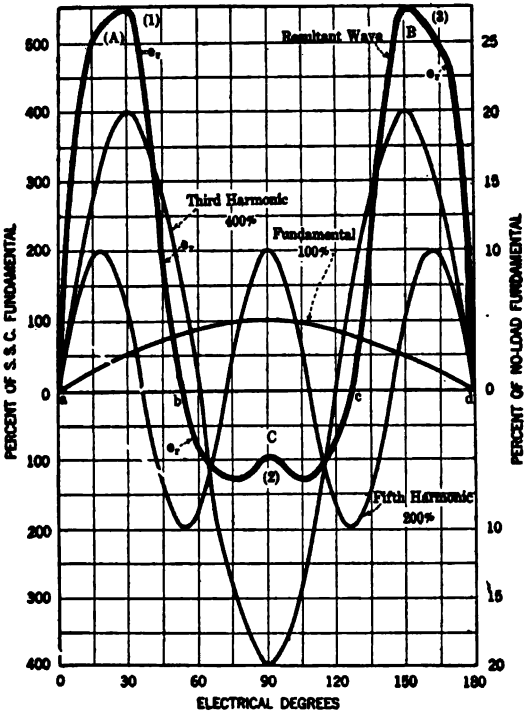

FIG. 26

Illustrating how the extremely distorted resultant wave forms are produced under sustained short circuit.

are drawn in Fig. 26 and their sum, e_r is seen to be extremely distorted and not unlike the waves obtained experimentally under s.s.c.

It is clear now why the flux distribution waves under synchronous motor operation deviate comparatively little from the no-load curve. Obviously at very low voltages such as are obtained under s.s.c., the harmonics of the field and armature reaction

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will assume a predominant rôle (unless they happen to be out of phase by ½ of their own period and thus neutralize each other).

In Fig. 27 the angular spacings between different crests are

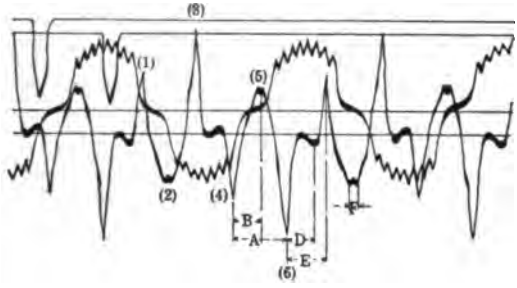

FIG. 27

Oscillogram No. O A 68. *Exposure No.* 1. First wave: contactor. Third wave e.m.f. induced in coil No. 5 at no load, 3.1 amperes field excitation, 52 cycles per second. *Exposure No.* 2: Machine series Y connected and short-circuited at same excitation and speed. Second wave: contactor. Fourth wave: E_{5B}. See Fig. 24.

designated, A, B, D, E and F and their values for different kinds of s.s.c. are given in Table XIV. From this it is seen that A is approximately 105 deg., or nearly equal to the pole-arc, as it would be expected—see Fig. 26; E, Fig. 27, is about 75 deg.

TABLE XIV.

Data relating to the e.m.fs. induced in the coils at the top and bottom of different slots under s.s.c. and the machine connected, series Y, series Δ and two-phase. Frequency = 60 cycles per second.

No. of oscill.	Field amps.	E_{1T} E_{5B}	Kinds of short-circuit	*A*	*B*	*D*	*E*	*F*
				in degrees see Figs. 27, 34, etc.				
202	4.95	E_{1T}	Y	105	51.3	40	75.5	19
O A 88	5	E_{1T}	Y	106.3	48	35.6	65.2	10
226	5	E_{1T}	Y neutral in	109	51	37	70	
O A 89	5	E_{5B}	Y	103.2	50.4	37.1	73.6	10
O A 88	5	E_{1T}	Δ	104.5	49.7	41.8	75.4	10
275	4.9	E_{5B}	Δ	106.2	50.8	39.2	73.5	
O A 89	5	E_{5B}	Δ	104.5	49.3	36.8	72.0	
O A 81	4.96	E_{1T}	two-phase	105	51.1	36.8	72.3	17.4
O A 86	4.96	E_{1T}	two-phase	105.3	47.8	48.2	76.1	15
O A 82	4.96	E_{5B}	two-phase	104.9	52.4	42.9	79.2	10.1

so that $(A + E) = 180$ deg. D is nearly $(1/2\,E)$ but B is less than $(1/2\,A)$; referring to the theoretical assumed curve e_r of Fig. 26, it will be easily seen that if the transverse armature reaction (due to the active or in-phase component of armature

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current with respect to the geometrical pole-axis) is not entirely negligible, the peaks (1) and (3) will be unequal, as it is the case in actual oscillograms Figs. 23 and 27, etc. A little consideration will show that the cross-magnetizing armature reaction

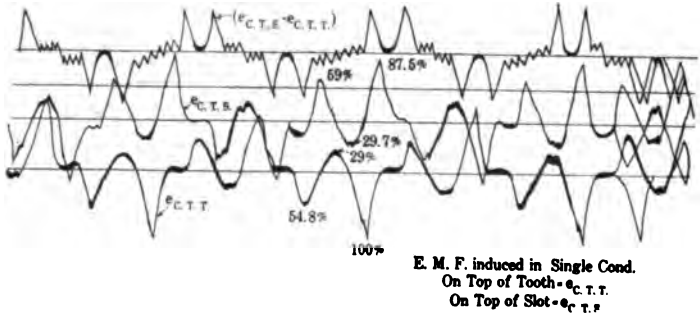

FIG. 27A

Oscillogram No. O A 76. Test No. 188. *Exposure No.* 1. Open circuit run at 5 amperes field excitation and normal speed. First wave = (E_{CTT} − E_{CTS}) = (e.m.f. induced in single conductor at center of tooth, No. 14, Fig. 3) − (e.m.f. induced in single conductor at top of succeeding wedge, No. 13, Fig. 3). This should be compared with Fig. 14. *Exposure No.* 2. s.s.c. run under same conditions. Second wave: E_{CTS}. Third wave: E_{CTT}. These two waves should be compared with Figs. 23, 27, 28a, etc., which give the e.m.fs. induced in full-pitch exploring coils.

will not only reduce, say crest (3), Fig. 26, but will shift crest (2) towards (3). Thus the angle B, between (2) and (3), Fig. 26, will be less than $(1/2\,A)$ = angle between crests (1) and (3), Fig. 27.

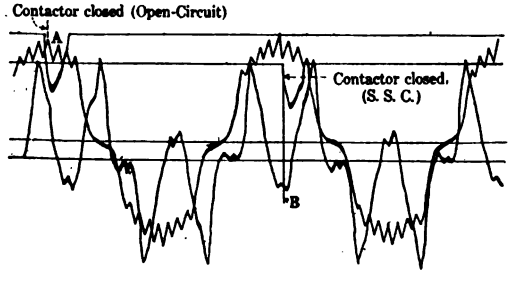

FIG. 28

Oscillogram No. O A 81. Test No. 193. Tables XIV and XV. First wave: Contactor Third wave: E_{1T} = e.m.f. of coil No. 1 at 4.96 amperes of field current, 60 cycles per second and no-load. Second wave: contactor. Fourth wave: E_{1T} under same conditions except machine short-circuited and connected two-phase.

In Fig. 27 the different crests are designated 1, 2, 3, etc. and their values in maximum volts for different kinds of s.s.c. are given in Table XV. Study of this table shows that for three-phase Y or delta s.s.c. the peaks on the same side of the zero line,

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i.e., (1), (3) or (4), (6) are quite unequal which might have been expected according to the preceding paragraph. Compare also Fig. 27A. In case of two-phase s.s.c., however, the corresponding crests (1), (3) or (4), (6) are very nearly equal. See Fig. 28.

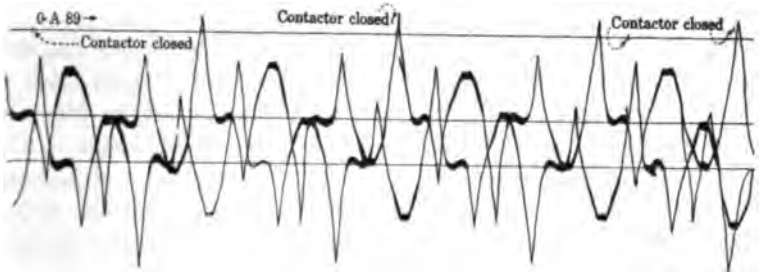
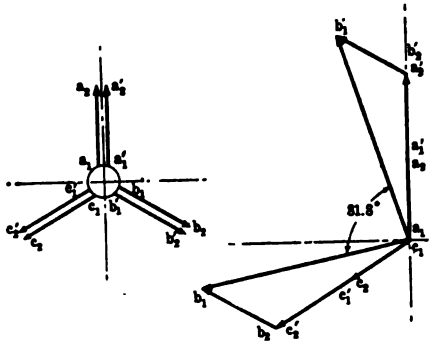


FIG. 28A

Oscillogram No. O A 89. Test No. 201. Table XIV and XV. First wave: Contactor. Third wave: E_{5B} = e.m.f. induced in search coil No. 5 under sustained short circuit at 5 amperes field excitation, 60 cycles per second and series delta connection. Second wave Contactor. Fourth wave: E_{5B} under same conditions, using same vibrator, etc., except machine series Y connected.

Before comparing further the s.s.c. phenomena with the alternator connected Y and delta, see Fig. 28A, also Y with the neutral in, and two-phase, attention should be called to the fact that the two-phase connection was only an imperfect one, the angle



Three-phase two-circuit armature. **Two-phase connection giving 81.8 degrees between phases**

FIG. 29

between the phases being 81.8 deg. instead of 90. This is clearly shown in Fig. 29 which is self-explanatory. Further as stated in the beginning of this section the armature current under three-phase s.s.c. is sinusoidal, which is not true in this case as may be seen from Fig. 4, first wave.

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In view of these two discrepancies it seems rather remarkable that the general wave shapes of the e.m.fs. induced in the search coils under two-phase short circuit, Fig. 28, are so similar to those obtained with three-phase connection, see Figs. 23, 27 etc., except for some small differences which will be considered presently.

In all the tests with two- or three-phase connection E_{5B} was always found to be considerably larger than E_{1T} so that in many cases it was necessary to insert extra resistance into the vibrator circuit. It will be recalled that this difference in the e.m.fs. induced in coils No. 1 and 5 has already been pointed out in connection with Figs. 7, 20, 22 and Table VII and IX, and it has been ascribed to the fact that the flux density at a surface bounding the bottom of the slots may be larger and more nearly perpendicular to the search coil conductors, than the flux

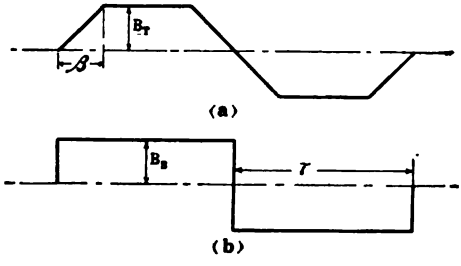

FIG. 30

Assumed flux distribution at top and bottom of slot.

density at the top of the slots. To illustrate this point quantitatively, assume Fig. 30A and 30B to represent the flux distribution at two surfaces bounded by the top and bottom of the slots respectively. For different values of β, the ratio of the 1st and 3rd harmonic of wave 30B to the 1st and 3rd harmonic of Fig. 30A, if $B_T = B_B$, is as follows; (B_B = flux density at bottom of slot. B_T = flux density at top of slot).

β in degrees	1	5	10	20	30
Ratio of firsts	1.022	1.005	1	1.02	1.11
Ratio of thirds	1.005	1.01	1.005	1.205	1.575

If $B_B = (1.09\ B_T)$ which is a fair assumption (see Fig. 20) the ratios are as follows:

β in degrees	1	5	10	20	30
Ratio of firsts	1.115	1.09	1.09	1.1	1.21
Ratio of thirds	1.092	1.103	1.37	1.31	1.91

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Further, with all connections the crests (2) and (5) columns (7) and (10) Table XV, Figs. 23 and 27 are distinctly larger for E_{5B} than for E_{1T}. Again, the amplitudes of these crests in volts do not differ greatly for delta or for Y connection but are smaller for two-phase connection. For example, crest (2) for E_{1T} is about 3.7 for delta and it is 4 volts for Y, but it is only about 1.8 or 2.6 for two-phase connection. The reason for this will be seen from Fig. 26 which shows that crest (2) is due

TABLE XV.

Data relating to the crest values of the e.m.fs. induced in the stator search coils No. 1, No. 3 and No. 5 under s.s.c. with the machine connected series Y, series Δ and two-phase.

No. of oscill.	Field amps.	Kind of s.s.c.	Frequency	Coil e.m.f.	Amplitudes in max. volts of: Crest (1)	Crest (2)	Crest (3)	Crest (4)	Crest (5)	Crest (6)
					See Figs. 27, 28, etc.					
1	2	3	4	5	6	7	8	9	10	11
202	4.95	Y	60	E_{1T}	6.62	4.01	7.88	6.4	3.7	7.56
O A 44	5	Y	54	E_{1T}	5.45	3.25	7.12	5.6	3.25	7.49
O A 88	5	Y	60	E_{1T}	7.21	6.12	11.24	7.17	6.21	11.24
O A 43	5	Y	54	E_{3M}	4.24	4.24	9.48	4.24	4.69	9.48
O A 72	5	Y	54	E_{5B}	6.43	5.01	10.62	6.3	5.28	10.43
O A 89	5	Y	60	E_{5B}	7.47	5.66	11.75	6.89	5.89	11.64
275	4.64	Δ	60	E_{1T}	7.86	3.73	9.59	8.06	3.73	9.45
275	4.9	Δ	60	E_{5B}	8.34	6.12	12.32	7.92	6.53	12.67
O A 88	5	Δ	60	E_{1T}	6.20	3.77	8.05	6.2	3.69	8.05
O A 89	5	Δ	60	E_{5B}	7.37	5.98	11.32	6.83	5.98	11.54
205	5 +	two-phase	60	E_{1T}	8.23	2.62	8.11	8.11	3.08	8.11
O A 81	4.96	two-phase	60	E_{1T}	7.89	1.85	8.32	7.68	1.85	7.89
O A 86	4.96	two-phase	60	E_{1T}	9.42	1.95	9.74	9 42	1.95	9.74
O A 52	3 16	Y	52	E_{1T}	3.1	1.94	4.08	3.1	1.75	4.37
O A 54	3.16	Y	52	E_{3M}	2.52	2.81	5.91	2.71	2.47	5.91

to the amplitude of the armature reaction which is the same for delta or Y but it is smaller for two-phase connection (imperfect) on account of the distribution factor of the winding.

Taking everything into consideration, it is found that the e.m.fs. induced in rotor coils with two-phase connection are fairly regular; this is further illustrated by wave analysis $V-4$ and $V-5$. Table XIII and also the e_a curves given in Fig. 31.

However, leaving aside, for a moment, all secondary effects it may be well to refer to a graphical explanation given by A.

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E. Clayton.[16] He considers an ideal alternator with no magnetic oscillations or secondary effects, no fringing, etc.; the no-load field form may then be assumed to be rectangular and the armature reaction to be portion of a sine wave as indicated in Figs. 32A and 32B. The resultant curve is given in Fig. 32c which resembles in general the waves obtained under s.s.c. conditions.

The rotor exploring coils under s.s.c. show $2\,q = 24$ ripples per cycle due to the teeth. See Table I, column 6 and Figs. 8B and 8c. They also indicate oscillations which repeat themselves

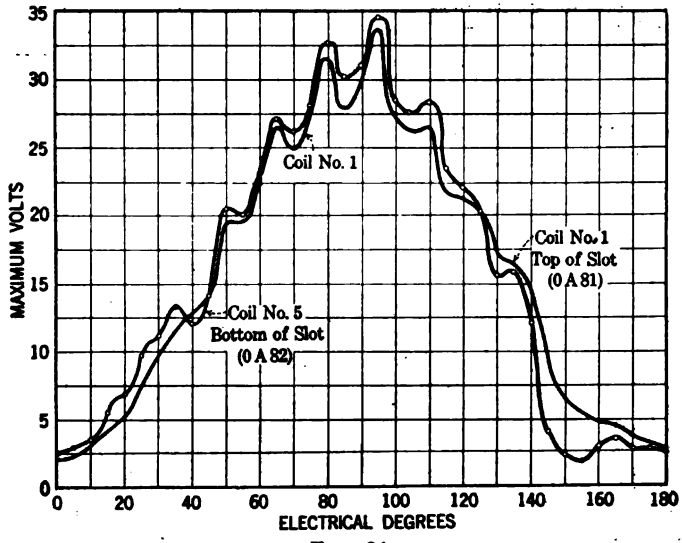

FIG. 31

E_a curves = $E_{fo} - E_r$ = difference between open circuit and s.s.c. e.m.fs. induced in full-pitch top and bottom search coils—field excitation = 5 amperes—periodicity = 60 cycles per second—connection = two-phase.

every 60 deg. See column 5. These as already explained, are due to armature reaction and their jagged unsymmetrical shape may be accounted for as suggested in Fig. 33 where A and B are due to armature reaction and the teeth, and have a frequency of $6\,f$ and $24\,f = 2\,q\,f$, respectively. Their resultant is seen to be similar to the E_{RF} waves Figs. 8B and 8c. In this connection it may be noted that the long flat oscillations which have been

16. The wave shapes obtained with a-c. generators working under steady short-circuit conditions. *Journal, Inst. E. E.*, (London), 1916, Vol. 54, p. 34. This is the first paper, so far as I am aware, to throw any light on this complex subject.

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ascribed to the eccentricity of the rotor disappear from the E_{fr} wave under s.s.c. Compare Figs. 4, 5 and 8. This may be explained if the peculiar flux distribution under s.s.c. be considered; taking this to be approximately as represented in Fig. 26 e_r wave, assume that at the position of minimum reluctance the area designated by (A), under crest (1) between $a\ b$, = 83 = B and C = 66 = area under crest (2) between $b\ c$; thus the total net flux per pole under s.s.c. will be 166 − 66 = 100 per cent. Next, at the position of minimum permeance, assume A = 75 = B and C = 50; the net flux per pole is again 150 − 50 = 100 per cent. Obviously under these conditions the e.m.f. due to the eccentricity of the rotor would disappear. However, the areas A, B and C when actually measured were not in the proportion assumed; but it is

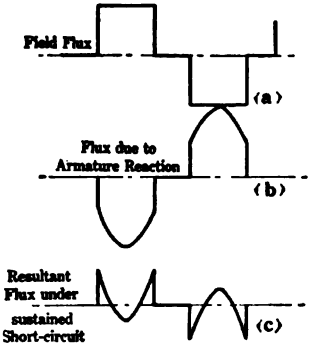

FIG. 32

Flux distribution curves of ideal salient-pole alternator under sustained short circuit

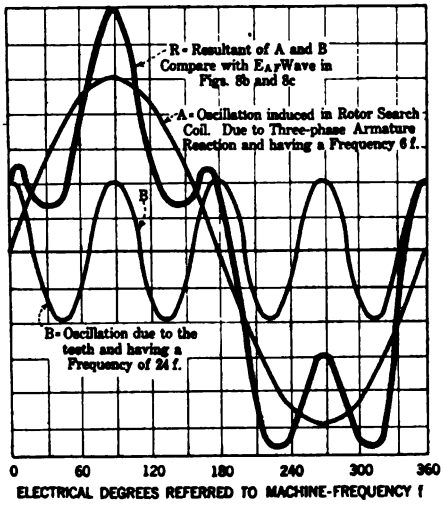

FIG. 33

Theoretical explanation of the production of the peculiar e.m.f. waves shown in Figs. 8b and 8c.

believed that some such explanation as the one just given is responsible for the disappearance of these oscillations which occur once in every revolution under no-load or synchronous motor operation.

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The e.m.fs. induced in rotor coils No. 8 and 7 with two-phase connection are given in Fig. 4 (third wave) and Fig. 34 (first wave) respectively. E_{RF} = e.m.f. induced in coil No. 7, is nearly 3 times (2.7) as large under two-phase s.s.c. as it is with Y connection. See Table I, columns 8 and 9. As to E_{RC} = e.m.f. induced in coil No. 8, see Fig. 34, it is found to be nearly 6 times (5.7) as large with two-phase s.s.c. as with three-phase s.s.c. Under synchronous motor operation Fig. 8A, E_{RC} is about 0.053 at 59.5 cycles against 0.03 at 50 cycles with three-phase s.s.c, Table I, Test No. 165.

In short, the flux oscillations with two-phase connection are

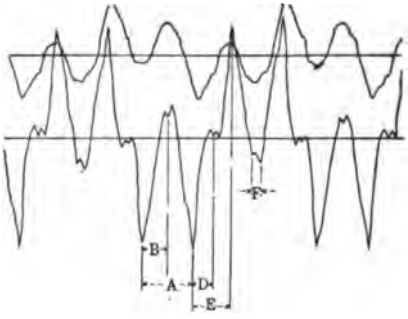

FIG. 34

Oscillogram No. O A 86. Test No. 198. Tables XIV and XV. Sustained short circuit at 4.96 amperes field excitation, and normal speed, two-phase connection. First wave: E_{RC} = e.m.f. induced in rotor coil No. 8. Second wave: E_{1T} = e.m.f. induced in coil No. 1.

very much larger than with three-phase connection. This is believed to be due mainly to the fact that the two phases were not exactly in quadrature.

It can be shown, (see Note C), that if the angle between the two phases is $(90 - \alpha)$ deg., (= 81.8 deg. in our case, see Fig. 29), the resultant armature reaction will consist approximately of (*a*) a regular travelling wave whose amplitude is approximately equal to the value obtained if $\alpha = 0$. See equation (**56**). (*b*), it will consist of a wave in *quadrature* to the first and travelling in the opposite direction and at the same speed but having an amplitude $(\sin 2\alpha)/2$ of (*a*). Hence it will be seen, that neglecting other effects, this second rotating field, which is *in quadrature to the first and maximum at* $x_1 = 0$, *i.e., at the coil-side of coils No.* 7 *and* 8, will be responsible for the e.m.fs. induced in them under two-phase s.s.c. Further it will account for the peculiarly distorted shape of the E_{RF} wave. This is illustrated

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in Fig. 35 where the e.m.f. induced in the rotor coil No. 7 is assumed to be due to H_{3R}, H_3^{α} and H_{5R}, *i.e.*, regular 3rd space harmonic travelling wave, 3rd space harmonic due to lack of quadrature and regular 5th space harmonic travelling wave. See equations (59) and (62). The amplitude of these waves is assumed to be 100, 27 and 66 per cent respectively, *i.e.*, $H_3^{\alpha} = 0.27\, H_{3R} = (\frac{1}{2} \sin 4\,\alpha)$, approximately. $H_{5R} = H_{3R}\,(4f/6f) = 0.66\, H_{3R}$, where (6 : 4) = ratio of their relative velocities with respect to the rotor. Thus it is seen that, Fig. 35 explains

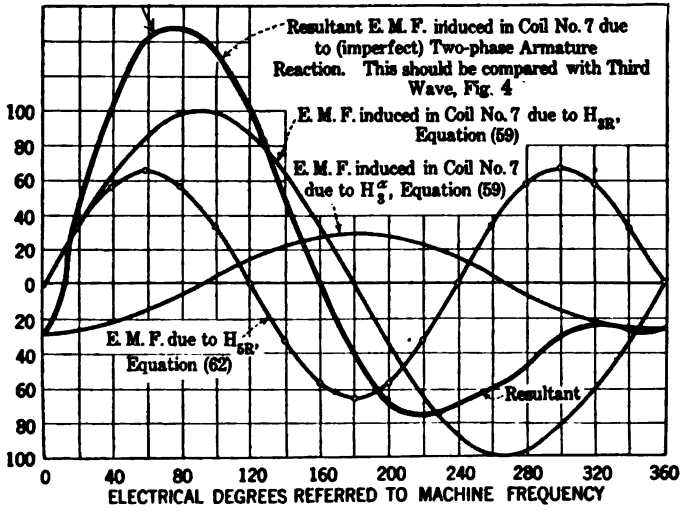

FIG. 35

Theoretical explanation of production of the peculiar E_{RF} wave under 2-phase s. s. c. See third wave Fig. 4.

approximately the peculiar shape of the E_{rf} wave given in Figs. 4 and 34.

Finally considering the e.m.fs. induced under s.s.c. with Y connection, in coils No. 9 and 10, Figs 3 and 3A, we find these to conform to the general wave shape we would expect, according to the explanation given in Section III. In Fig. 36, E_{TT} at no-load and at s.s.c. is given and it is seen that both are characterized by two prominent peaks, and they are quite similar to each other except for the decided difference in shape between a + crest and a − crest. The reason for this will be clear if it be recalled, as explained in detail in Section III, that the portion of the wave between a + and a − crest is produced while the coil is under the pole-arc; on open-circuit the portion of the *B*-curve under the pole is more or less toothed and it gives rise

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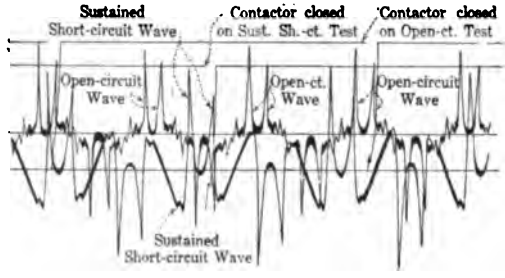
to little kinks see curve (*b*), Fig. 6. Under s.s.c., however, the *B*-curve is very small or even negative at the middle of the pole as indicated by crest (2) Figs. 26, 28, 37 etc. Consequently, as the coil passes from one pole tip to the next its enclosed flux



F G. 36

Oscillogram No. O A 61. Test No. 172. Table II. *Exposure No.* 1. Open-circuit run at 60 cycles per second and 4.95 amperes field excitation. First wave: contactor Third wave: E_{TT} = e.m.f. induced in concentrated exploring coil No. 10, on top of tooth. *Exposure No.* 2. Sustained short circuit with series Y connected and same field current and speed as before. Second wave: contactor. Fourth wave: E_{TT}: This shows that there is no displacement in E_{TT} wave under short-circuit condition.

undergoes considerable change, which produces the portion of the curve between a + and a − crest, as indicated in Figs. 36 and 37.

Space forbids considering single-phase s.s.c. phenomena; however, for the sake of completeness it may be stated that from a series of tests with the machine delta- or Y-connected

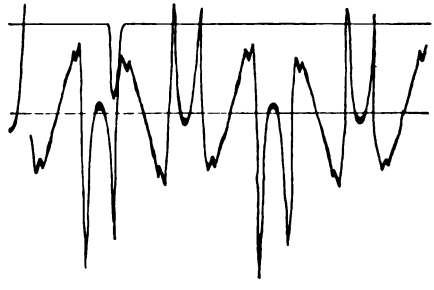


FIG. 37

Oscillogram O A 69. Test No. 180. Sustained short circuit at 5 amperes field excitation, and frequency of 60 cycles per second. Y series connection. First wave: E_{TT} = e.m.f. induced in coil No. 10 on top of tooth.

and short-circuited between two terminals, or one terminal and the neutral, (in case of Y) the exploring coil e.m.fs. were found to be fairly similar to those already described. The similarity is rather striking when the presence of complex secondary effects and pulsations of single-phase alternators is considered.

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V. Critical Resume and Conclusions

It seems fair to conclude from the above that the sustained short-circuit (s.s.c.) phenomena of alternators in spite of their complexity and the extremely distorted wave-forms to which they give rise, are amenable to the ordinary theories of synchronous generators as given by Blondel, Potier and others. It is only necessary to restrict the usual sweeping simplifying assumptions of so-called text-books in order to arrive at some of the general results given in the preceding pages. It has been shown that unless the field and armature flux waves are sinusoidal the resultant field under s.s.c. will be badly distorted, see for example Figs. 23, 27 etc., for the simple reason that with the very low voltages which obtain under s.s.c. conditions the fundamental is so greatly reduced that the higher harmonics assume a very predominant role. Starting with a field flux-wave consisting of, 1st 100 per cent, 3rd + 10 per cent, 5th + 5 per cent, and a corresponding armature flux curve containing the same harmonics with relative amplitudes of − 95 per cent, + 10 per cent and + 5 per cent, the resultant flux distribution will be given by e_r, Fig. 26; this is not unlike the actual s.s.c. oscillograms given in Figs. 23, 28, etc.

Furthermore it has been shown in Section IV, that with sinusoidal (in time) armature currents the H-curve of armature reaction will be sinusoidal, but not so with the B-curve; the latter, as is well known will be flat-topped depending upon the degree of saturation, and will further differ from the H-curve in that it will be toothed and have different values, and directions with respect to the armature conductors, at a surface bounded by the top of the slots than at a surface bounded by the bottom of the slots. Thus it is clear that from the view-point of s.s.c. phenomena from among the different schemes used for obtaining "sine-wave alternators" the best is the one which attempts to get to the root of the evil and obtain a sinusoidal flux distribution, under the poles to begin with. However, even then harmonics may and will be introduced by armature reaction. It is well in this connection to emphasize the fact that the B-curve of armature reaction will differ more or less from the H-curve producing it for the reasons just given. Consequently the elimination of the higher harmonics from the B-curve (of armature reaction) is very difficult, if not impossible, since even such devices as fractional slots per pole will avail little unless the permeance per unit area across the pole face is made uniform, constant

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and free from extreme effects of saturation. However, with a thorough understanding of the subject it should be possible to reduce the effects of the B-curve to a negligible quantity.

The magnetic oscillations under load, no-load and s.s.c. conditions are studied not only by means of full-pitch exploring coils placed on the stator and the rotor but also concentrated coils placed on top of a wedge or a tooth and having a pitch equal to a slot-pitch or fraction thereof. See Figs. 3 and 3A.

The principal oscillations observed are:

(1) *Those due to the teeth*; taking the amplitude (a), Fig. 13 as 100 per cent for the coil No. 1 on top of the slot, we find (a) to be 107 per cent for coil No. 3 and 108 per cent for coil No. 5. On the same basis of percentages, (d), Fig. 13, is 24, 31 and 32 per cent respectively for the same coils. See Fig. 7. These oscillations (due to the teeth) decrease under synchronous motor operation. Compare Figs. 7 and 22.

(2) *Oscillations due to the eccentricity of the rotor*. These have a frequency of $(1/p)$ machine-frequency, or one cycle per revolution.

(3) *Oscillations due to armature reaction* and having a frequency of 6 times and 4 times machine-periodicity in case of three-phase and two-phase connection. It seems rather remarkable that oscillations (2) disappears under s.s.c.; an explanation of this is given on p. 1183.

(4) *Oscillations due to the pulsations of magnetic reluctance* caused by the salient-pole construction of the machine. These are best illustrated by means of exploring coils placed on top of a slot and on top of a tooth and having a pitch equal to that of a tooth. See Figs. 10, 11, etc. These oscillations are analyzed in detail in Table II and it is seen that the e.m.f. induced in a concentrated coil on top of a tooth is about twice as large as the e.m.f. induced in an exactly similar coil placed on top of a wedge: These oscillations are studied in detail and they give a clear idea of the flux distribution under *actual running conditions*, which are, of course very important from the point of view of iron losses etc. In this connection it is well to call attention to the methods of analysis and investigation followed in this research; guided by a simplified general mathematical theory the flux pulsations and oscillations were separated and analyzed by taking oscillograms of the e. m. f.'s induced in different stator and rotor exploring coils as explained in detail in Section I.

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Certain pecularities and differences between the e.m.fs. induced in search coils at the top and bottom of slots are considered and, at least, tentative explanations are given. For these the reader can consult sections III and IV.

In regard to different kinds of short circuits it is shown that there is practically no difference between s.s.c. phenomena with the machine connected delta, Y, and Y with the neutral in. This might have been expected considering the elimination of triple frequency circulating currents due to the $\frac{2}{3}$ pitch armature winding. In case of two-phase connection, however, the armature current is not as sinusoidal as with three-phase connection; the variational e.m.fs. induced in the rotor coils under s.s.c. show very peculiar wave distortion as seen from Fig. 34, first wave, etc. An explanation of this is indicated in Fig. 35.

Before concluding these remarks it may be well to call attention to the bearing of the results given in the paper upon what are considered standard methods of testing large alternators. The thorough understanding of the underlying facts and the collection of quantitative data on the subject for salient as well as non-salient pole generators will throw a great deal of light on efficiency tests, cyclic heat run tests, short-circuit load loss tests, phase-shifting or feeding back method of load tests, etc. In such tests the flux distribution and the flux density at different points of the magnetic circuit is of considerable importance. It seems to the writer that, in such cases, even if it be extremely difficult to devise tests and means of predetermining by calculation certain characteristics of the machine, without actually loading it, it is very desirable to know and understand, as well as possible, the internal reactions so that proper rational allowances may be made. In any case, the detailed analysis of these phenomena seems to be a step in the right direction, and although the application of such information to design and test problems does not fall within the scope of this paper, it may be well to give a simple quantitative illustration of it.

Assume that the open-circuit core loss of an alternator is 2.8 per cent of its rating and that half of this can be charged to hysteresis and the other half to eddy current loss; let the s.s.c. test loss, excluding friction and windage, be 1.5 per cent. According to the ordinary assumption that the flux distribution under s.s.c. is sinusoidal and 10 per cent of the open-circuit value the hysteresis loss, P_h and the eddy current loss, P_e will be:

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$$P_h = \left(\frac{2.8}{2}\right) \cdot \left(\frac{0.10\,B}{B}\right)^{1.6} \cdot \left(\frac{f}{f}\right)$$

$$= 0.035 \text{ per cent}$$

$$P_e = \left(\frac{2.8}{2}\right) \cdot \left(\frac{0.10\,Bf}{Bf}\right)^{2}$$

$$= 0.014 \text{ per cent}$$

Consequently, $P_h + P_e = 0.049$ per cent.

Next assume that the fundamental is 5 per cent, the third harmonic is 15 per cent and the fifth 10 per cent; these are fair and very conservative values according to the results of this investigation. Then,

$$P_h = \left(\frac{2.8}{2}\right) \cdot \left(\frac{0.05\,B}{B}\right)^{1.6} \cdot \left(\frac{f}{f}\right) + \left(\frac{0.15\,B}{B}\right)^{1.6} \cdot \left(\frac{3f}{f}\right) + \left(\frac{0.10\,B}{B}\right)^{1.6} \cdot \left(\frac{5f}{f}\right)$$

$$= 0.114 \text{ per cent}$$

and,

$$P_e = \left(\frac{2.8}{2}\right) \cdot \left(\frac{0.05\,Bf}{B.f}\right)^{2} + \left(\frac{0.15\,B.\,3f}{B.f}\right)^{2} + \left(\frac{0.10\,B.\,5f}{B.f}\right)^{2}$$

$$= 0.45 \text{ per cent}$$

hence, $(P_h + P_e) = 0.564$ per cent.

Thus according to the usual assumption $(P_h + P_e)$ under s.s.c. tests is negligible, being $(0.049/1.5).\ 100 = 3.3$ per cent, nearly, in this example. But if we take the actual flux distribution under a s.s.c. test into consideration, and assume the very conservative values taken above for the 3rd and 5th harmonics of the flux-wave, we find that the eddy current and hysteresis losses amount to, $(0.564/1.5).\ 100 = 38$ per cent, nearly, of the total s.s.c. test losses, *i.e.*, the iron losses are not necessarily negligible.

Finally it seems almost unnecessary to call attention to the necessity of standardizing our terminology. In specifications or technical papers and discussions it is only fair that in case of least ambiguity some qualifying term be used to specify the kind of short circuit implied. The words "sudden" and "sustained" seem suitable. However, the question of importance is that we should appreciate the great difference between these

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two important and complex phenomena of sudden and sustained short circuits and never leave any doubt as to which one is meant.

In conclusion it is a pleasure to the author to acknowledge his indebtedness to the authorities of the Rice Institute for the facilities afforded and particularly the moral support they have lent him. He is also glad to take this opportunity and acknowledge the conscientious and very able assistance of Mr. J. S. Waters, Jr., of the Class of 1917.

SUPPLEMENTARY NOTES

Note A. General E.M.F. Equations

1. *E.M.F. Induced in Stator Search Coils.* As explained in the e.m.f. equations.

$$e = -N\frac{d\phi}{dt} \tag{1}$$

ϕ may be and is usually, a function of the time t, as well as the position x of the coil with respect to the pole. Thus,

$$\frac{d}{dt}(\phi_{x,t}) = \frac{\partial\phi}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial\phi}{\partial t}\cdot\frac{dt}{dt}$$

$$= v\frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial t} \tag{2}$$

But,

$$\phi_{(x,t)} = \int B_{(x,t)}\cdot l dx \tag{3}$$

whence,

$$e = -N\frac{d\phi}{dt} = -N\,l\left[v\frac{\partial}{\partial x}\left(\int B\,dx\right) + \frac{\partial}{\partial t}(B\,dx)\right] \tag{4}$$

$$= e_{\text{motional}} + e_{\text{variational}}$$

Strictly speaking, $\phi = \int B\,da.\cos(B, da)$, where $da = ldx$ = element of area at which the flux density is B, and $\cos(B, da) = \cos\theta$, is the angle between the vector B and the normal to the elementary surface da; $\cos\theta$ is, of course, ordinarily neglected, except where methods of vector analysis are used. In our attempt to explain certain phenomena, we shall find it necessary to take account of this—which at first may

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seem an unnecessary refinement. Thus strictly (4) should be written:

$$e = -N l v \frac{\partial}{\partial x}\left[\int B\,dx \cos\theta\right] - N l \frac{\partial}{\partial t}\left[\int B\,dx \cos\theta\right] \tag{4a}$$

$$= e_{\text{rotational}} + e_{\text{variational}}$$ [17]

By means of (4) or (4a), which are quite general, it is easy to obtain expressions for the rotational and variational e.m.fs. induced in any of the stator or rotor coils. Assuming the flux under the pole to be symmetrical with respect to the interpolar axis, we can write,

$$B = \left[B_{1m} \sin\left(\pi \frac{x}{\tau}\right) + B_{3m} \sin\left(3\pi \frac{x}{\tau}\right) + \text{etc.}\right.$$

$$\left. + B_{1m} \cos\left(\pi \frac{x}{\tau}\right) + B_{3m} \cos\left(3\pi \frac{x}{\tau}\right) + \text{etc.}\right] \tag{5}$$

The rotational e.m.f. induced in a stator coil of N concentrated turns and having an active length of l cm. will be:

$$e_{rot.st.} = -N l v \frac{d}{dx} \int_{x_1}^{x_1+\tau} (B\,dx) \tag{6}$$

$$= 2 N l v B = 2 N l v \left(B_{1m} \cdot \sin\left(\frac{x}{\tau}\pi\right) + \text{etc.}\right) \tag{7}$$

since according to the assumption made in (5) even harmonics are absent, and in case of a full-pitch coil B at $x_1 = -B$ at $(x_1 + \tau)$. For a short-pitch coil (7) must be multiplied by the pitch factor.

In order to find expressions for the variational e.m.fs. let us make the assumption that the *flux variations are sinusoidal.* This is a very convenient assumption, since our equations already involve sine functions, and it is reasonably correct, as indicated by experience, because we are dealing with a *correction term* to be *added* to the comparatively large rotational e.m.f. Thus if $2q$ = number of slots per pair of poles, p, and (f/p) = speed of the machine in rev. per sec., obviously, $2q\,(f/p)$ will be the number of slots per second passing a given point x_1; this will set

17. S. J. Barnett, Electromagnetic Theory, pp. 333 to 340.

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up an oscillation of small amplitude and of frequency $2qf$, which will be expressed in mathematical symbolism by:

$$B_{pul} = B\ (\eta \cos (2\,q\ \omega\, t)\ ) \tag{8}$$

Neglecting the higher harmonics of the B curve, see (5), we get,

$$B_{pul} = \eta\, B_1 \left(\sin \left(\frac{x}{\tau}\ \pi \right) \right) (\cos\ (2\,q\ \omega\, t)\ ) \tag{9}$$

whence,

$$\frac{\partial}{\partial t}\ (B_{pul}) = -\ \eta\, B_1\, 2\,q\ \omega \left(\sin \left(\frac{x}{\tau}\ \pi \right) \right) (\sin\ (2\,q\ \omega\, t)\ ) \tag{10}$$

and according to the second term of the right hand member of (4) the variational e.m.f. is proportional to:

$$\int_x^{x+\tau} \frac{\partial B_{pul}}{\partial t}\ dx = -\ \eta\, B_1\, 4\,q\ \omega\, \frac{\tau}{\pi} \left(\cos \frac{x}{\tau}\ \pi \right) (\sin\ 2\,q\ \omega\, t) \tag{11}$$

To find the e.m.f. induced in any armature conductor it must be remembered that owing to the relative movement between armature and field,

$$x = v\, t = (2\,\pi\, r)\ (\text{r.p.s.})\ t$$

$$= 2 \left(\frac{2\,\pi\, r}{2\,p} \right) (\text{r.p.s.})\ t = (2\tau f)\ t = \left(\omega\, \frac{\tau}{\pi} \right) t \tag{12}$$

where r = radius of rotor, so that,

$$e_{pul.st.} = -\ N\, l\ \frac{\partial}{\partial t} \int B\, dx$$

$$= 4\, N\, l\ \frac{\tau}{\pi}\ q\ \omega \cdot \eta\, B_1\, (\cos\ \omega\, t)\ (\sin\ 2\,q\ \omega\, t) \tag{13}$$

$$= 4\, N\, l\, q\ \omega\ \frac{\tau}{\pi}\ \eta\, B_1\, [\sin\ (2\,q\ \omega + 1)\ \omega\, t$$

$$+ \sin\ (2\,q - 1)\ \omega\, t] \tag{14}$$

To find analytical expressions for the static or variational e.m.f. in an armature conductor due to the to-and-fro swing of the flux, we shall assume that the B-curve is constant in magnitude, but it moves across the pole-face according to simple harmonic motion, at a frequency of $2\,qf$. Hence neglecting the higher harmonics of (5) we may write,

$$B_{sw} = B_1 \left[\sin \frac{\pi}{\tau}\ (x - \epsilon \sin\ 2\,q\ \omega\, t) \right] \tag{15}$$

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where $2\,q\,\omega$ is the angular velocity and ϵ the amplitude of the variations.

Differentiating (**15**) with respect to t and making the reasonable assumption that,

$$\cos\left(\frac{\pi}{\tau}\ \epsilon \sin 2\,q\,\omega\,t\right) = 1, \text{ and}$$

$$\sin\left(\frac{\pi}{\tau}\ \epsilon \sin 2\,q\,\omega\,t\right) \simeq \frac{\pi}{\tau}\ \epsilon\,(\sin 2\,q\,\omega\,t)$$

we obtain:

$$\frac{\partial B_{sw}}{\partial t} = -\,B_1\left[\cos\frac{\pi}{\tau}\,(x - \epsilon \sin 2\,q\,\omega\,t)\right]$$

$$\left[2\,\epsilon\,q\,\omega\,\frac{\pi}{\tau}\cos 2\,q\,\omega\,t\right] \qquad (16)$$

and

$$\int_{x}^{x+\tau} \frac{\partial B_{sw}}{\partial t}\,d\,x = [4\,\epsilon\,B_1\,q\,\omega\cos(2\,q\,\omega\,t)]$$

$$\left[\sin\frac{\pi}{\tau}\,(x - \epsilon \sin 2\,q\,\omega\,t)\cos\frac{x}{\tau}\,\pi\right] \qquad (17)$$

As before, to find the variational e.m.f. in a given armature conductor due to the flux-swing, substitute,

$$x = (2\,f\,\tau\,t) = \left(\omega\,\frac{\tau}{\pi}\,t\right)$$

Thus,

$$e_{sw.st.} = -\,4\,N\,l\,q\,\omega\,(\epsilon\,B_1)\,(\cos 2\,q\,\omega\,t)$$

$$[\sin(\omega\,t - \epsilon \sin 2\,q\,\omega\,t)\cos\omega\,t]$$

$$= -\,4\,N\,l\,q\,\omega\,(\epsilon\,B_1)\left[\{\sin(2\,q + l)\,\omega\,t - \sin(2\,q - l)\,\omega\,t\}\right.$$

$$\left. - \left\{\frac{\epsilon}{2}\sin(4\,q + 1)\,\omega\,t + \sin(4\,q - 1)\,\omega\,t\right\}\right] \qquad (18)$$

2. *E.M.Fs. Induced in Field Exploring Coils.* Equation (**7**) gives the rotational e.m.f. induced in a coil of N concentrated turns having a relative speed v with respect to B. Evidently this is zero for a coil placed on the rotor, and therefore,

$$e_{mot.r.} = 0 \qquad (19)$$

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To find the variational e.m.f. due to flux pulsations in a field (rotor) search coil whose coil-side is at any point x_1, simply substitute $x = x_1$ in (**13**). Thus,

$$e_{\overline{pul.r}} = 4\,N\,l\,q\,\omega\,(n\,B_1)\cos\left(\frac{\pi}{\tau}\,x_1\right)(\sin 2\,q\,\omega\,t) \qquad (\mathbf{20})$$

Similarly from (**17**) and (**4**) we get,

$$\begin{aligned} e_{sw.r.} &= -\,N\,l\,\frac{\partial}{\partial t}\int B\cdot d\,x \\ &= 4\,N\,l\,q\,\omega\,(\epsilon\,B_1)\,(\cos 2\,q\,\omega\,t) \\ &\quad \left[\left\{\sin\left(\frac{\pi}{\tau}\,x_1 - (\epsilon\sin 2\,q\,\omega\,t)\right)\right\}\cos\frac{\pi}{\tau}\,x_1\right] \end{aligned} \qquad (\mathbf{21})$$

For coils No. 7 and 8, $x_1 = 0$, whence we obtain directly (**22**) and (**23**) from (**20**) and (**21**) respectively, as given in the first part of the paper:

$$e_{sw.r.} = 2\,N\,l\,q\,\omega\,\epsilon^2\,B_1\sin(4\,q\,\omega\,t) \qquad (\mathbf{22})$$

and,

$$e_{pul.r.} = 4\,N\,l\,q\,\omega\,\eta\,B_1\sin(2\,q\,\omega\,t) \qquad (\mathbf{23})$$

Note B

1. *Theoretical Investigation of Armature Reaction.* Make the reasonable assumption (as shown on p. 1163) that the armature current under s.s.c. is sinusoidal (with respect to time). Further assume that the armature winding of say, a three-phase alternator is sinusoidally distributed (in space) as schematically represented in Fig. 17. If one of the phase windings were supplied with direct-current the magnetic field intensity H, at any point θ_{ger}, geometrical space radians or x centimeters, from the axis of reference, see Fig. 17, would be,

$$H_1' = \gamma\,I_{d.c.}\,(N_{mas}\cos p\,\theta_{gsr}) = \gamma\,I_{d.c.}\,N_{mas}\cos\left(\frac{x}{\tau}\pi\right) \qquad (\mathbf{24})$$

where, γ = constant, and $\left(\frac{x}{\tau}\,\pi\right) = p\,\theta_{gsr}$ = electrical space radians. Similarly for the other phases:

$$\left.\begin{aligned} H_1'' &= \gamma\,I_{d.c.}\,N_m\cos\left(\frac{x'}{\tau}\,\pi\right) = \gamma\,I_{d.c.}\,N_m\cos\left(\frac{x}{\tau} - \frac{2}{3}\right)\pi \\ H_1''' &= \gamma\,I_{d.c.}\,N_m\cos\left(\frac{x''}{\tau}\,\pi\right) = \gamma\,I_{d.c.}\,N_m\cos\left(\frac{x}{\tau} - \frac{4}{3}\right)\pi \end{aligned}\right\} \qquad (\mathbf{25})$$

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When these windings are supplied with balanced three-phase currents the resultant field intensity due to armature reaction will be:

$$H_1' = \gamma\,[I_m \cos \omega t]\left[N_m \cos \frac{x}{\tau}\pi\right] \tag{26}$$

$$H_1'' = \gamma\left[I_m \cos\left(\omega t - \frac{2}{3}\pi\right)\right]\left[N_m \cos\left(\frac{x}{\tau} - \frac{2}{3}\pi\right)\right] \tag{27}$$

$$H_1''' = \gamma\left[I_m \cos\left(\omega t - \frac{4}{3}\pi\right)\right]\left[N_m \cos\left(\frac{x}{\tau} - \frac{4}{3}\pi\right)\right] \tag{28}$$

whence by trigonometry,

$$\begin{aligned} H = H_1' + H_1'' + H_1''' &= \frac{3}{2} H_m \cos\left(\omega t - \frac{x}{\tau}\pi\right) \\ &= \frac{3}{2}\gamma I_m \cos\left(\omega t - \frac{x}{\tau}\pi\right) \end{aligned} \tag{29}$$

This gives the resultant field intensity due to armature reaction and it will be recognized to be the well known equation of a travelling wave which is a function of the time t, as well as the space position x, and whose maximum, H_m, occurs when $\omega t = \left(\frac{x}{\tau}\pi\right)$; in other words, whose maximum H_m travels at a speed $(x/t) = (\omega\tau/\pi) = (2f\tau)$, which is the peripheral speed of the rotor. See (**12**). Next, consider the case of N conductors per slot and one slot per pole per phase carrying I_{dc}. amperes. Evidently, as indicated in Fig. 18 the field intensity H, will be rectangular in shape and may be represented by the series:

$$H = \frac{4}{\pi} Y_m\left[\sin\left(\frac{x}{\tau}\pi\right) + \frac{1}{3}\sin\left(3\frac{x}{\tau}\pi\right) + \ldots\right] \tag{30}$$

Equation (**30**) takes the place of (**24**) and similar expressions can be written out for the other phases, corresponding to (**25**) and (**26**). However, without going any further into this, suffice to call attention to the references given in foot-note (**13**) and state that if the non-sinusoidally distributed winding represented in Fig. 18, be supplied with balanced alternating-current instead of direct-current, expressions exactly analogous to (**26**), (**27**) etc. will be obtained. Combining these trigonometrically, we get (**32**), (**33**) etc. which were given in the first part, in section IV.

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2. *E.M.F. Induced in Rotor and Stator Exploring Coils by the Armature Reaction.* It is a simple matter, now to obtain general expressions for the e.m.fs. induced in the different search coils, if we assume that the B-curve is sinusoidal; the rotational e.m.f. of a stator coil whose coil-side is at a fixed distance x_1 from the origin, see Fig. 6 will be according to (4):

$$e_{rot.st} = - N l v \frac{\partial}{\partial x} \int B \, d x \tag{36}$$

$$= - N l (2 f \tau) \frac{\partial}{\partial x} \int \frac{3}{2} B_a \cos \left(\omega t - \frac{x_1}{\tau} \pi \right) \cdot d x \tag{37}$$

$$= - N l (2 f \tau) \left(\frac{3}{2} B_a \right) \cos \left(\omega t - \frac{x_1}{\tau} \pi \right) \tag{38}$$

This is a sinusoidal e.m.f. of fundamental frequency and having a phase angle $\left(\frac{x_1}{\tau} \pi \right)$ with respect to the origin.

The fifth space harmonic of armature reaction will give:

$$e_{rot.st.} = - N l \frac{2 f \tau}{5} \cdot \frac{\partial}{\partial x} \int \frac{3}{2} B_a \cos \left(\omega t + \frac{5 x_1}{\tau} \pi \right) \tag{39}$$

$$= - N l \frac{2 f \tau}{5} \cdot \frac{3}{2} B_a \cos \left(\omega t + \frac{5 x_1}{\tau} \pi \right) \tag{40}$$

i.e., an e.m.f. of fundamental machine frequency and having a phase angle $(6 x_1/\tau) \pi$ with respect to the origin. Similar expressions may be derived for all other harmonics.

However, as already explained, the sinusoidal H-curve produced by the armature reaction will give rise to a distorted B-curve. This will produce in the stator coils a motional e.m.f. which will contain higher *time harmonics*. To find analytical expressions for these e.m.fs. (due to a distorted gliding or rotating B-curve) it will be convenient to assume the B-curve as stationary and the exploring coils revolving at the same speed in the opposite direction. In this manner the problem is reduced to the one already considered in Note A, and it seems hardly necessary to derive equations for them here.

As to the e.m.fs. induced in the rotor search coils (due to armature reaction), it is clear that the fundamental travelling

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wave given by (**32**), will induce no e.m.f. The 3rd harmonic will, of course, be zero for a three-phase machine. See (**33**).

To find the motional e.m.f. induced in a rotor coil due to the fifth space harmonic B_5,

$$B_5 = (1/2)\, B_a \cos\,(\omega\, t + 5\, x\, \pi/\tau) \tag{41}$$

it must be remembered that x increases at the rate of $2 f \tau$ with respect to the rotor. Thus substituting in (**41**),

$$\omega\, t = \pi\, x/\tau = (\pi/\tau)\; 2 f \tau\, t \tag{42}$$

we obtain,

$$B_5 = (3/2)\; B_a\; (\cos 6\; \omega\, t) \tag{43}$$

Similarly,

$$B_7 = (3/2)\; B_a\; (\cos 6\; \omega\, t) \tag{44}$$

That is, the 5th and 7th space harmonics though of fundamental frequency with respect to the stator are of sextuple frequency with respect to the rotor. This will be clear when it is considered that the 5th space harmonic glides backward at a speed $(2 f \tau/5)$ while the seventh travels forward at a speed $(2 f \tau/7)$. Assuming for convenience the rotor as stationary, it is evident, that the relative speed between the rotor and these gliding fields will be:

$$(2 f \tau) - \left(- \frac{2 f \tau}{5}\right) = 6 \left(\frac{2 f \tau}{5}\right) \tag{45}$$

and

$$2 f \tau - \left(+ \frac{2 f \tau}{7}\right) = 6 \left(\frac{2 f \tau}{7}\right) \tag{46}$$

respectively. The pole pitches τ_5 and τ_7 of the fifth and seventh fields are $(\tau/5)$ and $(\tau/7)$, and the corresponding number of pairs of poles are $5\, p$ and $7\, p$ respectively. Consequenty the frequency of the e.m.fs. induced in a rotor coil by them will be:

$$f_5 = (\text{r.p.s.})_5\; (p_5) = (\text{speed})_5 \left(\frac{1}{2 \tau\, p}\right) (5\, p) \tag{47}$$

$$= 6 \left(\frac{2 \tau f}{5}\right) \cdot \left(\frac{1}{2 \tau\, p}\right) (5\, p) = 6 f \tag{48}$$

Similarly for the 7th space harmonic,

$$f_7 = 6 \left(\frac{2 \tau f}{7}\right) \left(\frac{1}{2 \tau\, p}\right) 7\, p = 6 f \tag{49}$$

In general for the $(6\, n - 1)\; th$ space harmonic which travels backward with respect to the fundamental,

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$$f_{(6n-1)} = 6\,n\Big(2\tau f/(6\,n-1)\Big)\Big((6\,n-1)\,p/2\tau\,p\Big) = 6\,n f \qquad (50)$$

and for the $(6\,n+1)\,t\,h$ harmonic,

$$f_{(6n+1)} = 6\,n\Big(2\tau f/(6\,n+1)\Big)\Big((6\,n+1)\,p/2\tau\,p\Big) = 6\,n f \qquad (51)$$

Thus the 11th and 13th will give rise to rotor e.m.fs. having periodicities of $12f$, and so on for the others.

In case of two-phase alternators the 3rd and 5th will give rise to e.m.fs. of (4. machine frequency). The seventh and ninth will produce e.m.fs. of (8. machine frequency), etc.

However, as shown by the oscillograms given before, the rotor search coils indicate only pulsations having periodicities of $6f$ in case of three-phase connection and $4f$ in case of two-phase connection. In concluding this note attention may be called to the fact that the above analysis of armature reaction and the e.m.fs. induced by it in the stator and rotor coils is applicable to induction motors; as a matter of fact it is a little difficult for the writer to understand why standard texts do not treat armature reaction in alternators etc. in exactly the same way as the rotating fields of induction motors, as it has been done above, because it is more convenient and lends itself better to the derivation of quantitative expressions for the e.m.fs. induced in the stator and rotor by armature reaction alone.

Note C. Direct Component of Armature Reaction of an Imperfect Two-Phase Alternator

As already explained in detail the resultant magnetic field intensity due to balanced two-phase currents flowing through sinusoidally distributed two-phase windings will be given by:

$$H_1' = H_1' + H_1'' = H_m\Big[\Big\{\cos\omega t\Big\}\Big\{\cos\Big(\frac{x}{\tau}\pi\Big)\Big\} + \Big\{\cos\Big(\omega t + \frac{\pi}{2}\Big)\Big\}\Big\{\cos\Big(\frac{x}{\tau}\pi + \frac{\pi}{2}\Big)\Big\}\Big] \qquad (52)$$

Now in case of an imperfect machine which has a displacement between phases of $\Big(\frac{\pi}{2} - \alpha\Big)$ deg. instead of $\frac{\pi}{2}$, the sum of H_1' and H_1'' will be:

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$$H_1 = H_m \left[\{\cos \omega t\} \left\{ \cos \left(\frac{x}{\tau} \pi \right) \right\} \right.$$

$$\left. + \left\{ \cos \left(\omega t + \frac{\pi}{2} - \alpha \right) \right\} \left\{ \cos \left(\frac{x}{\tau} \pi + \frac{\pi}{2} - \alpha \right) \right\} \right] \quad (53)$$

$$= H_m \left[\frac{1}{2} \cos \left(\omega t + \frac{x}{\tau} \pi \right) + \frac{1}{2} \cos \left(\omega t - \frac{x}{\tau} \pi \right) - \right.$$

$$\left. \frac{1}{2} \cos \left(\omega t + \frac{x}{\tau} \pi - 2 \alpha \right) + \frac{1}{2} \cos \left(\omega t - \frac{x}{\tau} \pi \right) \right] \quad (54)$$

$$= H_m \left[\left\{ \cos \left(\omega t + \frac{x}{\tau} \pi \right) \right\} \left\{ \frac{1}{2} - \frac{\cos 2\alpha}{2} \right\} \right.$$

$$\left. - \left\{ \frac{\sin 2 \alpha}{2} \right\} \left\{ \sin \left(\omega t + \frac{x}{\tau} \pi \right) \right\} + \left\{ \cos \left(\omega t - \frac{x}{\tau} \pi \right) \right\} \right] \quad (55)$$

or approximately,

$$H_1 = H_m \left[\cos \left(\omega t - \frac{x}{\tau} \pi \right) - \left(\frac{\sin 2 \alpha}{2} \right) \left(\sin \left(\omega t + \frac{x}{\tau} \pi \right) \right) \right] \quad (56)$$

= regular fundamental (space) wave gliding forward + a second wave due to lack of quadrature, and gliding backwards.

Further it can be shown, as it was explained in Note B, that with ordinary non-sinusoidally distributed windings, supplied with alternating current of fundamental frequency, (with respect to time) there will be a 3rd space harmonic of the form, $H_3 = H_3' + H_3''$,

$$H_3 = H_o \left[\{\cos (\omega t)\} \left\{ \cos \left(\frac{3 x}{\tau} \pi \right) \right] \right.$$

$$\left. + \left\{ \cos \left(\omega t + \frac{\pi}{2} - \alpha \right) \right\} \left\{ \cos \left(\frac{3 x}{\tau} \pi + \frac{3 \pi}{2} - 3 \alpha \right) \right\} \right] \quad (57)$$

Translating (57) into words we shall find that it simply represents the field intensity due to a current of fundamental pulsation ω flowing through the 3rd harmonic of a non-sinusoidal winding. Simplifying this equation, we get,

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$$H_3 = H_e \Big[\left\{\cos\left(\omega t + \frac{3x}{x}\pi\right)\right\} \left\{\left(\frac{1}{2} + \frac{\cos 4\alpha}{2}\right)\right\}$$

$$+ \left\{\cos\left(\omega t - \frac{3x}{\tau}\pi\right)\right\} \left\{\left(\frac{1}{2} - \frac{\cos 2\alpha}{2}\right)\right\}$$

$$+ \left\{\sin\left(\omega t + \frac{3x}{\tau}\pi\right)\right\} \left\{\left(\frac{\sin 4\alpha}{2}\right)\right\}$$

$$- \left\{\sin\left(\omega t - \frac{3x}{\tau}\pi\right)\right\} \left\{\frac{\sin 2\alpha}{2}\right\} \Big] \tag{58}$$

or approximately,

$$H_3 = H_e \Big[\cos\left(\omega t + \frac{3x}{\tau}\pi\right)$$

$$+ \left(\sin\left(\omega t + \frac{3x}{\tau}\pi\right)\right)\left(\frac{1}{2}\sin 4\alpha\right)$$

$$+ \left(\sin\left(\omega t - \frac{3x}{\tau}\pi\right)\right)\left(\frac{1}{2}\sin 2\alpha\right)\Big] \tag{59}$$

$= H_{3R} + H_3^{\alpha} + H_{31}^{\alpha}$

= regular third space harmonic travelling backward at a speed $2f\tau/3$. + a second wave due to lack of quadrature and in quadrature to the first and travelling in the same direction and at the same speed as the first. + a third wave due to lack of quadrature and in phase with the second but travelling forwards, at the same speed as the first two.

If desired the third wave in (59), H_{31}^{α}, may be neglected, and H_3 be taken equal to H_{3R} and H_3^{α}.

Exactly as before for the 5th space harmonic, due to a sine wave of current of fundamental pulsation ($\omega = 2\pi f$), we have,

$$H_5 = H_e \Big[\{\cos(\omega t)\} \left\{\cos\left(\frac{5x}{\tau}\pi\right)\right\}$$

$$+ \left\{\cos\left(\omega t + \frac{\pi}{2} - \alpha\right)\right\} \left\{\cos\left(\frac{5x}{\tau}\pi + \frac{5\pi}{2} - 5\alpha\right)\right\}\Big] \tag{60}$$

Simplifying this, we find,

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$$H_5 = H_a \Big[\left\{ \cos\left(\omega t + \frac{5x}{\tau}\pi\right) \right\} \left\{ \frac{1}{2} - \frac{\cos 6\alpha}{2} \right\}$$

$$+ \left\{ \cos\left(\omega t - \frac{5x}{\tau}\pi\right) \right\} \left\{ \frac{1}{2} + \frac{\cos 4\alpha}{2} \right\}$$

$$- \left\{ \sin\left(\omega t + \frac{5x}{\tau}\pi\right) \right\} \left\{ \frac{1}{2} \cdot \sin 6\alpha \right\}$$

$$- \left\{ \sin\left(\omega t - \frac{5x}{\tau}\pi\right) \right\} \left\{ \frac{1}{2} \cdot \sin 4\alpha \right\} \Big] \qquad (61)$$

or approximately,

$$H_5 = H_a \Big[\cos\left(\omega t - \frac{5x}{\tau}\pi\right)$$

$$- \left(\sin\left(\omega t + \frac{5x}{\tau}\pi\right) \right) \frac{1}{2} \cdot \sin 6\alpha$$

$$- \left(\sin\left(\omega t - \frac{5x}{\tau}\pi\right) \right) \frac{1}{2} \cdot \sin 4\alpha \Big] \qquad (62)$$

$= H_{5R} + H_5^{\alpha}\ H_{51}^{\alpha}$

= a regular fifth space harmonic, travelling forward at a speed $2f\tau/5$. + a second wave in quadrature to the first and travelling backward at the same speed. + a third wave in quadrature to the first and gliding forward at the same speed.

As in (59) H_5 may be taken equal to $(H_{5R} + H_5^{\alpha})$ and the third wave be neglected. Similar expressions for other space harmonics may be derived by exactly the same procedure. Furthermore, in all the above we have assumed the current to be of the form $I_m \cos \omega t$. If the current contains higher time harmonics the resultant armature reaction may be found in a similar manner.

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DISCUSSION ON "SUSTAINED SHORT-CIRCUIT PHENOMENA AND FLUX DISTRIBUTION OF SALIENT-POLE ALTERNATORS" (DIAMANT), ATLANTIC CITY, N. J., JUNE 28, 1918.

F. D. Newbury: To a considerable extent, this paper appears to cover the same ground as the paper by A. E. Clayton on "Wave Shapes of A-C. Generators with Steady Short-Circuit Conditions," presented before the British Institution in 1915. The latter paper has an important advantage in that it shows the current waves and flux distributions of a number of different generators having a considerable range in size and of commercial design. This English paper treats this complex subject in a very clear and simple manner by showing the close correspondence between theoretical curves derived by graphical methods and the oscillograms.

The greater part of Prof. Diamant's paper is concerned with the results from a single 45-kv-a. generator, whose design proportions are not given. In considering these results, it cannot be too strongly emphasized that the significance of the various curves depends entirely upon the detail design proportions of the generator, and the location and spread of the search coils. For example, the double peaks in the curves of Fig. 11 are caused, as the author points out, simply by the reduction of flux density in the space between poles, and thus indicate a very familiar fact by an unfamiliar method.

It is dangerous to draw general conclusions from the results obtained from a single generator. I have in mind the results obtained from a 1500-kv-a., 12,000-volt synchronous motor, having very wide armature slots in comparison with the air gap. The ratio between armature slot opening and air gap was something over 3. There was a very pronounced ripple in both the field form and in the no-voltage e. m. f. wave form. A search coil of one tooth pitch indicated curves very similar to those shown by the author in Fig. 11. While there were only six slots per pole, the wave forms indicated a much higher frequency disturbance and analysis of the e. m. f. wave showed a large twenty-third harmonic. Further investigation of the design developed the fact that partially closed damper slots in the rotor had a pitch only slightly greater than half the slot pitch of the stator, and this conjunction of stator and rotor slots was the cause of the voltage pulsation. This conclusion was confirmed later by building another motor using the same stator punchings, but with a larger tooth pitch for the damper winding, in which the wave form had a relatively small ripple, in spite of the large ratio between stator slot opening and air gap. While this example illustrates a well-known cause of wave-shape distortion, it nevertheless emphasizes the point that the higher harmonics sometimes found in commercial generators are due to a variety of obscure causes, and the search coil e. m. f. waves shown in the paper may or may not have general significance.

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There are a number of specific points in the paper to which attention should be called.

It is stated that the subject of tooth ripples is of great practical importance. Tooth ripples are important in synchronous generators and motors only in connection with questions of telephone interference, and then only in connection with salient-pole alternators, in which air gaps are inherently small. One is apt to obtain an exaggerated idea of the importance of these variations in air gap reduction from this and other statements made by the author. For example, in Fig. 9 the variations in air gap flux were obtained with the rotor at rest, or moving slowly. Under these conditions, much greater variations are obtained than are obtained during normal operation when eddy currents very largely reduce the flux variations that tend to exist. In Fig. 9, the flux opposite a slot is 45 per cent of the flux opposite a tooth. In Fig. 7, obtained presumably from the same generator, this ratio for coil No. 1 is 76 per cent. In generators of commercial design, the ripples rarely, if ever, produce greater variations than those shown in Fig. 7.

Throughout the discussion of armature reaction, it is assumed that the space distribution of m. m. f. in a single concentrated armature coil is sinusoidal. At any instant of time, the space distribution of m. m. f. and flux, assuming uniform reluctance, is rectangular, as illustrated in Fig. 18. At equal intervals of time, the m. m. f. and flux may be represented by a series of rectangles, such as Fig. 18, the ordinates of which vary sinusoidally. This is important, because it means that the space distribution of the armature reaction flux depends on the space distribution of the armature coils, and the latter should be taken into account in all calculations concerning armature reaction.

This assumption of sine-shaped armature reaction flux, and the assumption recorded that the field form of most modern machines approaches the sine shape, leads to the statement that the resultant of the main field and armature field under sustained short-circuit conditions must also be approximately sine shaped. These assumptions are immediately shown to be false, by the statement that "we find indications of an extremely distorted flux distribution." The reason for this distortion is explained by Fig. 32, taken from Clayton's British paper, in which the main flux is assumed to be rectangular, and the armature flux a segment of a sine curve. It seems hardly worth while to build up a theory upon assumptions that are immediately shown to be wide of the fact.

In general, harmonics in the distribution curve of flux have a considerably reduced effect on the voltage generated in the armature winding, because the voltages generated by any particular harmonic of the flux distribution curve are in most cases (except the third and multiples) opposed in different conductors connected in series. Furthermore, the current on short circuit

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is still closer to a sine wave on account of the increasing reactance of the winding for the higher harmonics. Therefore, while the flux distribution on short circuit, as shown in the paper, varies widely from the flux distribution at normal voltage and no load, the generated voltage on short circuit and the current resulting therefrom, depart less and less from the sine form.

One matter of practical importance in connection with the sustained short-circuit operation is this distortion of the flux, due to the relatively weak field excitation under this condition.

This matter has been quite completely treated in Clayton's paper previously referred to, and is moreover quite simply treated graphically, even when actual flux distributions are taken into account. One important result of this distorted flux is the increase in losses under short-circuit conditions in some generators. The short-circuit losses of large two and four-pole turbo-generators, when operated on short circuit, are very much larger than can be accounted for by the losses in the armature winding and the core loss corresponding to the internal voltage. The departure of the flux distribution from the sine increases rapidly as the ratio of field to armature m. m. f. decreases, and with the ratio of unity or less, now commonly used, it is quite probable that the large short-circuit losses in these machines are largely due to the flux distortion produced by the armature reaction. This is a matter of considerable practical importance, as the present Institute Standardization Rules require that the total short-circuit losses be included in the loss existing under normal operating conditions. There is opportunity here for experimental and analytical work that will enable us to more completely determine the source of loss under the condition of sustained short circuit.

R. E. Doherty: I wish to agree with practically all that Mr. Newbury has said in regard to this paper, but there is one point which I think he has put a little too strong. I think it is not Mr. Diamant's intention to give quantitative results to be used as a guide in designing the machines, but rather to give a qualitative explanation of some of the curious wave irregularities that we all recognize as existing in sustained short-circuits. Of this he has given the very clear explanation that while under sustained short-circuit the fundamental is practically annihilated, the harmonics are not very much affected and, therefore, the relative importance of the harmonics as compared to the fundamental has been increased hundreds of times in some cases. This produces those curious waves shown on the oscillogram of coil voltage at sustained short circuit.

William F. Dawson: In the average modern turbo-alternator the field form is trapezoidal until saturation is reached (in the field teeth nearest the pole) when the corners of the trapesoid are rounded off. The trapezoid is a close approach to a sine wave and is more so when rounded at the corners by saturation.

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With a satisfactory flux wave as the foundation, conditions are further improved by a generous number of slots per phase and pole, and by the practical elimination of the third harmonic by making the armature coil pitch approximately two thirds, the resulting e. m. f. and current waves at the armature terminal are very close to true sine waves at all loads. Considerations of ventilation, regulation and the minimizing of core loss insure such large air gaps as compared with slot widths that while tooth ripples still exist their amplitude is too small to create any objectionable disturbance even under sustained short circuit.

C. M. Davis: I wish to emphasize what Mr. Doherty has said, that this paper presents the results only of tests upon a particular machine, and it is not the author's intention in any way to imply that the case mentioned is a general one. He is merely seeking a way of explaining, to his own satisfaction, some of the actions which are not evident on first examination.

N. S. Diamant: In answer to Mr. Newbury's remarks, may I say that I am rather disappointed in that I was expecting to hear something worth while and profitable considering his experience which I am sure is very wide. I was expecting to have him tell us something about the more complex aspects of the subject, which are carefully taken up in my paper.

As Mr. Newbury does not seem to be sure to what extent my paper and that of A. E. Clayton cover the same ground, I may add that reference is made to A. E. Clayton's 1916 paper on 1181 and 1182. There, in a general way Clayton's explanation is given, and a further note made that his paper; namely, Clayton's, is the first one, as far as I am aware, to throw any light on this complex subject. As explained in my introduction, I had started this work before Clayton's paper appeared. The explanation and theory of Clayton, which is graphical, is very good; so good in fact that I have included it in my paper. My explanation of the same phenomenon is a little different as may be seen by referring to Figs. 24, 24A, 25 and 31, also oscillograms Figs. 23, 27 etc., and Tables VII, VIII and XI.

Furthermore, I have taken questions of flux distribution under load and no-load conditions as well as under sustained short circuit. It is disappointing that Mr. Newbury, with his wide experience, doesn't tell us more definitely in regard to several points, at least twelve of which are very specifically referred to in the abstract and very carefully treated, in the paper itself. For example, Mr. Newbury states that the "paper is concerned with results from a single 45-kv-a. generator", and that it is dangerous to draw general conclusions from the results obtained from a single machine. This may be true enough but it would be very much more useful to me and to the reader for Mr. Newbury, instead of making these general remarks which are of no practical value, to tell us more specifically just what points in the paper he thinks could be generalized and what points should not. Frankly, it is not quite clear to me just what is the

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relation between the example he gives about a 1500-kv-a. machine and the danger of generalizing, unless it be to show that it is a rather extravagant way of investigating flux pulsations, "by building another motor using the same stator punchings but with a larger tooth-pitch for the damper winding." To be more specific, may I ask Mr. Newbury to tell us what he thinks is the origin and cause of to-and-fro flux oscillations and pulsations. From his experience is the general theory given by Worall and referred to in my paper acceptable to him. What does Mr. Newbury think about:

1. My method of determining experimentally the armature reaction by subtracting no-load and sustained short-circuit exploring coil voltages, as given in Figs. 23, 27, etc.

2. What does he think is the explanation of the fact that the third harmonic seems to increase with increasing excitation as given in Table IX.

3. What does Mr. Newbury think about my experimental and mathematical analysis of armature reaction of imperfect two-phase alternator as given in Fig. 4, Fig. 35 and the supplementary note C page 1199.

(A tentative explanation of this is referred to in foot note 12.)

I don't quite agree with his statement that "tooth ripples are important only in connection with questions of telephone interference, and then only in connection with salient-pole alternators." Is not the question of tooth ripples and higher harmonics important in connection with iron losses and sometimes dielectric losses also. However, I may add that my paper did not attempt to state just how important higher harmonics may become in certain cases, or of how little importance they may be in other cases. The object is more to consider the fundamental theory of the origin and production of ripples in m. m. f. curves, in flux waves and induced voltages, and I, for one, will appreciate it if I could have Mr. Newbury's ideas on the subject.

Mr. Newbury states that the assumption of sine-shaped armation reaction flux is made in the paper. This is not quite so. As stated on p. 1163, the armature current under short-circuit conditions is assumed to be sinusoidal and then very clear distinction is made as to whether the m. m. f. will be sinusoidal or not and whether the flux, due to the m. m. f. will be sine-shaped or not. If I am not mistaken, in most discussions this distinction, namely, whether a sinusoidal m. m. f. may or may not give a sinusoidal flux, is not quite as clearly adhered to as it is in the paper.

Although outside of the scope of the paper, in connection with the "critical resume conclusions" attention is called to the practical bearing of the results of the paper on standard methods of testing alternators by means of cyclic heat run tests; short-circuit load loss tests, phase-shifting load tests etc. It will be interesting to hear something in regard to this.

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There is little to add to Mr. Doherty's discussion except that as in Mr. Newbury's case, considering his wide experience covering a great many types and sizes of machines, it will be of considerable interest to the profession to have his views on the many different specific points in connection with flux distribution that are taken up and at least tentatively explained in the paper.

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REACTANCE OF SYNCHRONOUS MACHINES AND ITS APPLICATIONS

BY R. E. DOHERTY AND O. E. SHIRLEY

Abstract of Paper

Part I treats of the calculation and application of the armature self-inductive reactance of synchronous machines. A short, reliable method is given in the form of curves, Figs. 20A, B, C, making the calculation from design sheet data a matter of a few minutes. Table I shows a comparison of calculated and test values (obtained from saturation and synchronous impedance curves) for 138 machines, ranging from high-speed turbine generators to the low-speed engine type.

Three points were brought out during the investigation:

(1) That in polyphase machines, the armature self-inductive reactance, just as the armature reaction, is a polyphase, not a single-phase phenomenon, and therefore the mutual induction of phases in a three-phase machine increases the effective self-induction of each phase by approximately 50 per cent over the single phase value, while in two-phase machines, in which the mutual induction of phases is zero, the effective self-induction of the phase is the same for two-phase or single-phase operation.

(2) That the variation of armature reactance during the cycle, due to salient-pole construction, is practically eliminated in Y-connected, three-phase machines for the reason that the variation, consisting almost entirely of a third harmonic, is cancelled in such machines. This leaves, in effect, a uniform reluctance for the leakage flux emanating from the tooth tips.

(3) That in the familiar method of obtaining the armature self-induction from the saturation and synchronous impedance curves (*i.e.*, by subtracting the armature reaction, that is, the demagnetizing ampere turns of normal current under sustained short circuit, from the corresponding field ampere turns), a very large error in this test value of self-induction may occur, if, as is usually done, the armature reaction is calculated on sine wave assumptions. A set of curves shown in Fig. 20, which are plotted from results derived in Appendices A and B, give values of the correction factor which applies to calculations based on sine wave.

An approximate, but convenient, method of applying the armature reactance in the calculation of field excitation under load is given in Appendix C.

In Part II it is shown that the initial short-circuit current of synchronous machines is determined not only by the armature self-inductive reactance, as is often assumed, but also by the field self-inductive reactance. Neglecting the field reactance in calculation may give a calculated short-circuit current 50 per cent or more, too high. A formula is derived for calculating the field reactance which, added to the armature reactance, gives the total which determines the initial short-circuit current and which it is proposed should be called, as previously recommended by

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other authors, *transient reactance*. Table IV shows a comparison on eleven machines of the actual short-circuit currents as determined by oscillograph, and as calculated by methods proposed in the paper.

The calculations of *transient reactance* apply strictly to salient laminated pole alternators without amortisseur winding. However, such experience as is recorded in Table IV, if analyzed in connection with theoretical considerations, affords a basis for estimating the *transient reactance* of turbine generators with massive steel rotors and of salient-pole machines with amortisseur windings. These points are summed up in "Summary and Conclusions".

An attempt is made to describe the apparently complicated physical phenomena of sudden short circuits in terms as free as possible from mathematics. One interesting and important point which the authors establish from the physical interpretation of the problem is that there is a very significant rise in flux at the bottom of the pole at short circuit. This may be (if the rotor is entirely laminated) 30 or 40 per cent of normal flux, and apparently explains why, in machines with solid steel rotor rims, the field attenuation factor, α_f, changes radically after the first few cycles after short circuit.

Introduction

As the march of time obliges designers to become ever more efficient in the use of materials, it becomes more and more necessary that they should extend design calculations to further completeness. The self-inductive reactance of synchronous machines stands in very important relation to this problem. In the predetermination of field excitation, of voltage regulation, and of the initial short-circuit current, the reactance is, of course a very essential factor.

Three years ago the authors undertook a survey of the work which had been done on the calculation of armature reactance in the hope of finding, if not in any particular existing formula, at least in a combination of different ones, an expression for reactance that would give reliable calculations—reliable, we mean when applied to the entire range of machines as built by one of the large manufacturing companies. The result was probably very much the same as that which others have obtained who may have made a similar attempt. Particular expressions applied well to particular classes of machines, but none was found, and no combination of existing expressions was found, that would apply generally. Moreover, on account of the many factors in the problem, the equation for reactance is a very clumsy thing to handle—too clumsy, indeed, to make practical the calculation of reactance on each new, proposed design. It is nevertheless desirable to have this calculation. By methods described later an expression was finally arrived at which does apply generally

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to all classes of machines from two-pole turbine-generators to 136-pole water-wheel driven generators, of any voltage and frequency, three-phase, two-phase or single-phase, and the results are put in convenient shape for the designer's use, thereby making the calculation for each machine a practical matter. A tabulated comparison of calculated and test results (taken from ordinary commercial tests for open and short-circuit characteristics) is shown in Table I.

The importance of predetermining the initial short-circuit current of synchronous machines need not be dwelled upon. Mechanical stress involved, the rupturing capacity of circuit breakers, heating, etc., all make it a matter of supreme importance. The theory underlying short circuits has been thoroughly worked out by Steinmetz, Berg and Bouchert; and Diamant[1] has extended the mathematics involved and investigated the possibility of determining experimentally the attenuation factors, r/L, which determine the rate of decrease of the transients. That is, the complete equation for the instantaneous values of short-circuit current has been available for some time. The difficulty in its use has been that the factors involved could not be predetermined, and, to some extent, there has been uncertainty regarding what constitutes the reactance which determines the short-circuit current. The authors will show that this reactance includes the field self-induction as well as that of the armature, and will derive a formula by which the total reactance may be calculated. To the author's knowledge this has not been previously worked out. Table IV gives a comparison of calculated and actual values of short-circuit current.

In the process of obtaining the above results, another closely related problem was incidentally solved, namely, the effect of the harmonics in the no-load flux wave upon the field excitation required for normal voltage and also upon the armature reaction. This is given in Appendices A and B.

The object of the paper, therefore, is (1) to present a reliable, general formula for the calculation of self-inductive armature reactance of synchronous machines. (Given in Part I).

(2) To give a simplification of the general equation, permitted by standard construction, accompanied by working curves making it possible to calculate the per cent armature reactance from the design sheet in less than five minutes. (Given in Part I).

1. TRANS., A. I. E. E. 1915, Vol. XXXIV, p. 2237.

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TABLE I

COMPARISON OF CALCULATED AND TEST VALUES AT ARMATURE SELF-INDUCTIVE REACTANCE

SALIENT POLE MACHINES							
	Kv-a.	ϕ	Freq. ~	R.p.m.	Voltage	x_{pa}	
						Calc.	Test
1	33	3	60	1200	440	13.4	8.06
2	50	3	60	300	2300	16.0	13.9
3	125	3	62½	750	2300	13.5	11.0
4	180	3	60	900	220	10.9	12.3
5	240	3	60	150	440	17.1	15.2
6	265	3	25	150	400	28.1	34.8
7	300	3	60	1200	4000/2300	15.6	13.0
8	320	3	50	200	5000	12.5	15.5
9	345	3	60	720	4000	13.1	10.7
10	370	3	50	187½	500	14.5	20.0
11	375	3	60	150	2300	14.7	13.2
12	375	3	60	150	3100	14.4	11.4
13	400	3	25	300	2300	15.2	21.0
14	400	3	60	300	4000	27.5	30.1
15	400	3	60	200	600	18.9	19.3
16	400	3	60	150	6600	24.6	21.4
17	450	3	25	750	13200	8.8	3.1
18	475	3	50	214	440	16.4	15.3
19	500	3	60	600	2300	7.6	7.5
20	500	3	60	150	2300	16.0	13.7
21	500	3	60	100	240	14.4	13.9
22	540	3	60	164	6600	31.0	27.3
23	560	3	25	750	480	6.6	5.8
24	560	3	60	720	13200	15.1	12.0
25	560	2	60	720	2400	18.6	23.2
26	570	3	25	750	13200	9.0	8.9
27	580	3	60	720	2300	13.4	15.6
28	585	3	60	90	2000	14.4	15.6
29	605	3	60	150	2200	16.8	18.1
30	625	3	60	150	2300	10.7	10.0
31	625	3	60	100	480	21.1	15.8
32	675	3	60	90	6600	36.8	33.5
33	700	3	25	750	2200	10.1	6.00
34	700	3	25	750	13200	13.2	16
35	700	3	25	500	6600	11.4	7.3
36	700	3	60	900	4000	13.8	12.7
37	700	3	40	300	600	10.5	10.2
38	750	3	60	300	600	22.5	23.6
39	750	3	60	150	2300	18.4	19.5
40	780	3	60	120	2300	21.7	21.0
41	800	3	60	90	2300	41.9	44.0
42	850	3	25	500	6600	5.6	8.4
43	900	3	60	360	600	20.5	18.6
44	900	3	60	150	2300	22.1	18.2
45	1000	3	60	450	2300	22.5	19.8
46	1000	3	60	200	600	21.2	16.8
47	1200	3	25	500	4400	16.3	16.9
48	1250	3	25	750	13200	10.5	10.5
49	1250	3	50	500	5500	12.3	12.8
50	1250	3	60	300	2300	12.0	10.5
51	1250	2	62.5	375	5400	12.1	10.2

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TABLE I. (*Continued*)

COMPARISON OF CALCULATED AND TEST VALUES AT ARMATURE SELF-INDUCTIVE REACTANCE

SALIENT POLE MACHINES							
	Kv-a.	ϕ	Freq. ~	R.p.m.	Voltage	x_{pa}	
						Calc.	Test
52	1350	3	25	375	6600	11.3	12.8
53	1400	3	25	500	2200	10.0	8.9
54	1400	3	25	375	13200	11.5	13.8
55	1400	3	60	514	13200	10.8	10.9
56	1400	3	60	514	13200	12.9	11.9
57	1420	3	25	300	2300	11.0	5.8
58	1425	3	60	240	2200	21.1	22.0
59	1450	3	60	720	2300	11.3	8.6
60	1500	3	25	500	2300	7.9	10.2
61	1500	3	60	514	2200	14.5	15.0
62	1500	2	62.5	750	2300	14.3	20.0
63	1625	3	25	750	3300	10.5	10.5
64	1800	3	60	300	2300	10.6	12.6
65	2000	3	60	720	2200	13.8	14.0
66	2000	3	60	100	2300	8.4	8.0
67	2000	2	60	360	2400	9.6	7.2
68	2100	3	60	514	4600	15.7	15.7
69	2125	3	40	200	600	15.3	14.1
70	2200	3	25	750	3300	9.1	15.0
71	2200	3	25	375	2300	9.1	9.7
72	2250	3	25	214	2300	5.9	4.4
73	2250	3	60	514	6000	15.3	16.9
74	2250	3	60	450	3450	17.8	17.1
75	2500	3	25	250	2200	5.8	4.0
76	2800	3	25	300	2300	5.5	4.9
77	2000	3	60	514	13200	18.9	19.2
78	2800	3	60	450	4000	15.0	18.0
79	3000	3	50	500	6600	13.1	17.4
80	3000	3	60	450	2300	8.25	11.0
81	3125	3	25	750	14000	16.1	15.8
82	3500	3	50	500	11500	19.2	20.0
83	4000	3	60	600	2300	13.8	11.2
84	5000	3	25	300	2300	8.9	10.2
85	5000	3	25	88.3	6600	13.9	13.8
86	5650	3	60	360	6600	19.8	21.3
87	6000	3	50	500	3450	7.65	6.1
88	6000	3	50	500	16500	21.6	26.5
89	6600	3	25	375	14000	9.5	11.0
90	6250	3	60	600	6600	16.8	13.9
91	6750	3	50	600	11000	19.3	25.6
92	7050	3	40	400	600	6.6	8.5
93	7050	3	50	375	6600	13.9	13.0
94	7500	3	60	600	13800	13.3	16.9
95	8750	3	50	500	600	15.1	19.4
96	8750	3	50	500	6600	6.9	7.1
97	9000	3	25	57.7	11000	13.9	16.4
98	10000	3	60	514	6600	6.81	7.9
99	10000	3	62.5	55.6	6600	20.4	19.4
100	12000	3	25	375	6600	13.2	12.6
101	15000	3	50	375	6600	10.2	12.3

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TABLE I. (*Continued*)

TURBINE ALTERNATORS							
	Kv-a.	ϕ	Freq. ~	R.p.m.	Voltage	x_{pa} Calc.	x_{pa} Test
1	3750	3	60	3600	2300	9	14.5
2	4375	3	60	1800	2400	5.9	2.4
3	4375	3	60	1800	4500	6.7	3.9
4	4375	3	60	1800	2300	7.7	8.2
5	4375	2	60	1800	2300	6.25	5.1
6	5000	3	25	1500	5500	10.6	10.2
7	5000	3	25	1500	6600	6.4	6.8
8	5000	3	25	1500	6600	7.3	5.5
9	5000	3	60	3600	2300	9.0	8.9
10	5625	3	25	1500	2300	6.1	3.3
11	6250	3	25	1500	6600	10.1	10.6
12	6250	3	60	3600	2300	8.6	12.2
13	6250	3	60	1800	6600	8.6	7.8
14	7500	3	25	1500	2300	7.1	6.9
15	7500	3	25	1500	6600	10.5	7.1
16	7500	3	25	1500	6600	10.5	11.8
17	7500	3	60	1800	4000/2300	9.4	5.0
18	7500	3	60	1800	4160/2300	9.1	8.2
19	7500	3	60	1800	2300	7.6	9.1
20	7812	3	25	1500	2300	7.7	7.8
21	7812	3	60	1800	2300	10.1	9.5
22	7812	3	60	1800	4000/2300	10.6	8.6
23	7812	3	60	1800	4000/2300	11.8	10.9
24	9375	3	25	1500	13200	8.9	9.3
25	9375	3	60	1800	4000/2300	8.0	8.6
26	9375	3	60	1800	5000	11.0	9.2
27	10000	3	25	1500	6600	9.9	9.8
28	10000	3	40	2400	10000	12.5	13.8
29	10000	3	60	1800	6600	14.2	9.2
30	12500	3	25	1500	11000	8.4	8.2
31	12500	3	60	1800	4000/2300	9.1	7.4
32	12500	3	60	1800	11000	13.0	17.1
33	12500	3	60	1800	13200	19.2	13.5
34	12500	3	60	1800	2300	10.2	8.6
35	18333	2	37.3	2240	4469	6.3	7.4
36	18750	3	60	1800	2300	13.8	12.0
37	25000	3	60	1800	13200	14.4	15.4

(3) To derive a formula for calculating the field self-induction which combines with the armature self-induction to determine the initial short-circuit current; and further to describe in part the physical phenomena which exist in the machine at short circuit. (Given in Part II).

(4) To derive a formula for calculating armature reaction, taking into account the effect of field distribution. (It is necessary to determine the armature reaction accurately, before the armature self-induction can be determined from standard open and short-circuit tests). (Given in Appendix B).

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(5) To show the effect of flux distribution (*i.e.* the effect of the harmonics in the no-load flux wave) on the field excitation. (Given in Appendix A).

(6) To give a simplified method of calculating field excitation which has worked out well in connection with the *per cent* armature reactance. (Given in Appendix C).

PART I.

Armature Reactance

Of all the work which had been done on armature reactance, that of Gray[2] was, in our opinion, the most complete. While it involved certain familiar factors, such as that for the slot reactance and the leakage from tooth tip to tooth tip which have appeared in other reactance formulas, it yet took into account for the first time, we believe, the variation of tooth-tip reactance in salient-pole machines for different positions of the pole, which is very important. Fechheimer introduced a factor to account for the effect of fractional pitch armature coils. The results given in this paper have been obtained by a modification and extension of the work done by Gray, Fechheimer[3] and others. A bibliography which may be of historical interest is included at the end of the paper.

There were three difficulties of fundamental importance which it was necessary to overcome before consistent results were obtained. The first was the influence of the harmonics in the no-load flux wave upon the effective armature magnetomotive force or armature reaction. The armature self-inductive reactance, as obtained from the saturation and synchronous impedance curves (the open-circuit and short-circuit characteristics) involves the difference between the actual field m.m.f. on short circuit and the effective armature m.m.f. for the corresponding current. Since the armature reaction is calculated, this difference must also, to that extent, be considered as calculated. The familiar, approximate formula usually employed in determining effective armature m.m.f. per pole,

$$A = \frac{2.12\ N_q\ I}{K_p\ K_d} \quad \text{(three-phase)}$$

is based upon sinusoidal distribution of flux and armature m.m.f.

2. Gray's Elec. Machine Design, 1st Edition, page 215.
3. Trans. A. I. E. E., Vol. 31, p. 578.

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N_q = series turns per pole per phase
I = amperes per turn
K_p and K_d = coefficients, greater than unity, which take account of the effect of fractional pitch coils and the distribution of coils respectively.

Often the flux distribution is very different from a sine wave. In such cases the actual effective armature m.m.f. may be different by 8 to 10 per cent from the value based on sine-wave assumptions—*i.e.*, as calculated from the above formula. This error in armature m.m.f., of course causes a much larger percentage

FIG. 1—TOOTH-TIP LEAKAGE FLUX

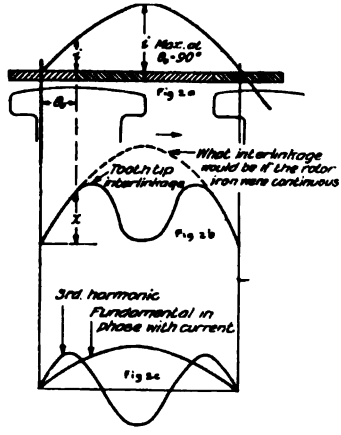

FIG. 2—TOOTH-TIP INTERLINKAGES AT ZERO POWER FACTOR

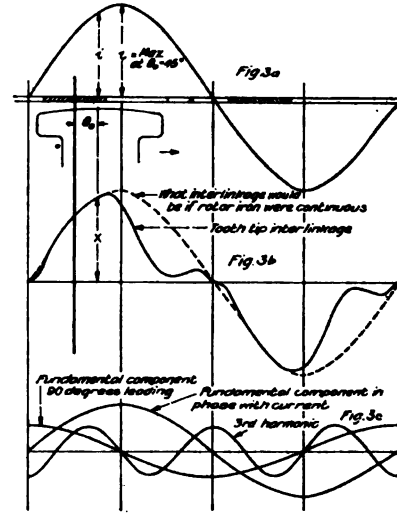

FIG. 3—TOOTH-TIP INTERLINKAGES AT 80 PERCENT POWER FACTOR

error in the difference between field and armature m.m.f's. on short circuit, which difference, correctly determined, represents the armature self-induction. It is therefore obvious that the effect of flux distribution must be taken into account. This has been done by a single factor, K_ϕ, which is derived in Appendix A, and is given in Fig. 20.

The second difficulty was met in accounting correctly for the, variation in the permeance of the "tooth-tip" leakage path for different positions of the pole. Gray's method of accounting for this neglected certain serious factors, as will be shown later. Fig. 1 shows *tooth-tip* leakage of one-phase belt at a time when

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the pole-face iron forms part of the leakage path. It is obvious from Fig 1, that as the pole moves on, a part, and finally all, of the path between tooth-tips will be air. At zero power factor the current in any particular phase group is a maximum when that phase group is half way between poles, when the angle, θ_0, between the center of the phase group and the center of the pole Fig. 2A, is 90 deg.; and is zero when the group is over the middle of the pole, when θ_0 is zero. This produces a curve of leakage interlinkages shown as x in Fig. 2B. The dotted curve indicates what the leakage curve would be if the iron of the rotor were continuous. Now for any other power factor, say $\cos \theta = 0.7$, if the distortion of the magnetic field is neglected, the current will be a maximum when

$$\theta_0 = \theta = 45 \text{ deg.}$$

and zero when $\theta_0 = -45$ deg. But distortion can not be neglected. By shifting the flux toward the trailing tip (in a generator) the position of the pole is relatively advanced, thereby increasing the total angle between the center line of the pole and the position of the phase group when the current is a maximum, to

$$\theta_0 = \theta + \theta_d$$

where θ_d is the distortion angle, shown in Fig. 18. For illustration let the current be a maximum when θ_0 equals 45 deg. Fig. 3A. This would correspond in certain machines to a power factor of about 80 per cent and would produce a curve of leakage interlinkages, x, shown in Fig. 3B. The dotted curve indicates the leakage which would exist if the rotor iron were continuous.

An analysis of the curves of interlinkages shown in Figs. 2B and 3B discloses an interesting and very important fact: that these curves as shown in Figs. 2C and 3C, are made up principally of a fundamental and a large third harmonic, all other harmonics being negligible; and that since in a Y-connected 3-phase machine the third is eliminated, the fundamental alone remains for consideration in such machines—which probably comprise 95 per cent of all that are now being manufactured. And even in two-phase or Δ-connected three-phase machines the effective results are much the same. Experience has been that the short-circuit characteristic and saturation curves show practically the same value of reactance in the same machine

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whether it is connected Y or Δ. The probable explanation of this is that in Δ-connected or two-phase machines the current, especially under short circuit, is not a sine wave, as it is for Y-connected machines, but instead, contains a negative third harmonic, as shown in Fig. 4. This operates to lower the peaks in Fig. 2B, that is, to decrease the third harmonic in the reactance. And although, of course, the effect of the remaining, decreased third harmonic in the reactance voltage is to restrict the flow of current at certain points of the cycle, and therefore produce a current of smaller average value than would exist if the third harmonic were eliminated, nevertheless the form factor of the resulting peaked wave shown in Fig. 4, is correspondingly higher, that is, the ammeter will read correspondingly higher, and therefore show in this case about the same reading for sustained short-circuit current as would be shown in the case of a Y-connected machine. In general, then, it is necessary to consider only the fundamental in the tooth-tip leakage.

The salient pole construction, which is the cause of the large third harmonic mentioned above, is therefore, in effect replaced by a continuous rotor of uniform reluctance, which is greater than that of iron but lower than that of air, and which is different for different power factors, being lower at high power factor. This is obvious from a comparison of Figs. 2B and 3B.

At high power factor, an interesting circumstance arises. In addition to that component of the fundamental of the tooth-tip leakage which is in phase with the current and which therefore produces a reactive voltage lagging behind the current by 90 deg., there is also a cosine term, leading the current by 90 deg., which produces a reactive voltage in phase with the current. This has important significance in the matter of field excitation required under load, as pointed out later.

Gray's value for the tooth-tip factor is based on a root-mean-square of instantaneous values of a curve somewhat similar to that shown in Fig. 2B., the principal difference in the curves being that Fig. 2B., takes account of the distribution of the coils in the phase belt. Being based on the root-mean-square and neglecting the spread of the phase belt, Gray's value for this factor is approximately 100 per cent higher than that given in this paper, and does not take account of the in-phase component of reactive voltage, or of the effect of the distortion of the magnetic field—both of which occur under watt load.

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The third difficulty involved the conception that on a polyphase machine, the self-induction of the armature is a polyphase, not a single-phase phenomenon; and that therefore the reactance per phase of the same machine, connected three phase in one case, two phase in another, would be approximately 1.5 times the single-phase value in the former, 1.0 times the single-phase value in the latter. In other words the magnetic field of self-induction is a rotating polyphase field just as is the field of armature reaction, and the resulting overlapping of phase produces greater leakage interlinkages in any particular phase of a three-phase machine than would exist by the m.m.f. of the phase alone. For instance, in a three-phase machine, when the

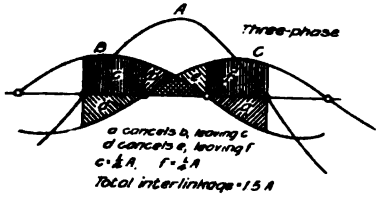

FIG. 5A

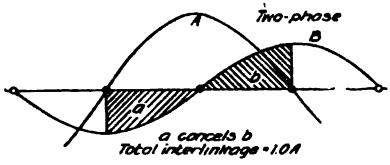

FIG. 5B
MUTUAL INTERLINKAGES—POLYPHASE MACHINE

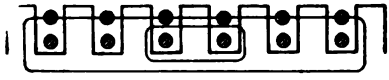

FIG. 6 A

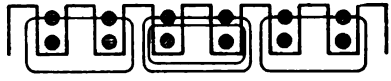

FIG. 6B
SLOT LEAKAGE IN THREE-PHASE MACHINE

current is maximum in phase A, it is one-half maximum in B and C. The increase of interlinkages in A, due to B and C, is shown in Fig. 5A. For two-phase, the current is zero in B when a maximum in A, and moreover they are displaced in space so that the mutual interlinkages must be zero at all times as shown in Fig. 5B.

Unless this fact is taken into account when considering three-phase machines, the calculated value, which is the reactance per leg, will be only two-thirds as large as the test value, unless some of the factors in the reactance equation are overburdened to make up the difference. Then, obviously, the formula would not apply to two-phase machines; it would give calculated results 50 per cent too high. This is just what had happened

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during this investigation. By using Gray's formula modified by Fechheimer's "Short Pitch Factor", calculations checked fairly well on certain classes of three-phase machines, but not on two-phase. The factor of tooth-tip leakage being 100 per cent too high and the other factors being approximately right, made up for neglecting the increase of 50 per cent on the total, and naturally made two-phase calculations too high. By using the proper value of tooth-tip leakage, and allowing for the polyphase action, the equation applies equally well to three-phase, two-phase and single-phase machines.

There is, however, one term of the reactance equation which is not affected by the mutual induction of phases, namely, the slot leakage. Fig. 6B shows the three different phases of a two-slot per pole per phase machine, and the slot leakage which exists when the current is a maximum in the middle group, one-half maximum in the other two. Fig. 6A is equivalent to Fig. 6B, and it is clear that the interlinkages of the middle group have not been increased by the m.m.f. of the other two groups. The same reasoning can be applied to other instants, when the currents have different values, to show that there is no mutual induction. This operates to reduce the ratio of three-phase to single-phase (terminal to neutral) reactance to a value slightly lower than 1.5.

It is not obvious at first that this overlapping of phases occurs in the case of tooth-tip leakage. Consider, however, that the elimination of the third harmonic in this term, in effect produces a path of uniform reluctance (similar to a path of air only having lower reluctance) for the tooth-tip leakage. In other words, the case is similar to that of the end winding whose leakage path is air, or to that of the induction motor rotating main field. And, as in the two latter cases, the polyphase field is greater than that produced by one phase alone by about 50 per cent, as shown in Fig. 5A.

Turning to the question of applying the reactance in the calculation of field excitation required under load, experience has demonstrated that the use of the zero power factor value of reactance (that is, the value determined from open-circuit and short-circuit test) gives reasonably reliable calculation of field current for loads of any power factor. Why this could be true when the reactance at 80 per cent power factor is, by reason of the different pole position when the current is a maximum, about 60 per cent higher than that at zero power factor, was a perplex-

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ing question. It is answered by the appearance of the in-phase component of reactive voltage in the tooth-tip factor. The rI drop is in phase opposition to the current; and the voltage consumed by it is in phase with the current. The $x_1 I$ drop, produced by leakage interlinkages in phase with the current, is 90 deg. behind the current; and the voltage consumed by it is 90 deg. ahead of the current. The $x_2 I$ drop produced by leakage interlinkages 90 deg. ahead of the current, as shown in Fig. 3c, is in phase with the current; and the voltage which it consumes is in phase opposition to the current. Fig. 7 shows these relations for approximately 80 per cent power factor, and makes clear why the internal voltage E_1', determined by the use of the zero power factor reactance, is practically the same as E_1 which is actually required by the much higher reactance, corresponding to 80 per cent power factor, the effect of the latter being mitigated by the in-phase component $x_2 I$. There is a difference in phase between E_1' and E_1 which, by using the zero power factor value of reactance, will give a calculated excitation slightly too low, but for practical purposes the difference is considered negligible.

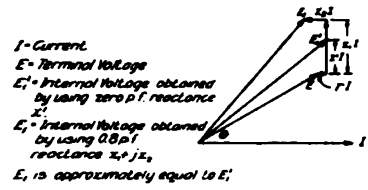

FIG. 7—VOLTAGE DIAGRAM AT 80 PERCENT POWER FACTOR

In the interest of simplicity and convenience in calculation, therefore, it is proposed that for the present at least, the zero power factor value of reactance be used. The method of calculating field excitation which is given in this paper follows that assumption. In the future it may become desirable to make correction for the distortion angle and power factor; hence the problem remains open to this further refinement.

CALCULATION OF ARMATURE REACTANCE

A = effective armature reaction in ampere turns per pole

a = conductors per slot; for a double layer winding a is twice the number of turns per coil.

C = air-gap coefficient based on Carter's fringing coefficient.

c = number of circuits in multiple in armature winding.

C_a = reduction factor for Φ_a at zero power factor, depending on ratio of pole arc to pole pitch.

C_i = reduction factor for Φ_i at zero power factor, depending on ratio of pole arc to pole pitch.

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C_1 = 1.5 C_i for three-phase
= C_i for two-phase.
C_2 = 1.5 C_a for three-phase
= C_a for two-phase.
d_1 = distance from top of copper to bottom of copper in the armature winding Fig. 8.
d_2 = distance from top of copper to armature face Fig. 8.
e = voltage per phase
f = frequency in cycles per second.
g = average air gap.
I_n = normal current per phase in amperes
i = instantaneous value of current in conductor
K = reduction factor for Φ_s Φ_i and Φ_a depending on pitch of armature coils.
K_d = factor for effect of distribution of armature coils in calculation of induced voltage, depending on slots per pole and phase.

TABLE II—VALUES OF K_d

s		1	2	3	4	5	6	7	8up
K_d	Three-phase	1.0	1.035	1.043	1.044	1.045	1.046	1.047	1.047
K_d	Two-phase	1.0	1.083	1.098	1.103	1.106	1.108	1.109	1.11

K_i = factor for Φ_i depending on slots per pole and phase.
K_p = factor for effect of short pitch of armature coils in calculation of induced voltage.

$$K_p = \frac{1}{\text{Sin } (90° \times p)}$$

where p is pitch of coil in per cent of full pole pitch.

K_ϕ = factor for effect of form of flux wave shape in calculation of induced voltage. Values of K_ϕ are given in Fig. 20.
L_e = length of one phase belt of end connections.
l = gross stacked length of armature core (iron + ducts)
N_s = total number of stator coils
n = number of phases.
p = per cent pitch of armature winding (= 0.8 for 80 per cent pitch)
q = number of poles
s = slots per pole and phase.
w_s = width of slot

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w_t = width of tooth at armature face.
For w_s and w_t see Fig. 8.

x_a = reactance in ohms

x_{pa} = armature reactance in per cent.

α = ratio of pole arc to pole pitch

τ = pole pitch at armature core surface.

Φ = flux per pole in maxwells.

Φ_a = effective tooth-tip interlinkages per phase belt per ampere conductor for unit length of armature, when the entire phase belt is between the poles.

ϕ_a = effective instantaneous value of Φ_a.

Φ_e = effective end connection interlinkages per phase belt per ampere conductor for unit length of phase belt of end connections.

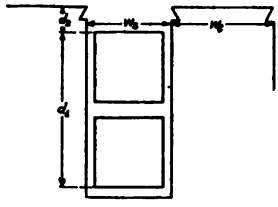

FIG. 8—SLOT DIMENSIONS

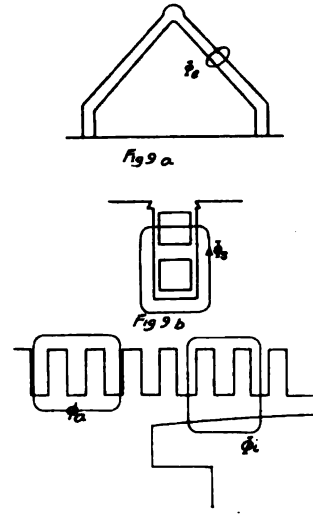

FIG. 9C
ARMATURE LEAKAGE FLUX DIAGRAMS

Φ_i = effective tooth-tip interlinkages per phase belt per ampere conductor for unit length of armature, when the entire phase belt is under a pole.

ϕ_i = effective instantaneous value of Φ_i.

Φ_s = effective slot interlinkages per phase belt per ampere conductor for unit length of armature.

ψ = electrical angle by which the armature coil span is less than 180 deg.

Theory. The armature leakage flux in an alternating-current generator may be divided into three factors.

(1) End connection leakage flux, represented by $\Phi_e L_e$, the effective flux per ampere conductor interlinking one phase belt of end connections. (Fig. 9 A).

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(2) Slot leakage flux represented by Φ_s, the effective flux per ampere conductor through the slot interlinking unit length of one phase belt. (Fig. 9 B).

(3) Tooth-tip leakage flux represented by

(a) Φ_i, the effective flux per ampere conductor, crossing the air gap into the pole face and returning across the gap and interlinking unit length of one phase belt, when the entire phase belt is under the pole. (Fig. 9C.)

(b) Φ_e, the effective flux per ampere conductor emerging from the tooth tips and interlinking unit length of one phase belt when the entire phase belt is between the poles. (Fig. 9c.)

The expression for reactance has been derived by a number of writers, and may be expressed in our notation by the following equation:

$$x = \frac{2\,\pi\,f\,s^2\,a^2\,q\,l}{10^8\,c^2}\,L_0 \tag{1}$$

Where L_0 is the total interlinkages per ampere conductor in the phase per unit length of the armature[4].

The factor $\frac{2\,\pi\,f\,s^2\,a^2 q\,l}{10^8\,c^2}$ will be replaced by M in the following derivation.

End Connection Leakage. The end connection leakage can only be approximated, but the value of $\Phi_e\,L_e$ is obviously between the following two limits.

The first limit assumes that the length of the path around the phase belt varies directly as the width of the belt. See Fig. 9A.

Then

$$\Phi_e = K_1\,\frac{n}{\tau}$$

where K_1 is a constant.

The second limit assumes that the length of the path is independent of the width of the belt.

Then Φ_e would be constant.

Obviously Φ_e can not change directly as $\frac{n}{\tau}$ because the depth of coil is a factor of the length of path; on the other hand, the length of path *must* increase some as τ increases, that is, it can not be independent of τ. Hence the variation of Φ_e must be between the first power of $\frac{n}{\tau}$ and the zero power of $\frac{n}{\tau}$. The

4. Electrical Machine Design, Gray Edition 1913, p. 21.

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analysis of tests for entire armature reactance on over 100 machines, as well as exploring coil tests for end leakage reactance on a 125-kv-a. generator indicates that Φ_e varies approximately as $\sqrt{\frac{n}{\tau}}$ that is

$$\Phi_e = K_2 \sqrt{\frac{n}{\tau}}$$

The length of the phase belt $L_e = p\,\tau\,K_3$.

Then $$\Phi_e\,L_e = K_4 \sqrt{n}\,p\,\sqrt{\tau}$$

For a three-phase double-layer winding, and dimensions in centimeters, $K_4\sqrt{n} = 2.8$ was found to give satisfactory results.

The factor for a two-phase two-layer winding will be reduced in the ratio of $\frac{\sqrt{2}}{\sqrt{3}}$ and also by the factor 0.667 on account of zero mutual induction between phases in this case, and will be 1.6 in round numbers.

The end connection reactance for a double-layer winding will be

$$x = \frac{2.8\,M\,p\,\sqrt{\tau}}{l} \qquad \text{Three-phase} \quad (2)$$

$$= \frac{1.6\,M\,p\,\sqrt{\tau}}{l} \qquad \text{Two-phase} \quad (3)$$

For a single-layer winding the value of $\Phi_e\,L_e$ will be doubled as proposed by Gray.

Slot Leakage. Three-phase coil pitch between 66.66 and 100 per cent. The expression for slot reactance with short-pitch armature winding was derived by Fechheimer[5], and is used with a correction in the number of A and B coils, which is $\frac{N_s\,\psi}{180}$ instead of $\frac{N_s\,\psi}{2 \times 180}$ as given in his paper.

Omitting the factors for the top of the partially closed slot and modifying for our notation, the expression for inductance of open slots becomes

$$L = 0.4\,\pi\,a^2 \left[\left(\frac{19}{96}\,\frac{d_1}{w_s} + \frac{3}{8}\,\frac{d_2}{w_s}\right)\frac{N_s\,\psi}{3 \times 60} + \left(\frac{7}{96}\,\frac{d_1}{w_s} + \frac{3}{8}\,\frac{d_2}{w_s}\right)\frac{N_s\,\psi}{3 \times 60} + \frac{N_s}{3}\left(1 - \frac{\psi}{60}\right)\left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right)\right] \qquad (4)$$

5. Fechheimer "Synchronous Motors" A. I. E. E., Vol. XXXI, 1912, p. 580.

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Simplifying

$$L = 0.4\,\pi\,a^2 N_s \left[\left(1 - \frac{\psi}{320}\right)\frac{d_1}{3\,w_s} + \left(1 - \frac{\psi}{240}\right)\frac{d_2}{w_s}\right] \qquad (5)$$

$$x = M\,\frac{0.4\,\pi}{s}\left[\left(1 - \frac{\psi}{320}\right)\frac{d_1}{3\,w_s} + \left(1 - \frac{\psi}{240}\right)\frac{d_2}{w_s}\right] \qquad (6)$$

The derivation of this expression in the above paper omits the allowance for the out of phase component of the reactive voltage induced in one phase by the half coils of another phase in the same slot. The final expression however is correct as it can readily be seen by reference to Fig. 10, the reactance component of E_2, the voltage induced in phase one by phase two,

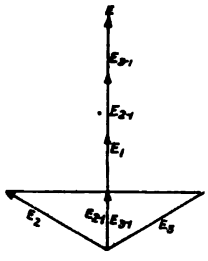

FIG. 10—MUTUAL INDUCTION IN SHORT-PITCH ARMATURE WINDINGS

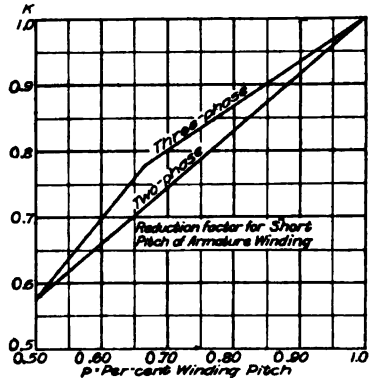

FIG. 11

will be exactly balanced by the reactive component of E_3 the voltage induced in phase one by phase three.

For simplicity and with very little decrease in accuracy, we may replace the two short-pitch factors in equation (6) by an average value K from Fig. 11.

Then $$x = M\,\frac{0.4\,\pi}{s}\,K\left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right) \qquad (7)$$

Pitch below 66⅔ per cent.

Following Fechheimer's method with the correction for number of half coils, the expression for reactance where the pitch is less than 66⅔ per cent will be

$$x = M\,\frac{0.4\,\pi}{s}\left[\left(1.187 - \frac{\psi}{160}\right)\frac{d_1}{3\,w_s} + \left(1.25 - \frac{\psi}{120}\right)\frac{d_2}{w_s}\right] \qquad (8)$$

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Replacing the two factors by an average value K from Fig. 11

$$x = M \frac{0.4\,\pi}{s} K \left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right) \tag{9}$$

Two phase.

Pitch between 50 and 100 per cent.

The expression for two-phase slot reactance with short pitch derived by a similar process is

$$x = M \frac{0.4\,\pi}{s} \left[\left(1 - \frac{\psi}{240}\right)\frac{d_1}{3\,w_s} + \left(1 - \frac{\psi}{180}\right)\frac{d_2}{w_s}\right] \tag{10}$$

Using an average value of reduction factor from Fig. 11

$$x = M \frac{0.4\,\pi}{s} K \left(\frac{d_1}{3\,w_s} + \frac{d_2}{w_s}\right) \tag{11}$$

Tooth-Tip Leakage. The tooth-tip leakage may be most conveniently calculated by dividing it into two components; the effective leakage flux around the part of the phase belt opposite the iron of the pole face (maximum value Φ_i); and the effective leakage flux around the part of the phase belt between the poles (maximum value Φ_a).

The values of Φ_i and Φ_a for a complete phase belt were determined by Gray.[6] His method has been followed for Φ_i except in determining the area of the path across the air gap. The leakage flux will fringe from the teeth in almost exactly the same way as does the no-load flux, hence the area of the path across the air gap is the length of armature times the tooth pitch divided by the air gap coefficient, Fig. 12B.

For one and two slots per pole and phase,

$$\Phi_i = 0.4\,\pi \frac{w_s + w_t}{2\,g\,C} \tag{12}$$

Following Gray's derivation, it will be found that for higher numbers of slots per pole and phase

$$\Phi_i = 0.4\,\pi\,K_i \left(\frac{w_s + w_t}{2\,g\,C}\right) \tag{13}$$

A curve for K_i plotted against slots per pole and phase is given in Fig. 12A.

Gray's expression for Φ_a has been adopted without change,

6. Electrical Machine Design, Gray Edition, 1913, p. 220.

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and simplified by assuming $w_s = w_t$, which gives a definite value of Φ_a for any given value of slots per pole and phase.

Φ_a plotted against slot per pole and phase is shown in Fig. 12.

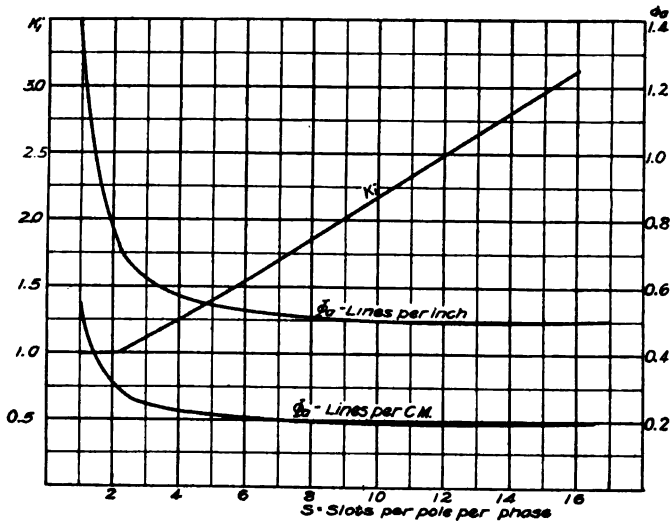

FIG. 12A

Modification of Φ_i and Φ_a by Definite Pole Construction. It should be noted that Φ_i, would be the leakage flux per ampere conductor in the phase belt, if the phase belt were opposite the

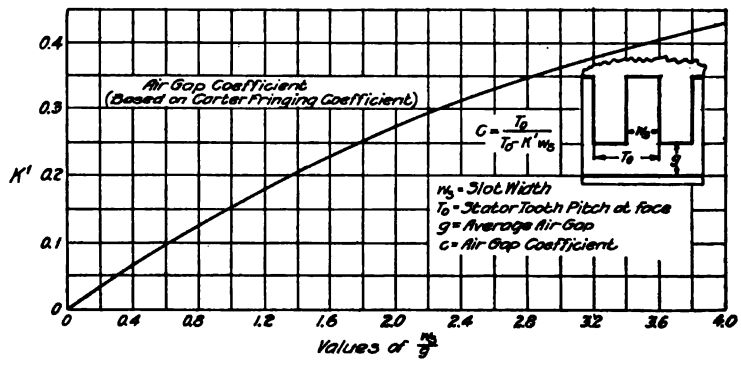

FIG. 12B

pole face iron throughout the cycle. Also Φ_a would be this leakage flux if there were no iron in the tooth-tip leakage path. The actual condition is a path partly iron and partly air and varying

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in proportion of air and iron during the cycle. This condition may be approximated by calculating the interlinkages in the iron part of the path, and also the interlinkages in the air part of the path. The ratio of the former to Φ_i, and of the latter to Φ_a will be reduction factors to be applied to Φ_i and Φ_a in calculating the reactances due to tooth-tip leakage. The actual tooth-tip leakage will be obtained by adding the two parts as calculated in this way.

The general theory of the variation of Φ_i and Φ_a at zero power factor has been discussed in the first part of this paper, and it will now be applied to the actual calculation.

The instantaneous values of leakage flux $\phi_i\, i$ vary from zero when the current is zero (Fig. 13A position *a*) to a maximum approximately where the pole begins to leave the phase belt (Fig. 13A position *b*) and then decreases to zero where the phase

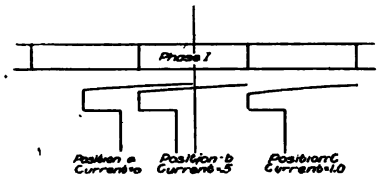

FIG. 13A—DIAGRAM OF POLE POSITIONS

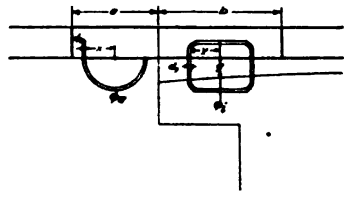

FIG. 13B—TOOTH-TIP LEAKAGE

belt is entirely between the poles. (Fig. 13A position *c*). The instantaneous value of leakage flux $\phi_a\, i$ is zero when the current is zero, and remains zero until the pole begins to leave the phase belt, after which it increases gradually to a maximum where the current is a maximum at 90 deg. Obviously the curves for $\phi_i\, i$ and $\phi_a\, i$ are symmetrical about the 90 deg. axis. Fig. 14.

The values of $\phi_i\, i$ and $\phi_a\, i$ for complete distribution of the armature winding were calculated by the following methods.

Tooth-tip leakage $\phi_a\, i$

For one ampere in the phase belt and unit length of phase belt the m. m. f. at x (See Fig. 13B) is

$$\text{m. m. f.} = \frac{2x}{\frac{\tau}{n}} = \frac{2nx}{\tau} \tag{14}$$

where n = number of phases

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and τ = pole pitch

$$d\,\phi = 0.4\,\pi\,\frac{2\,n\,x}{\tau}\;\frac{d\,x}{\pi\,x} \tag{15}$$

$$= 0.4\,\pi\,\frac{2\,n}{\pi\,\tau}\,d\,x \tag{16}$$

Let effective interlinkages due to $d\,\phi = d\,\phi_a$

Then
$$d\,\phi_a = \frac{2\,x}{\frac{\tau}{n}}\,d\,\phi \tag{17}$$

$$= \frac{2\,n\,x}{\tau}\,0.4\,\pi\,\frac{2\,n}{\pi\,\tau}\,d\,x \tag{18}$$

$$\phi_a = \int_0^{a/2} d\,\phi_a = 0.4\,\pi \times \frac{2\,n^2}{\pi}\,\frac{x}{\tau^2} + C\Big]_0^{a/2} \tag{19}$$

$$= 0.4\,\pi\,\frac{n^2}{2\,\pi}\left(\frac{a}{\tau}\right)^2 + C \tag{20}$$

For $a = 0$ $\phi_a = 0$ Hence $C = 0$

$$\phi_a = 0.4\,\pi\,\frac{n^2}{2\,\pi}\left(\frac{a}{\tau}\right)^2 \tag{21}$$

for one centimeter of phase belt.

$$\phi_a = 3.2\,\frac{n^2}{2\,\pi}\left(\frac{a}{\tau}\right)^2 \tag{22}$$

for one inch of phase belt.

For the special case of three-phase and where a is expressed as a per cent. of τ.

$$\phi_a = 4.6\,a^2 \text{ for one inch of phase belt.} \tag{23}$$

Tooth-tip Leakage $\phi_i\,i$

Referring to Fig. 13B

$$\text{m. m. f.} = \frac{2\,y\,n}{\tau} \tag{24}$$

Let g_b = average air gap over b.

$$d_\phi = 0.4\,\pi\,\frac{2\,y\,n}{\tau}\,\frac{d\,x}{2\,g_b} \tag{25}$$

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Let effective interlinkages due to $d\,\phi = d\,\phi_i$

$$d\,\phi_i = \frac{2\,y\,n}{\tau}\,d\,\phi \tag{26}$$

$$= \frac{0.4\,\pi\,2\,n^2\,y^2}{g_b\,\tau^2}\,d\,y \tag{27}$$

$$\phi_i = 0.4\,\pi\,\frac{2\,n^2}{g_b\,\tau^2}\int_0^{a/2} y^2\,d\,y \tag{28}$$

$$= 0.4\,\pi\,\frac{2\,n^2}{3\,g_b\tau^2}\,y^3 + c\Big]_0^{b/2} \tag{29}$$

At $y = 0$ $\phi_i = 0$ hence $c = 0$

$$\phi_i = 0.4\,\pi\,\frac{n^2\,b^3}{12\,g_b\tau^2} \tag{30}$$

Where b and g_b are expressed as a per cent. of τ

$$\phi_i = 0.4\,\pi\,\frac{n^2\,b^3}{12\,g_b} \tag{31}$$

per centimeter of phase belt.

$$= 3.2\,\frac{n^2\,b^3}{12\,g_b} \tag{32}$$

per inch of phase belt.

For three-phase and one-inch length of phase belt.

$$\phi_i = 2.4\,\frac{b^3}{g_b} \tag{33}$$

The area of leakage path across the air gap in actual machines will be reduced due to the effect of armature slots and this will be allowed for by the introduction of the air gap coefficient, C, given in Fig. 12B.

$$\phi_i = 2.4\,\frac{b^3}{C\,g_b} \tag{34}$$

The derivation of the actual values of $\phi_i\,i$ and $\phi_e\,i$ will be illustrated by an example for three-phase where α is $66\frac{2}{3}$ per cent.; the minimum gap is 1.5 per cent of the pole pitch; and the ratio of maximum to minimum gap is 2 to 1, giving an average gap of approximately 2 per cent of the pole pitch. Average values of air gap coefficient will be assumed. The armature winding will be assumed to be completely distributed.

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Table III gives the calculation of $\phi_i\, i$ and $\phi_a\, i$ as the current varies through 90 deg. calculated from the formulas.

$$\phi_i = 2.4 \frac{b^3}{C\, g_b} \qquad \text{from} \quad (33)$$

$$\phi_a = 4.6\, a^2 \qquad \text{from} \quad (23)$$

TABLE III.

θ	b	g_b	C	b^3	Cg_b	ϕ_i	a	a^2	ϕ_a	i	$\phi_i\, i$	$\phi_a\, i$
0	0.333	0.0165	1.09	0.036	0.018	4.76	0	0	0	0	0	0
15	0.333	0.0177	1.09	0.036	0.0193	4.43	0	0	0	0.26	1.15	0
30	0.333	0.0188	1.08	0.036	0.204	4.2	0	0	0	0.50	2.1	0
45	0.25	0.0196	1.08	0.0156	0.0212	1.75	0.083	0.007	0.032	0.71	1.24	0.023
60	0.167	0.020	1.08	0.0047	0.0216	0.52	0.167	0.028	0.128	0.87	0.45	0.111
75	0.083	0.023	1.07	0.006	0.0246	0.06	0.250	0.063	0.29	0.97	0.06	0.28
90	0	0.027	1.07	0	..	0	0.333	0.110	0.505	1.0	..	0.505

Dimensions are expressed as per cent of pole pitch.

Average gap = 0.02

$$\Phi_i = 2.4 \frac{0.333^3}{1.08 \times 0.02} = 4.08$$

$$\Phi_a = 4.6 \times 0.333^2 = 0.505$$

Fig. 14 shows the above values of $\phi_i i$ and $\phi_a i$, and the total tooth-tip leakage over 180 deg. These waves were analyzed for harmonics and the equation is given in the figure. The fundamentals only are plotted.

It will be noted that the third is the most important harmonic, and since the third harmonics cancel in the terminal voltages of star connected three-phase machines, the effect of only the fundamental needs to be considered. It is obvious from the equation in Fig. 14 that the higher harmonics may be neglected without introducing any appreciable error.

C_i is the ratio of the maximum value of the fundamental of $\phi_i\, i$ Fig. 14 to the value of Φ_i that is,

$$C_i = \frac{0.89}{4.08} = 0.218$$

Similarly

$$C_a = \frac{0.22}{0.505} = 0.435$$

In the derivation of C_i and C_a a sine wave of armature current has been assumed. This is very nearly the case with a star-

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connected machine, and, as has been explained, in a Δ-connection the short-circuit current as read on an ammeter is practically equal to that with star-connection for the same field current. The values of C_i and C_a as derived for star connection may therefore be used for delta connection, even though the current is not a sine wave in the latter case.

The values of C_i and C_a were calculated for complete distribution of the armature winding with various ratios of pole arc to

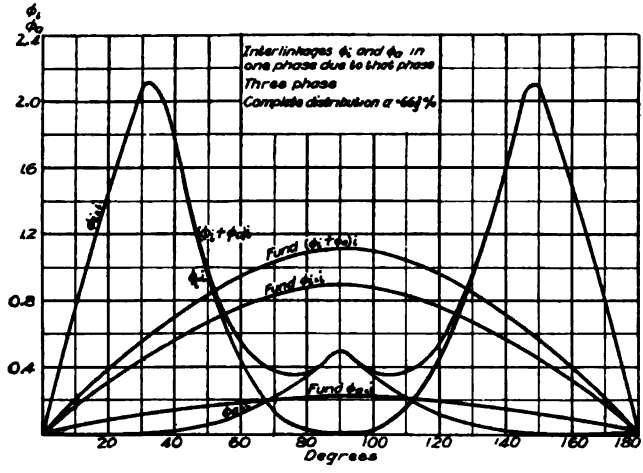

FIG. 14

$$\phi_i = .89 \sin \theta + 1.22 \sin 3\theta + .29 \sin 5\theta - .15 \sin 7\theta - .18 \sin 9\theta - .06 \sin 11\theta$$

$$\phi_a = .22 \sin \theta - .13 \sin 3\theta + .06 \sin 5\theta - .04 \sin 7\theta + .04 \sin 9\theta - .01 \sin 11\theta$$

$$\phi_i + \phi_a = 1.11 \sin \theta + 1.09 \sin 3\theta + .35 \sin 5\theta - .19 \sin 7\theta - .14 \sin 9\theta - .07 \sin 11\theta$$

pole pitch for three phase and two phase by the method shown in Table III, and are given in Fig. 15.

The variation of C_i and C_a with distribution of the armature winding was investigated and it was found the results obtained by the application of these factors C_i and C_a to Φ_i and Φ_a would give practically correct results when applied to the case of one slot per pole and phase. It is evident that there is just one leakage path around the coil in the case of one slot per pole and phase, and this path will be partly through the pole face iron and partly through air, but the length of path in each will vary as the pole moves along. The total interlinkages were calculated over 90 deg. for this condition and the waves analyzed. It was found

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that the ratio of the fundamental of the total interlinkage wave in this case to the value of $\Phi_i + \Phi_a$ was practically the same as the ratio of $C_i\,\Phi_i + C_a\,\Phi_a$ to $\Phi_i + \Phi_a$, that is the values of C_i and C_a derived for complete distribution apply also to one slot per pole and phase.

It is very important to note that while the values of C_i and C_a do not change with distribution, Φ_i and Φ_a do show a large variation.

The polyphase effect in tooth-tip reactance has been discussed in the first part of this paper and is taken into account by increasing the values of C_i and C_a by 50 per cent for three phase. For two phase the values of C_i and C_a are used as calculated for one phase of the winding.

Let C_1 be the factor by which Φ_i is multiplied to obtain the effective interlinkages per ampere, and C_2 be the factor for Φ_a Then

$$C_1 = 1.5\ C_i \text{ for three phase}$$
$$= C_i \quad \text{for two phase}$$
$$C_2 = 1.5\ C_a \text{ for three phase}$$
$$= C_a \text{ for two phase}$$

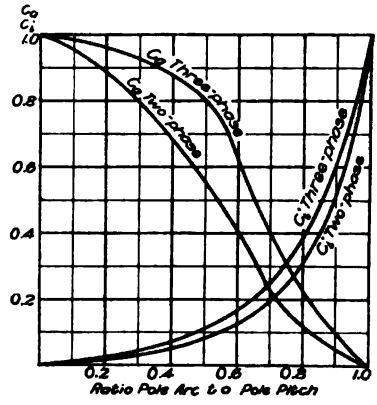

FIG. 15—VALUES OF C_i AND C_a

It will be noted that the values of C_i and C_a are not the same for two phase as for three phase.

The factors C_1 and C_2 for both three phase and two phase are plotted against ratio of pole arc to pole pitch in Fig. 16.

The reduction factor for tooth-tip reactance accounting for short pitch coils may be taken as the average factor K from Fig. 11.

Reactance Formulas. The various factors of the reactance formula may be combined to give the following equation for a double-layer armature winding.

For dimension in centimeters.

Three Phase.

$$x_a = \frac{2\pi f\, s^2 a^2\, q\, l}{10^8\, c^2} \left\{ \overbrace{\frac{2.8\, p\sqrt{\tau}}{l}}^{\text{End}} + \right.$$

$$\left. K \left[\overbrace{\frac{0.4\pi}{s}\left(\frac{d_1}{3\, w_s} + \frac{d_2}{w_s}\right)}^{\text{Slot}} + \overbrace{0.4\pi\, C_1\, K_i \frac{w_s + w_t}{2gC}}^{\text{Tooth Tip}} + C_2\, \Phi_a \right] \right\} \quad (35)$$

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Two Phase.

$$x_a = \frac{2\,\pi f s^2 a^2 q\, l}{10^2 c^2} \left\{ \overset{\text{End}}{\frac{1.6\, p\, \sqrt{\tau}}{l}} + \right.$$

$$\left. K \left[\overset{\text{Slot}}{\frac{0.4\,\pi}{s}\left(\frac{d_1}{3\,w_s}+\frac{d_2}{w_s}\right)} + 0.4\,\pi\, C_1\, K_i\, \overset{\text{Tooth Tip}}{\frac{w_s + w_t}{2\,g\,C}} + C_2 \Phi_a \right] \right\} \quad (36)$$

For single layer armature winding, multiply end connection factor by 2.0 and use $K = 1.0$.

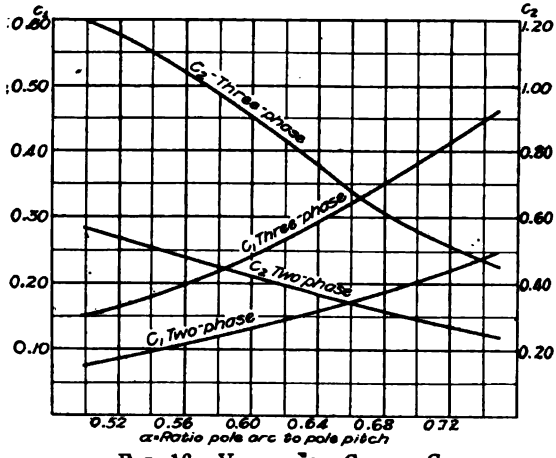

FIG. 16—VALUES OF C_1 AND C_2

REDUCTION FACTORS FOR TOOTH-TIP LEAKAGE

$C_1 = 1.5\, C_i$ for three phase
$= \quad C_i$ for two phase.
$C_2 = 1.5\, C_\alpha$ for three phase
$= \quad C_\alpha$ for two phase.

For dimensions in inches.

Substitute 4.5 for 2.8 in factor for three-phase end leakage; 2.5 for 1.6 in case of two-phase: and 3.2 for 0.4π in slot leakage and first factor of tooth-tip leakage. Use Φ_a in lines per inch.

Where x_a = armature reactance per phase in ohms.

f = frequency in cycles per second

s = slots per pole and phase

a = conductors per slot; for a double layer winding a is twice the number of turns per coil.

q = number of poles

l = gross stacked length of armature core (iron + ducts)

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c = number of circuits in multiple in armature winding
p = per cent pitch of armature winding (=0.8 for 80 per cent pitch)
τ = pole pitch at armature core face
K = reduction factor depending on pitch of armature coils
d_1 = distance from top of copper to bottom of copper in the armature winding. Fig. 8
d_2 = distance from top of copper to armature face Fig. 8
C_1 = 1.5 C_i for three phase
= C_i for two phase
C_2 = 1.5 C_a for three phase
= C_a for two phase
See Fig. 16 for C_1 and C_2
K_i = factor for Φ_i depending on slots per pole and phase
w_s = width of slot Fig. 8
w_t = width of tooth at armature face Fig. 8
g = average air gap
C = air gap coefficient based on Carter's fringing coefficient
Φ_a = effective tooth interlinkages from Fig. 12

Reactance in Per Cent. It is very convenient to have the reactance expressed in per cent and in terms of quantities easily available from the design of the machine.

Three Phase

Per cent reactance is given by $x_{pa} = \dfrac{I_n x_a}{e}$ **(37)**

(x_{pa} = 0.20 means 20 per cent reactance)
Where I_n = normal current per phase
x_a = reactance per phase in ohms
e = voltage per phase

$$\text{Armature reaction } A = \frac{2.12\ s\ a\ I_n}{2\ K_p\ K_d\ K_\phi\ c} \qquad \textbf{(38)}$$

$$\text{Flux per pole } \Phi = \frac{e \times 10^8\ K_p\ K_d\ K_\phi\ c}{2.22\ s\ a\ q\ f} \qquad \textbf{(39)}$$

Let L_0 = bracket expression from reactance formulas **(35)** or **(36)**

$$x_a = \frac{2\pi f\ s^2 a^2 q}{10^8\ c^2}\ L_0 \qquad \textbf{(40)}$$

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From (38)

$$I = \frac{2\ K_p\ K_d\ K_\phi\ A}{2.12\ s^1\ a} \tag{41}$$

and from (39)

$$e = \frac{2.22\ s\ a\ q\ f}{10^8\ K_p\ K_d\ K_\phi\ c}\ \Phi \tag{42}$$

$$x_{pa} = \frac{I\ x_a}{e} = \frac{2.67\ K_p^2\ K_d^2\ K_\phi^2 A\ l}{\Phi}\ L_0 \tag{43}$$

Two Phase

Replacing the factor 2.12 in the armature reaction formula by 1.41 the formula for two phase becomes

$$x_{pa} = \frac{4.\ K_p^2\ K_d^2\ K_\phi^2\ A\ l}{\Phi}\ L_0 \tag{44}$$

Where K_d, and L_0 are based on the two-phase curves and formulas. See reactance formula (36).

Working Formulas. The expression for

$$\Phi_i = 0.4\ \pi\ K_i\ \frac{w_s + w_t}{2\ g\ C}$$

for dimensions in centimeters and

$$3.2\ K_i\ \frac{w_s + w_t}{2\ g\ C}$$

for dimensions in inches may be simplified by assuming $w_s = w_t$ and then a series of curves for Φ_i plotted against the values of $\frac{w_s}{g}$ may be derived for different values of slots per pole and phase. The curves are given for dimensions in inches in Fig. 17. The working formulas for reactance of double layer windings with lengths expressed in centimeters are as follows:

Three Phase

$$x_{pa} = \frac{2.67\ (K_p\ K_d\ K_\phi)^2\ A\ l}{\Phi}\ \left\{ \overset{\text{End}}{\frac{2.8\ p\ \sqrt{\tau}}{l}} + \right.$$

$$\left. K\left[\overset{\text{Slot}}{\frac{0.4\pi}{s}\left(\frac{d_1}{3\ w_s} + \frac{d_2}{w_s}\right)} + \overset{\text{Tooth Tip}}{0.39\ (C_1\ \Phi_i + C_2\ \Phi_a)}\right]\right\} \tag{45}$$

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Two Phase

$$x_{pa} = \frac{4.\,(K_p\,K_d\,K_\phi)^2\;A\;l}{\Phi}\left\{\overset{\text{End}}{\frac{1.6\,p\,\sqrt{\tau}}{l}} + \right.$$

$$\left. K\left[\overset{\text{Slot}}{\frac{0.4\pi}{s}\left(\frac{d_1}{3\,w_s}+\frac{d_2}{w_s}\right)} + \overset{\text{Tooth Tip}}{0.39\,(C_1\,\Phi_i + C_2\,\phi_a)}\right]\right\} \tag{46}$$

Note that factor 0.39 is introduced in the tooth-tip leakage because Φ_i and Φ_a in the curves are expressed in leakage lines per inch length of core.

For dimensions in inches.

Three Phase.

$$x_{pa} = \frac{2.67\,(K_p\,K_d\,K_\phi)^2\,A\,l}{\Phi}\left\{\overset{\text{End}}{\frac{4.5\,p\,\sqrt{\tau}}{l}} + \right.$$

$$\left. K\left[\overset{\text{Slot}}{\frac{3.2}{s}\left(\frac{d_1}{3w_s}+\frac{d_2}{w_s}\right)} + \overset{\text{Tooth Tip}}{C_1\,\Phi_i + C_2\,\Phi_a}\right]\right\} \tag{47}$$

Two Phase

$$x_{pa} = \frac{4.0\;(K_p\;K_d\,K_\phi)^2\;A\;l}{\Phi}\left\{\overset{\text{End}}{\frac{2.5\,p\,\sqrt{\tau}}{l}} + \right.$$

$$\left. K\left[\overset{\text{Slot}}{\frac{3.2}{s}\left(\frac{d_1}{3\,w_s}+\frac{d_2}{w_s}\right)} + \overset{\text{Tooth Tip}}{C_1\,\Phi_i + C_2\,\Phi_a}\right]\right\} \tag{48}$$

See Fig. 11 for K, Fig. 17 for Φ_i, Fig. 12 for ϕ_a and Fig. 16 for C_1 and C_2. ($x_{pa}=0.2$ means 20 per cent reactance.)

Reactance Curves. The formula may be still further simplified by assuming average values of d_1, d_2 and w_s for a certain narrow range of slot pitches. A series of five curves covering the ordinary ranges of slot pitches, and giving values of end, slot and tooth-tip factors as a function of the slots per pole and phase are given in Figs. 20A, 20B, etc. The curves will give the accuracy necessary for most calculations of reactances and make the work quite simple.

Reactance of Non-Salient Pole Generators. The reactance of turbine generators, the most common type of non-salient pole machines, was calculated by assuming the ratio of pole arc to

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pole pitch to be the ratio of the unslotted pole centre to pole pitch. Since practically all machines of this type now have five or more slots per pole and phase, the expression for Φ_i was simplified by substituting in equations (**35**) and (**36**) the value of K_i expressed as a function of the slots per pole and phase.

That is $$K_i = 0.625 + 0.156\, s \text{ (above } s=4) \tag{49}$$

With a ratio of pole arc to pole pitch of 33 per cent, an approximate average for turbine generators, the value of C_1 is 0.075 for three phase and 0.035 for two phase, and C_2 is 1.32 for three phase and 0.60 for two phase.

Three Phase

$$C_1\, \Phi_i = 0.075\, (0.625 + 0.156\, s)\; 0.4\, \pi\, \frac{w_s + w_t}{2\, g\, C} \tag{50}$$

in lines per centimeter.

$$C_1\, \Phi_i = 0.075\, (0.625 + 0.156\, s)\; 3.2\, \frac{w_s + w_t}{2\, g\, C} \tag{51}$$

in lines per ampere cond'r per inch.

Allowing the approximation $w_s = w_t$ and simplifying we have

$$C_1\, \Phi_i = (0.059 + 0.015\, s)\, \frac{w_s}{g\, C} \tag{52}$$

in lines per centimeter.

$$C_1\, \Phi_i = (0.15 + 0.038\, s)\, \frac{w_s}{g\, C} \tag{53}$$

in lines per inch.

$$C_2\, \phi_a = 1.32 \times 0.198 = 0.26 \text{ line per centimeter}$$
$$= 1.32 \times 0.50 = 0.66 \text{ line per inch.}$$

Two Phase

Similarly

$$C_1\, \Phi_i = (0.0275 + 0.007\, s)\, \frac{w_s}{g\, C} \tag{54}$$

lines per centimeter.

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$$C_1 \Phi_i = (0.07 + 0.018\, s)\ \frac{w_s}{g\, C} \tag{55}$$

lines per inch.

$C_2 \Phi_a = 0.12$ line per centimeter.

$= 0.3$ line per inch

Substituting these values in equations (**45**), (**46**), (**47**), and (**48**) will give the formulas for turbine generator reactances.

Application of Formula. The use of the reactance formulas will be illustrated by the application of equation (**47**) to a 6600-kv-a. three-phase 25-cycle 14,000-volt generator.

Example:

$s = 5.25 \quad p = 82.5\% = 0.825 \quad A = 11{,}000 \quad g = 0.925$

$\tau = 36.5 \quad \alpha = 0.698 \quad w_s = 1.125$

$d_1 = 3.2 \quad d_2 = 1.05 \quad \Phi = 47.8 \times 10^6$

$K_p = 1.04 \quad K_d = 1.046 \quad K_\phi = 1.016$

$$\frac{2.67\,(1.04 \times 1.046 \times 1.016)^2\ 11{,}000 \times 38}{47.8 \times 10^6} = 0.0283$$

$$\frac{4.5\, p\, \sqrt{\tau}}{l} = \frac{4.5 \times 0.825\ \sqrt{36.5}}{38} = 0.59$$

$$\frac{3.2}{s}\left(\frac{d_1}{3\, w_s} + \frac{d_2}{w_s}\right) = \frac{3.2}{5.25}\left(\frac{3.2}{3.375} + \frac{1.05}{1.125}\right) = 1.15$$

$$K = 0.885 \qquad K \times 1.15 = 1.02$$

$$\frac{w_s}{g} = \frac{1.125}{0.925} = 1.22 \qquad \Phi_i = 5.1 \qquad C_1 = 0.37$$

$$K\, \Phi_i\, C_1 = 5.1 \times 0.37 \times 0.885 = 1.67$$

$$\Phi_a = 0.54 \qquad C_2 = 0.57$$

$$K\, \Phi_a\, C_2 = 0.54 \times 0.57 \times 0.885 = 0.27$$

$$3.55$$

$$x_{pa} = 0.0283 \times 3.55 = 0.10$$

$$= 10 \text{ per cent}$$

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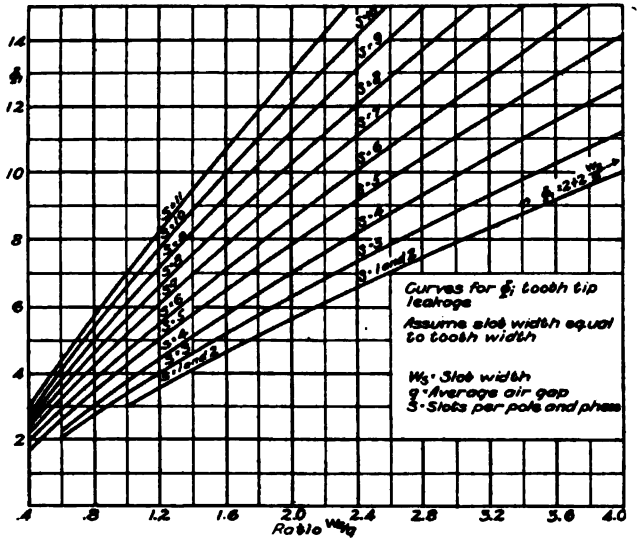

FIG. 17

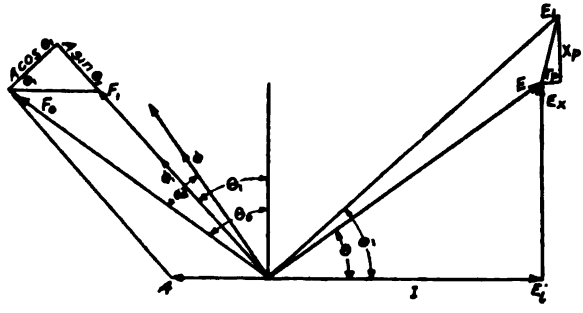

FIG. 18—EXCITATION DIAGRAM

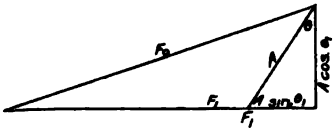

FIG. 19A—M. M. F. DIAGRAM FOR OVER-EXCITED MACHINE

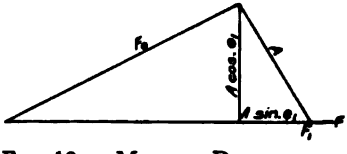

FIG. 19B—M. M. F. DIAGRAM FOR UNDER-EXCITED MACHINE

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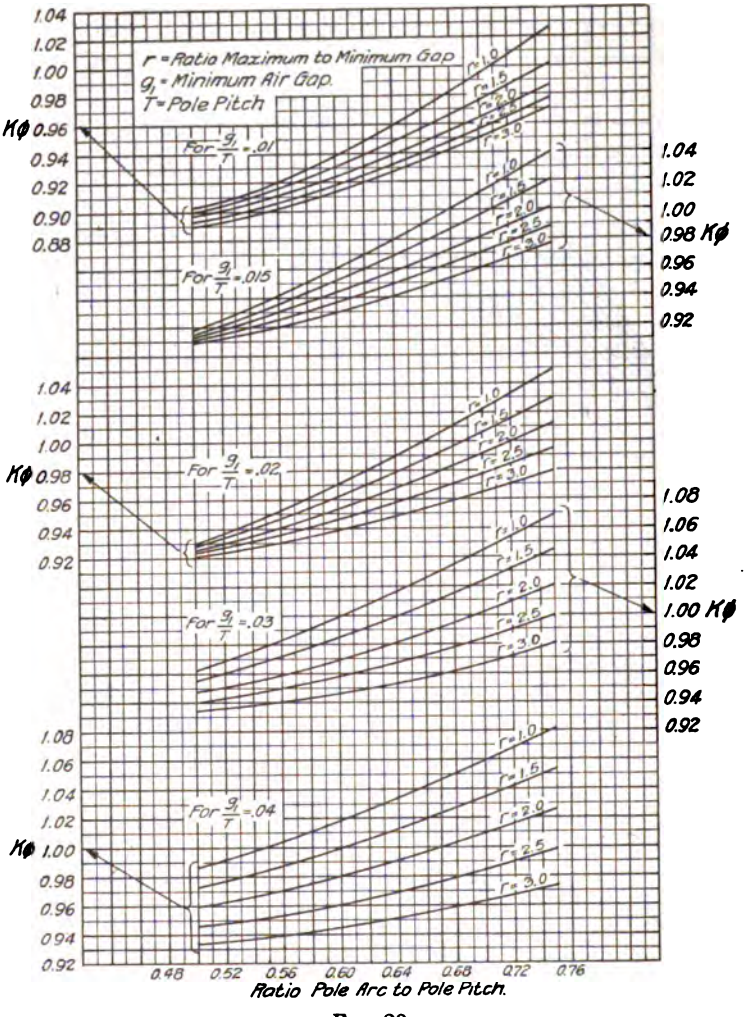

FIG. 20

FLUX DISTRIBUTION COEFFICIENT, K_ϕ

Note: Curves apply to poles having a constant radius of pole arc for all points of the arc except the extreme tip, which is rounded off by small radius. The maximum air gap is taken as the radial intercept, at the pole edge, between the armature face and the pole arc extended. That is, the rounded tip of small radius is neglected in determining the maximum gap.

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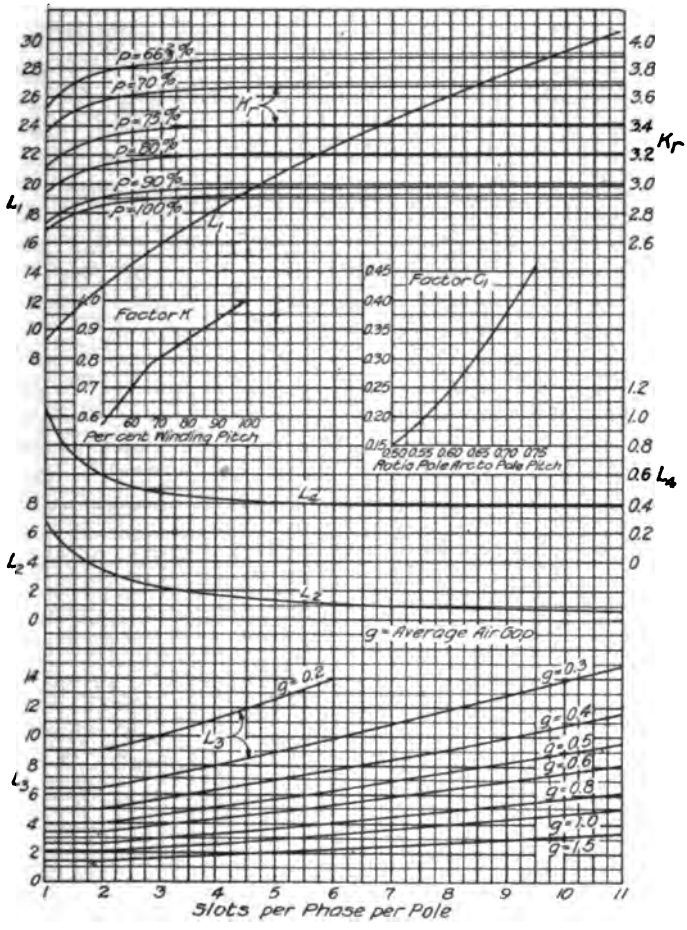

Fig. 20c

Armature Reactance of Three-Phase Alternators

$$\text{Armature Reactance } x_{pa} = \frac{K_r\, K_\phi^2\, A\, l}{\Phi}\left[\frac{p\, L_1}{l} + K\,(L_2 + C_1\, L_3 + L_4)\right]$$

(For 20% Reactance $x_{pa} = 0.20$)

K_r, K, L_1, L_2, L_3, L_4 and C_1 are given by curves.

A = $A\ T$ armature reaction per pole.

Φ = magnetic flux per pole in maxwells.

l = gross length of armature in inches (iron + ducts).

p = arm. winding pitch (0.8 for 80 per cent)

K_ϕ = flux distribution factor (Fig. 20)

Curves for Slot Pitch 1.3 to 1.5. Two-Layer Windings.

For Single-Layer Windings multiply L_1 by 2.0 and use K = 1.0.

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PART II

Transient Reactance and Short Circuits

A great deal of work has been done toward placing the phenomena of short circuits upon a calculable basis, but there is much yet to be done. Diamant's paper[7] reviewed historically the development of the theory underlying short circuits giving in detail the equations of Berg and Boucherot, and with some extensions in the mathematics, gave an expression from which, if all the factors were known, the entire wave of instantaneous current, from the instant of short circuit until the sustained value is approached, might be predicted. The potential usefulness of such an expression is obvious; but it is equally obvious to what extent its usefulness is restricted until all of the factors are known.

This paper offers methods by which the factors may be calculated with practical accuracy on certain classes of machines, (those with salient, laminated poles without amortisseur winding) but there are refinements yet to be made, and extensions to be worked out for other types of machines.

Diamant's equation (**26**)[7], which gives the curve of positive crests, is perhaps the most practical form for the engineer's use. In our notation and simplified for the condition of short circuit occurring at zero voltage (*i.e.* wave completely offset, which must be assumed in most engineering calculations) is,

$$i_c = \sqrt{2}\,(I_0 - I_s)\,\epsilon^{-\alpha_f t} + \sqrt{2}\,I_0\,\epsilon^{-\alpha_a t} + \sqrt{2}\,I_s$$
$$= i_1 + i_2 + \sqrt{2}\,I_s$$

where

I_0 = *alternating component* of short-circuit current $= \dfrac{E}{x_0}$

E = voltage per leg.

x_0 = total reactance effective on sudden short circuit. This should be called *transient reactance*, as suggested by Durgin and Whitehead[9] and by F. D. Newbury[10].

I_s = value of sustained short-circuit current.

α_f = field attenuation factor, defined later.

7. Trans. A. I. E. E., 1915, Vol. 34, p. 2237.

8. Term proposed by Hewlett, Mahoney and Burnham, Trans. A. I. E. E., 1918, Vol. XXXVII, p. 128.

9. Trans. A. I. E. E., Vol 31, 1912, p. 1662.

10. *Elec. Journal*, April 1914, p. 196.

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α_a = armature attenuation factor, defined later.
t = seconds after short circuit.

For calculation, a convenient form is:

1st term, $$i_1 = \sqrt{2}\,(I_0 - I_s)\,\epsilon^{-\alpha_f t}$$

$$\log_e \frac{\sqrt{2}\,(I_0 - I_s)}{i_1} = \alpha_f t$$

2d term, $$i_2 = \sqrt{2}\,I_0\,\epsilon^{-\alpha_a t}$$

$$\log_e \frac{\sqrt{2}\,I_0}{i_2} = \alpha_a t$$

Fig. 21 shows the skeleton thus derived. Beyond the ordinate A, when i_2 is practically zero, the wave is symmetrical about

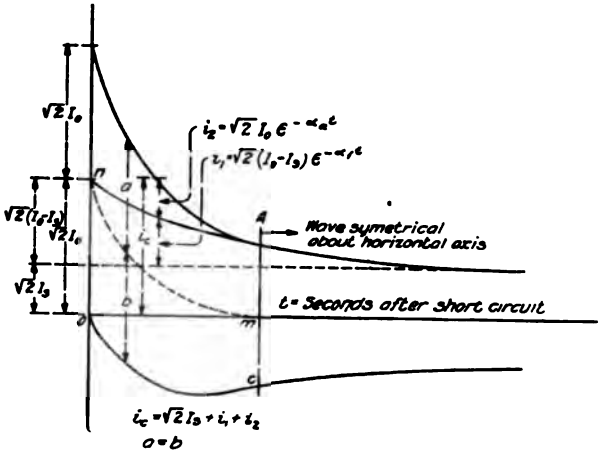

FIG. 21—SHORT-CIRCUIT ARMATURE CURRENT

the horizontal axis. The curve of negative crests between o and c can be obtained by plotting i_2 from the zero line as shown by dotted curve $n\ m$. This may be then taken as the zero of the alternating component; *i.e.* make $a = b$ at all points between o and m.

With the curve shown in Fig. 21, the three questions which the designing engineer is asked, can be more accurately answered than in the past; namely, what is the first rush of current; what is the current at, say, ¼ or ½ second after short circuit, this interval depending usually upon the operating time of the circuit breaker; and what is the sustained value.

The sustained value is easily taken from the design sheet by the well known method. The value of I_0 is determined by the

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transient reactance, x_0. And it will be shown later that α_f and α_a depend upon x_0 and the resistance of field and armature circuits.

In the past, the rate of decrease of the current has been predetermined by empirical curves, based upon oscillographic data, such as given by Hewlett, Mahoney and Burnham[11]. After all, for commercial engineering work, such curves will still be used; but in their preparation or in the investigation of special cases, the above equation will be very useful. But before those curves can be used, the generator transient reactance which adds directly to the external reactance, must be known.

The authors would make it clear at this point that they are not in agreement with conception given in Diamant's paper, of the factors which determine the first rush of current, and of what constitutes the factors α_f and α_a. He states that the initial value of the alternating component (crest value), which he calls $(A+B)$, "is equal to the maximum phase voltage before short circuit divided by the armature impedance, with good approximation." As brought out in the following paragraph, this assumption would lead to serious error in many cases. The matter of α_f and α_a will be discussed in subsequent pages.

Turning now to the problem in hand, the reactance which limits the initial short-circuit current in most synchronous machines is not the armature self-inductive reactance only; it is rather the combined self-inductive reactance of both the armature and field circuits. The calculated short-circuit current, based on the armature self-induction, is therefore higher, in some cases 40 to 50 per cent higher, than the actual short-circuit current. In salient, laminated pole alternators, in which the field leakage flux, that is, the self-induction, is large and in which rotor eddy currents are practically negligible, this difference is more pronounced than in the continuous rotor construction of turbine generators, in which the effective field self-induction is largely reduced by the distribution of the field winding and also by the solid steel rotor,[12] or in salient pole machines with low resistance amortisseur winding. Hence in the latter two cases, as might be expected and as experience indicates, the armature self-inductive reactance is a reasonably accurate measure of the short-circuit current; and it is proposed, for the present at least, that this basis be used for

11. Trans. A. I. E. E., 1918, Vol. XXXVII, p. 130.
12. A. B. Field, Trans. A. I. E. E., Vol. 31, Part 2, p. 1651.
13. Trans. A. I. E. E., Vol. 31, Part 2, p. 1657.

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these cases. This is not the point of view taken by Durgin and Whitehead[13] in 1912. Their conclusion, based on tests of a 12,000-kw., 9000-volt, 25-cycle turbo-generator in the Fisk Street Station of the Commonwealth Edison Company, was that "the short-circuit currents of alternators are limited by reactance much more complex and much higher than the self-inductive reactance of the armature." While the authors agree, as stated above, that this is true for alternators with salient, laminated poles, they submit that it is not true for many turbo-generators, as now ordinarily constructed, or in the case of salient pole machines with low resistance amortisseur windings. Tests in Table IV, confirm this.

In order to establish a conception of the effect of armature and field self-induction, consider in part the phenomena which occur in an alternator under short circuit, and the fundamental principle involved. *A closed electric circuit, without resistance, must persist magnetically in the same condition as at the instant of closing; that is, must contain, so long as it is closed, the same number of magnetic interlinkages.* If the flux tends to increase or decrease, this will produce a current of sufficient magnitude to maintain the interlinkages which existed when the circuit was closed. This law determines, in a large measure, what happens under short circuit; because, although both the armature and field circuits actually contain some resistance, and the currents therefore die down in the familiar transients, Fig. 22, yet its effect during the first cycle or so after short circuit is, for qualitative considerations at least, negligible.

It follows that the flux linked with the field winding before short circuit must persist after short circuit; that likewise whatever flux is caught in the armature circuit at the instant of short circuit, must there persist; and that the flux from the rotating field poles can not now enter the armature, *i. e.*, can not change the magnetic interlinkages of the armature circuit.

Consider a simple case of closed circuit in Fig. 23. The closed-ring conductor, without resistance, is moved from position *a* to the center of the magnetic field, ϕ, in *b*. This causes a current I_0 in the ring—a current sufficient to produce around the leakage paths of the ring a flux equal to $-\phi$. This is illustrated hypothetically in *c*. But I_0 in the ring in *b* tends to demagnetize the field. This requires a corresponding increase in m.m.f. of the field coil, to maintain the flux ϕ linked with the coil. The result, as shown on *b*, is to force the flux around the ring.

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Suppose the ring is closed around the flux as shown in *d*. Then if the ring is moved to position in *f*, the result will be as shown: the magnetic flux ϕ will remain linked with the field

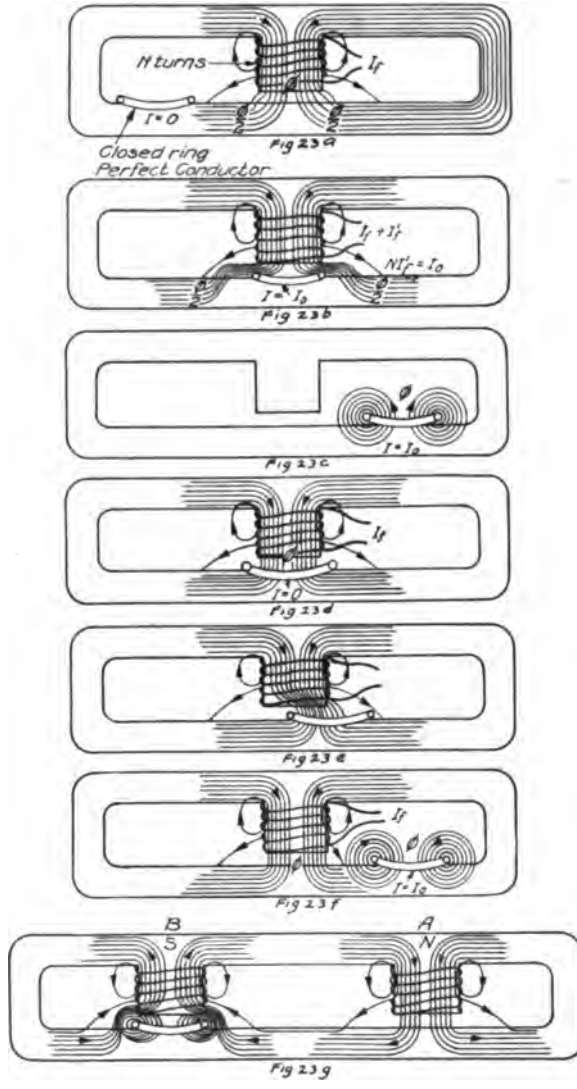

FIG. 23—DIAGRAMMATIC REPRESENTATION OF SHORT-CIRCUIT FLUXES

coil, as in *d*, and in addition, another magnetic field of equal value will be maintained around the ring by a current I_0, the same value, (assuming equal reluctance of leakage path),

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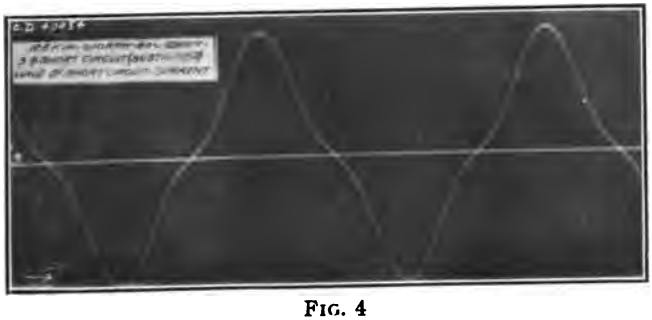

FIG. 4

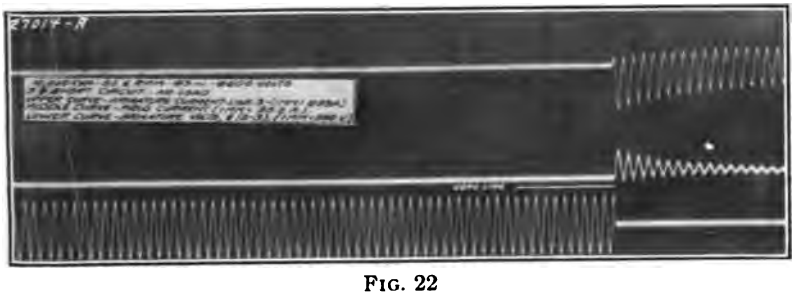

FIG. 22

[DOHERTY AND SHIRLEY]

FIG. 25B.—ACTUAL MAGNETIC DISTRIBUTION OF A FOUR-POLE FIELD—DERIVED DISTRIBUTION SHOWN IN FIG. 25A

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which was necessary in *b*. Force is required to move the ring from *d* to *f*, as shown by the distortion in *e*. The work done in moving the ring is the energy stored in the new magnetic field around the ring in *f*. If the ring containing zero flux, as in *a*, were moved along in *g*, the current would rise to I_0 when under A, decrease to zero between A and B, and rise to $-I_0$ under B; in other words, an alternating current would be produced. If, however, the ring, enclosing the flux ϕ and containing the direct current I_0, as in *f*, were moved as above, the current would decrease to zero under A, rise again to I_0 between A and B, and reach $2I_0$ under B. In this case, the resultant current, varying from zero to $2I_0$ is made up of an *alternating component* and a *direct component*, the latter being equal to the maximum of the former.

The events occurring in an alternator immediately after a no-load single-phase short circuit have thus been illustrated. The short circuit at maximum voltage is represented by the ring enclosing zero flux, the short circuit at zero voltage, by the ring enclosing the flux ϕ. Fig. 23e illustrates how a large alternating torque is produced when the short circuit occurs at zero voltage.

A three-phase short circuit at no load is different from the single-phase short circuit in this important respect, that it is impossible, three-phase, to have the short circuit occur when zero flux is enclosed in the short-circuited armature winding. The entire flux is at all times enclosed by some parts of the armature winding, and therefore in whatever position the field structure happens to place the flux when the short circuit occurs, there it must remain, a series of polar regions, stationary in space, while each field pole with its equal value of flux moves on These stationary polar regions of normal intensity, of course generate in the rotating field winding an alternating current of normal frequency, and of a maximum value in m.m.f. equal to the direct current which maintains the stator polar regions. That is, the flux, once intact through the entire normal magnetic path is now literally torn apart at the air gap, and at 180 electrical degrees from the instant of short circuit, forms two separate magnetic circuits, closing through the leakage paths between armature and field windings. At the end of one cycle the flux is again intact through the normal path. All the work which, during the first half cycle, was done on the field in tearing it apart, in actually establishing normal interlinkages in circuits of much lower permeance than that of the normal circuit, is returned to the rotor during the last half cycle. The torque

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which accompanies this transfer of energy has been calculated by S. H. Weaver, *G. E. Review*, Nov., 1915.

Return to the question of the self-inductive reactance existing at sudden short circuit. It has been generally understood

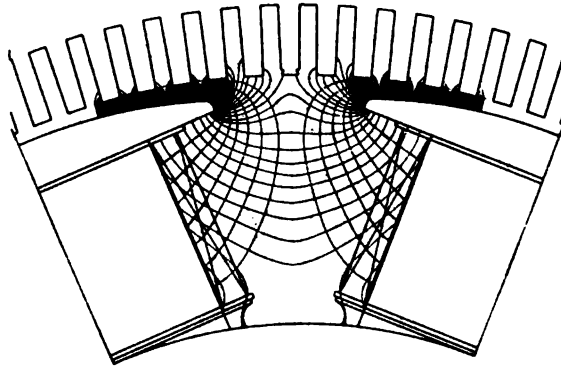

FIG. 24A—DERIVED DIAGRAM OF MAGNETIC DISTRIBUTION OF SALIENT-POLE MACHINE

how the armature self-induction limits the initial short-circuit current; that is, how, if a machine had 10 per cent armature self-induction, normal armature current would cause flux through

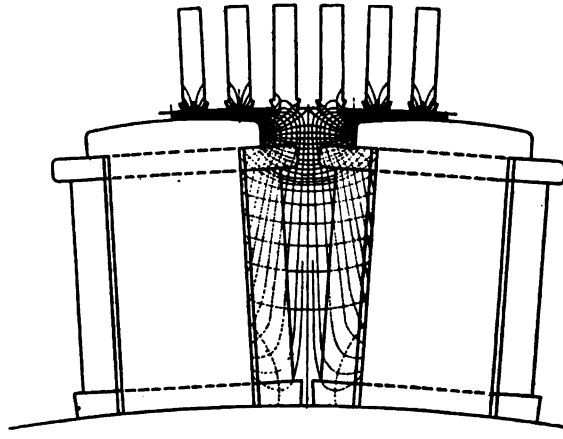

FIG. 24B—DERIVED DIAGRAM OF MAGNETIC DISTRIBUTION OF SALIENT-POLE MACHINE

the armature leakage paths equal to 10 per cent of normal flux, and that therefore on sudden short circuit, when (neglecting field self-induction) the total normal flux must be absorbed in the leakage paths, the armature current would have to rise to

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ten times normal. It may not have been so clear how and to what extent the field self-induction affects the short-circuit current. From foregoing considerations and from an approximate knowledge of the field leakage paths, it is possible to determine this point. The leakage paths have been approximately determined by plotting equi-magnetic potential lines as shown in Figs. 24A and 24B. The flux must be at right angles to these lines. The entire flux emanating from field poles of different proportions is thus illustrated in these diagrams. Figs. 25A and 25B show the graphically determined and actual magnetic distribution of a 4-pole rotor with an iron plate bridging the poles, wooden blocks supporting the ends and forming an air gap. It will be noted that a large portion of the leakage flux emanates

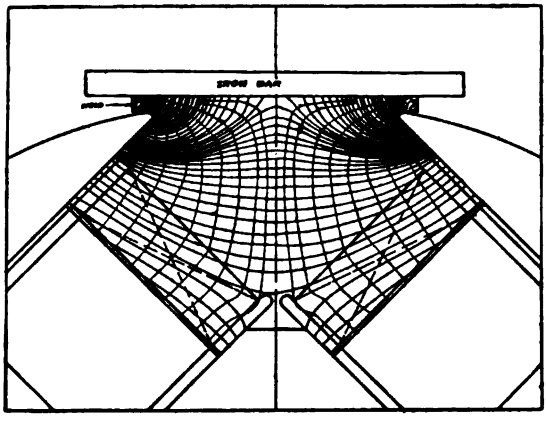

FIG. 25A—DERIVED DIAGRAMS OF MAGNETIC DISTRIBUTION OF THE FOUR-POLE FIELD SHOWN IN FIG. 25B

from the under side of the pole tip, which, we believe has not generally been considered in leakage calculations. In Fig. 25B the paths as indicated by the iron filings can not be traced to the pole body and to all of the under surface of the pole tip, on account of the field winding. But it is clear from the direction of the paths at the edge of the field winding that they will terminate approximately as shown in Fig. 25A. These figures of course do not show quite what the distribution would be if the rotor were placed in the stator. Then, both the derived and actual distribution would be different, but not greatly different. This experiment was made to confirm the method. Fig. 26 shows the distribution on a machine with a large number of poles.

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Fig. 27A represents the distribution of flux at no load on a three-phase generator. *A*, *B* and *C* represent the different phase belts. Fig. 27B represents the same machine with open

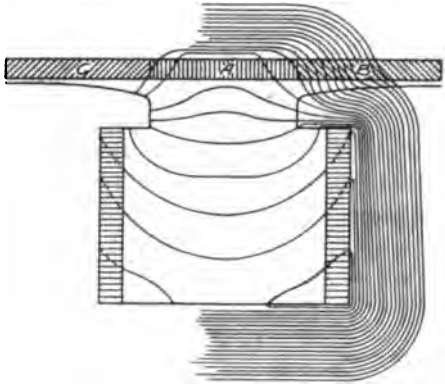

FIG. 27A—DIAGRAM OF MAGNETIC FIELD AT NO-LOAD EXCITATION ON FIELD

field circuit, the field poles rotating synchronously in position shown, and with a three-phase current in the armature. The entire magnetic flux shown in Fig. 27B represents the synchronous reactance. All magnetic lines in Fig. 27B which

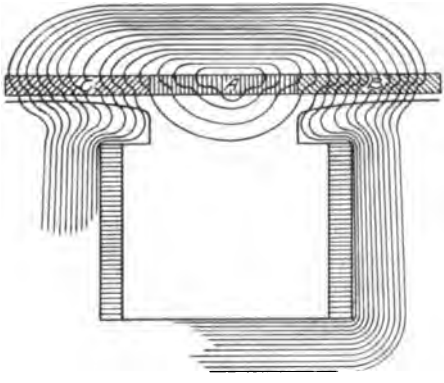

FIG. 27B—DIAGRAM OF MAGNETIC FIELD AT NO-LOAD—THREE-PHASE EXCITATION ON ARMATURE FIELD REVOLVING AT SYNCHRONOUS SPEED

do not link with the field winding represent the armature self-induction, or armature reactance. Likewise in Fig. 27A, that part of the field flux which does not link with the armature winding represents field self-induction, or field reactance. Just

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as on sustained short circuit, the field current causes the armature current to flow, and must therefore be greater in m.m.f. than the armature current, by the armature self-induction, that is greater in the ratio,

$$\frac{\text{synchronous reactance } AT}{\text{effective armature reaction}}$$

$$= \frac{\text{total flux linked with armature, Fig. 27B}}{\text{mutual flux, Fig. 27B}}$$

so also on sudden short circuit, when the armature current causes the rise in field current, the armature m.m.f. must be

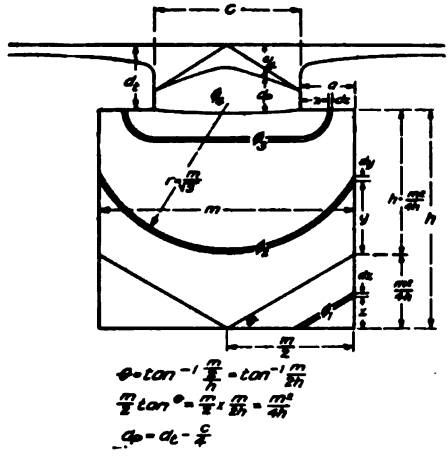

FIG. 28—DIAGRAM FOR CALCULATING FIELD LEAKAGE

greater than the rise in field m.m.f. by the field self-induction, that is greater in the ratio,

$$\frac{\text{total field winding interlinkages, Fig. 27A}}{\text{mutual flux interlinkages, Fig. 27A}}$$

For convenience in developing the point, assume that all of the field leakage is concentrated between pole tips shown as ϕ_4 in Fig. 28. On sudden short circuit, since the normal flux which is linked with the field winding and which before short circuit entered the armature, can now neither enter the armature nor die out through the field winding, it must be forced through the leakage paths. At the instant at which the armature current in its rise at short circuit passes through normal value, the ac-

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companying rise in field m.m.f. will be approximately equal (actually less) to the effective armature reaction, A, corresponding to normal current. This increase in field m.m.f. must cause a corresponding increase in ϕ_4, Fig. 28; but since ϕ_4 links with the field circuit, and the field interlinkages can not increase, it follows that the increase in ϕ_4 represents the portion of the normal flux Φ, which is absorbed on short circuit by the field self-induction when the field and armature m.m.f's. have risen to A. As an illustration, let the flux ϕ_4 corresponding to a field m.m.f., A, be 4 per cent, and let the armature self-induction be 10 per cent. Then as the armature current passes through normal value, 14 per cent of normal flux Φ, will have been absorbed in the leakage paths; and the armature current will rise to 7.15 times normal value.

But ϕ_4 does not, as above assumed, represent all of the field leakage. Other paths of ϕ_1, ϕ_2, ϕ_3, etc., Fig. 28, exist. It is not so obvious how this additional leakage from the sides and ends of the pole, behaves, and how it operates to limit the short-circuit current. It would appear that none of the normal flux, Φ, could sink into leakage paths of ϕ_1, and ϕ_2, since in doing so it would have to cut through the closed field winding; and also, and for the same reason, that the leakage ϕ_1 and ϕ_2 could not increase. Actually, both happen. The condition which must be fulfilled is that *the total effective interlinkages of the field circuit shall not change*. Two lines linking one half of the field turns is equivalent to one line linking all of the turns; four lines linking one fourth of the turns is equivalent to one line linking all, etc., etc. What actually happens, therefore, is that as the short-circuit current rises, some of the normal flux, Φ, sinks into the paths of ϕ_1 and ϕ_2; *and to just the extent to which interlinkages are decreased in this process at the top of the winding, new flux, compensating for the loss, appears at the bottom of the winding*. In machines which have rotors entirely laminated, this increase in flux may be 30 to 40 per cent. So that the total field leakage flux, Φ_l, existing under sudden short circuit, is the same (or nearly so) as would exist on open circuit with the same field m.m.f.; and the effective interlinkages which Φ_l represents is the portion of the normal flux Φ, which would be absorbed in the total field self-induction on short circuit.

Let ϕ'_{lm} be the effective field leakage interlinkages per unit length of machine, corresponding to a m.m.f. on the field equal to normal armature reaction, A. Then, *if* a rise to normal

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current in the armature would cause a rise in field m.m.f. equal to A, the ratio

$$x'_{pf} = \frac{l\phi'_{ln}}{\Phi} \tag{1}$$

would represent the per cent field self-induction. Actually the rise in field m.m.f. corresponding to normal armature current is less than A, as already pointed out, by the ratio

$$\frac{\text{mutual flux interlinkages, Fig. 27A}}{\text{total field winding interlinkages, Fig. 27A}}$$

$$= \frac{\Phi}{\Phi + \frac{F}{A}\, l\, \phi'_{ln}} = \frac{1}{1 + \frac{F}{A}\frac{l\,\phi'_{ln}}{\Phi}} = \frac{1}{1 + \frac{F}{A} x'_{pf}} \tag{2}$$

where

F = no-load field m.m.f.

l = length of machine

It is still further reduced by the field self-induction in another way: the portion of normal flux which at short circuit is absorbed in the field leakage paths no longer requires, as it did before short circuit, a corresponding portion of the no-load field m.m.f. F to maintain it across the air gap. But all of F remains at short circuit. Hence the difference between F and the portion of F which is actually needed for the flux which is not absorbed in field leakage paths, is balanced by a correspondingly decreased rise in field m.m.f. Hence, a rise in armature m.m.f. equal to A, produces a rise in field m.m.f. equal to,

$$F_a = \frac{A}{1 + \frac{F}{A} x'_{pf}} - x_{pf} F \tag{3}$$

Where X_{pf} represents the per cent actual field self-induction or per cent field reactance. This is

$$x_{pf} = \frac{F_a}{A} x'_{pf} \tag{4}$$

But

$$x'_{pf} = \frac{l\,\phi'_{ln}}{\Phi}$$

and $\phi'_{ln} = A\,L$ (from equation 35)

Hence

$$x'_{pf} = \frac{A\,l\,L}{\Phi} \tag{5}$$

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Substituting (3) and (5) in (4) and simplifying

$$x_{p_f} = \frac{A}{F} \frac{\dfrac{F\,l\,L}{\Phi}}{\left(1 + \dfrac{F\,l\,L}{\Phi}\right)^2} \tag{6}$$

Where

A = effective polyphase armature reaction AT corresponding to normal current I_n

F = no-load field AT

Φ = normal flux per pole (in maxwells) entering the armature

$$L = \left[1.42\,\frac{h}{m} + 0.16\left(\frac{m}{h}\right)^5 + 1.6 \log\left(1 + \pi\,\frac{a}{c}\right) + 5.0\,\frac{d_p}{c}\left\{1 + 0.62\,\frac{c}{I}\log\left(1 + \frac{\pi\,b}{4\,c}\right)\right\} + 0.55\,\frac{d}{l}\right]$$

If length is in inches, the expression for L becomes

$$L = \left[3.6\,\frac{h}{m} + 0.04\left(\frac{m}{h}\right)^5 + 4\log\left(1 + \pi\,\frac{a}{c}\right) + 12.8\,\frac{d_p}{c}\left\{1 + 0.62\,\frac{c}{l}\log\left(1 + \frac{\pi}{4}\,\frac{b}{c}\right)\right\} + 1.4\,\frac{d}{l}\right]$$

a, b, c, d, h, m, d_p, see Fig. 28 and Fig. 34

m = distance between pole cores at $h \div 2$

l = gross stacked length of machine (core + ducts)

d = width of pole core

L = applies for all values of $\frac{h}{m}$ above 0.5, and is the effective leakage lines per unit length of machine, linking the entire field winding when the field m.m.f. is one ampere turn. The expression for L is derived in the following pages.

The terms $4 \log\left(1 + \pi\frac{a}{c}\right)$ and $0.62 \log\left(1 + \frac{\pi}{4}\,\frac{b}{c}\right)$ are plotted in Fig. 29.

The total per cent reactance therefore which determines the initial short-circuit current (that is, the transient reactance) is given by

$$x_{p_0} = x_{p_a} + x_{p_f} \tag{7}$$

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where x_{p_a} = per cent armature self-inductive reactance (or armature reactance) calculated by equation (47) Part I.

x_{p_f} = per cent field self-inductive reactance (or field reactance) calculated by equation (6) Part II

(x_{p_0} = 0.2, means 20 per cent reactance)

Hence the effective value of the alternating component of the initial short-circuit current is

$$I_0 \frac{I_n}{x_{p_0}} \tag{8}$$

where I_m = normal current.

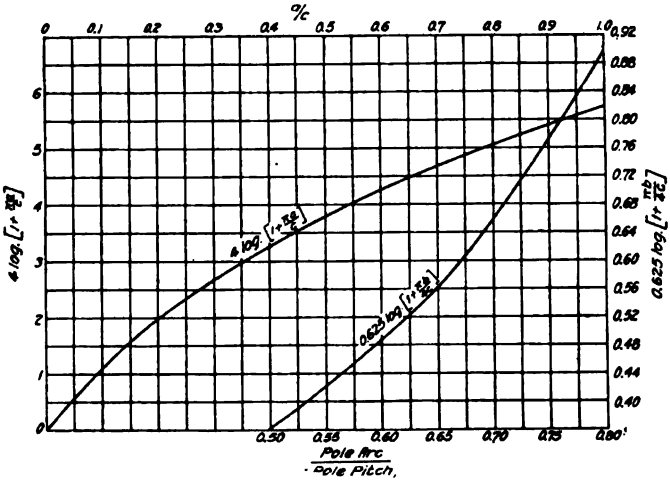

FIG. 29

The above discussion has considered only the alternating component of the initial short-circuit current. Actually a direct component of armature current, approximately equal in m.m.f. to the alternating component of armature current must always exist on a three-phase short circuit, and *may* exist on a single-phase short circuit, as already mentioned. That is at 180 degrees after short circuit, it is necessary, in such a case, for twice normal flux to exist across the leakage paths of the armature and field. It is obvious that this additional flux of normal value would be absorbed in the leakage paths in the same manner as above described.

Assuming a sine distribution of both the field flux Fig. 27A and

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of the armature flux Fig. 27B, the direct component of current would be equal to the crest of the alternating component; that is

$$I_{dc} = \sqrt{2}\, I_0 \tag{9}$$

The distribution is, however, usually not sinusoidal. Since the effect of the important harmonics on short-circuit current (alternating component) is practically eliminated, as discussed in Part I, it follows that the magnitude of the alternating component is determined by the fundamental of the flux wave. The direct component, however, is determined by the total flux, that is by the entire flux which the armature encloses, and may therefore be greater or less than $\sqrt{2}\, I_0$ by a small amount. Fig. 30 and Fig. 31 show both cases. In Fig. 30 the direct component is greater than the maximum of the alternating component; in Fig. 31, less.

Fig. 30 also shows the third harmonic voltage, terminal to neutral, existing on short circuit.

The formula for x_{p_0} applies to polyphase short circuits on salient laminated pole machines without amortisseur windings. (The effect of amortisseur windings is discussed later). It will apply to single-phase short circuit, terminal to neutral, if: (1) A, in the formula for x_{pf} is taken as the maximum value of the pulsating armature m.m.f. of normal current, *i.e.*

$$A_1 = \sqrt{2}\, K'\, N_q\, I_n \tag{10}$$

where

N_q = series turns per phase (*i.e.* per leg) per pole.

I_n = normal current per turn

K' = factor accounting for short coil pitch, coil distribution, and flux distribution. (2) the armature reactance is taken equal to two thirds of the three-phase value, since as pointed out in Part I, three-phase reactance is approximately 50 per cent higher than the single-phase. That is, for single-phase short circuit, terminal to neutral,

$$x_{p_{01}} = \frac{2}{3}\, x_{p_a} + x_{p_{f1}} \tag{11}$$

where x_{pf1} is field reactance for single-phase short circuit. But since the three-phase armature reaction is

$$A_3 = 1.5\, \sqrt{2}\, N_q\, I_n\, K' \tag{12}$$

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FIG. 26—ACTUAL MAGNETIC DISTRIBUTION

FIG. 30

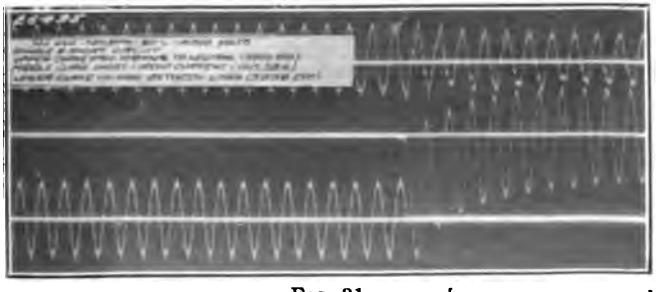

FIG. 31 [DOHERTY AND SHIRLEY]

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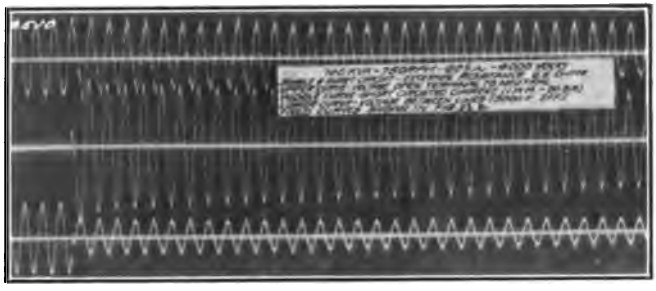

FIG. 32A

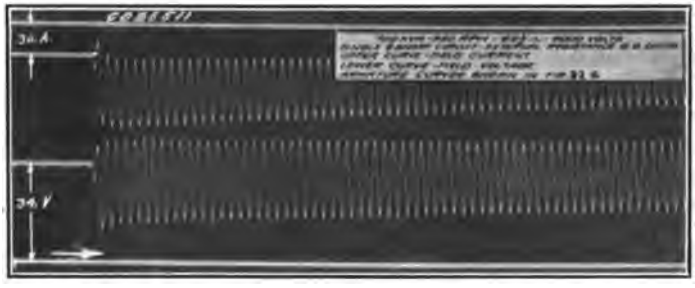

FIG. 32B

[DOHERTY AND SHIRLEY]

FIG. 35B—FIELD LEAKAGE PATH AT END OF POLE

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the equation for x_{p01} can be written

$$x_{p_{01}} = \frac{2}{3}\left(x_{p_a} + x_{p_f}\right) = \frac{2}{3} x_{p_0} \tag{13}$$

That is, on a single-phase short circuit, terminal to neutral, the current should be about 50 per cent larger than on a three-phase short circuit. The authors have not made tests to confirm this point.

The action of the field and armature on single-phase short circuit is as follows: if the short circuit occurs when the armature encloses zero flux, that is, at maximum voltage, Figs. 32A and 32B, the pulsating armature reaction produces in the field, for well known reasons, a direct m.m.f. equal to about one-half (actually less) of the maximum armature m.m.f., and a double frequency m.m.f. whose maximum value equals the direct m.m.f. The double frequency m.m.f. is of course at negative maximum at instant of short circuit, so that at the end of the first half cycle the total field m.m.f. is about equal to the maximum armature m.m.f.; that is, the armature and field absorb in their leakage paths at this instant, the total normal flux in proportion to their respective self-inductions.

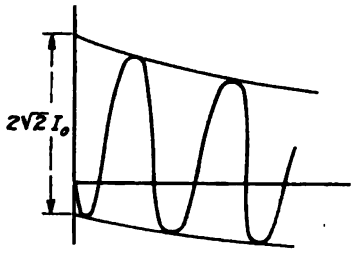

FIG. 33—SHORT CIRCUIT ARMATURE CURRENT

A single-phase short circuit between terminals gives, in most cases, practically the same current as a three-phase short circuit, as experience has shown, and may therefore be calculated by the formula for three-phase until a more accurate method is worked out.

The total self-inductive reactance effective at short circuit expressed in ohms, that is, the transient reactance, is

$$x_0 = \frac{x_{p_0} \times \text{normal voltage per leg}}{\text{normal current per leg}} \tag{14}$$

Table IV shows a comparison of calculated and actual values of the alternating component of short-circuit current in terms of normal current. The actual values were determined by projecting to the axis of zero time the curve of positive crests and also the curve of negative crests, Fig. 33. The intercept on the vertical axis represents $2\sqrt{2}\,I_0$, *i.e.* two times the crest value of the alternating component.

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TABLE IV—SALIENT-POLE MACHINES

Machine					Conditions of short circuit			Type of rotor	Amortisseur winding	Ratio—Normal cur. to R. M. S. of alternating component of short-circuit current†	
Kv-a.	Phase	Cyc.	R. P. M.	Voltage	Phase	Load	Voltage			Calc.	Test
10000	3	63	55 6	6600	3	None	6600	Laminated salient pole	None	0.27	0.25
"	"	"	"	"	3	None	"	"	"	"	0.25‡
"	"	"	"	"	3	None	"	"	"	"	0.26‡
"	"	"	"	"	3	None	"	"	"	"	0.25
"	"	"	"	"	3	7500kw	"	"	"	"	0.22
"	"	"	"	"	3	7500kw	"	"	"		0.24
9000	3	25	57.7	11000	3	None	11000	"	None	0.22	0.26
"	3	"	"	"	3	"	11000	"	"	"	0.26
"	"	"	"	"	3	"	11000	"	"	"	0.25
"	"	"	"	"	3	"	8250	"	"	"	0.25
"	"	"	"	"	1	"	8250	"	"	"	0.26
"	"	"	"	"	1	"	8250	"	"	"	0.29
6600	3	25	375	14000	3	None	16000	"	None	0.15	0.12
"	"	"	"	"	3	"	14000	"	"	"	0.13
"	"	"	"	"	3	"	14000	"	"	"	0.13
"	"	"	"	"	3	"	8000	"	"	"	0 14‡
4300	1	25	300	11000	1	None	11000	"	Low-Res.	0.19*	0.20
"	"	"	"	"	1	"	11000	"	"	"	0.22
"	"	"	"	"	1	"	11000	"	"	"	0.19
"	"	"	"	"	1	"	8250	"	"	"	0.20
"	"	"	"	"	1	"	8250	"	"	"	0.19
"	"	"	"	"	1	"	8250	"	"	"	9.23
"	"	"	"	"	1	"	5500	"	"	"	0.23
"	"	"	"	"	1	"	5500	"	"	"	0.23
1500	3	50	500	6600	3	None	6600	"	None	0.16	0.12
1500	3	50	500	6600	3	"	4950	"	"	"	0.13
1500	3	50	500	6600	3	"	3300	"	"	"	0.14
1500	3	50	500	6600	3	"	1650	"	"	"	0.15
1400	3	60	720	2300/4000	3	None	4000	"	High- Res.	0.18	0.21
"	"	"	"	"	3	"	"	"	"	"	0.16
"	"	"	"	"	3	"	"	"	"	"	0.21
"	"	"	"	"	3	"	"	"	"	"	0.21
"	"	"	"	"	1	"	"	"	"	"	0.21
"	"	"	"	"	1	"	"	"	"	"	0.18
"	"	"	"	"	1	"	"	"	"	"	0.21
"	"	"	"	"	1	"	"	"	"	"	0.22
"	"	"	"	"	1	"	"	"	"	"	0.20
"	"	"	"	"	1	"	3000	"	"	"	0.21

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SALIENT-POLE MACHINES (*Continued*)

Machine					Condition of short circuit			Type of rotor	Amortissuer Winding	Ratio—Normal cur. to R. M. S. of alternating component of short-circuit current†	
Kv-a	Phase	Cyc.	R. P. M.	Voltage	Phase	Load	Voltage			Calc.	Test
700	3	60	720	2300/4000	1	None	3996	Laminated salient pole	High Res	0.18	0.20
"	"	"	"	"	1	"	3996	"	"	"	0.23
"	"	"	"	"	1	"	3996	"	"	"	0.16
"	"	"	"	"	1	"	3036	"	"	"	0.18
"	"	"	"	"	1	"	3036	"	"	"	0.20
"	"	"	"	"	1	"	2016	"	"	"	0.17
"	"	"	"	"	1	"	2016	"	"	"	0.19
"	"	"	"	"	3	"	3996	"	"	"	0.15
"	"	"	"	"	3	"	3996	"	"	"	0.14
"	"	"	"	"	3	"	3036	"	"	"	0.15
"	"	"	"	"	3	"	2016	"	"	"	0.17
"	"	"	"	"	3	"	2016	"	"	"	0.15
125	3	60	900	2300	3	None	2300	"	High-Res	0.18	0.19
"	"	"	"	"	3	"	2300	"	High-Res.	"	0.19
"	"	"	"	"	3	"	2300	"	None	"	0.18
"	"	"	"	"	3	"	2300	"	None	"	0.18
"	"	"	"	"	3	"	2300	"	None	"	0.17
"	"	"	"	"	3	"	1150	"	None	"	0.20
"	"	"	"	"	3	"	1150	"	None	"	0.21

TURBINE ALTERNATORS

10000	3	40	2400	10000	3	None	12000	Continuous solid steel	None	0.12*	0.12‡
"	"	"	"	"	"	"	10000	"	"	"	0.13‡
"	"	"	"	"	"	"	5000	"	"	"	0.16‡
"	"	"	"	"	"	Full Load 10 P. F.	10000	"	"	"	0.15‡
"	"	"	"	"	"	"	10000	"	"	"	0.15‡
"	"	"	"	"	"	Full Load O P. F.	10000	"	"	"	0.12‡
6250	3	60	3600	2300	3	None	2300	"	"	0.10*	0.09
3750	"	60	3600	2300/4000	3	None	4000	"	None	0.09*	0.10
"	"	"	"	"	"	"	4000	"	"	"	0.08
"	"	"	"	"	"	"	3000	"	"	"	0.11
"	"	"	"	"	"	"	3000	"	"	"	0.09
"	"	"	"	"	"	"	2000	"	"	"	0.12

†In order to have ratios on same basis, normal current is decreased or increased according as voltage at short circuit is less or greater than normal.
*Transient reactance taken equal to armature reactance.
‡Average from three waves on one film.

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Example: 6600-kv-a., three-phase, 25-cycle, 14,000-volt generator.

$$l = 38 \quad A = 10970 \quad \Phi = 47.7 \times 10^6$$

$$F = 15600 \quad \frac{h}{m} = 0.687 \quad \frac{a}{c} = 0.413$$

$d_t = 2.0$ $\quad d_p = 2.0 - \frac{11.5}{4}$ (neglect d_p if it is zero or negative) $\quad \frac{d}{l} = 0.407$

Ratio $\frac{\text{pole arc}}{\text{pole pitch}} = 0.675 \quad \frac{b}{c} = 2.08$

$$L = \left[3.6 \times 0.687 + 0.04 \left(\frac{1}{0.687}\right)^5 + 4 \log (1 + \pi \times 0.413) + 1.4 \times 0.407 \right]$$

$$= [2.47 + 0.26 + 3.3 + 0.57] = 6.6$$

$$x_{pf} = \frac{10970}{15600} \frac{\dfrac{38 \times 15600 \times 6.6}{47.7 \times 10^6}}{\left(1 + \dfrac{38 \times 15600 \times 6.6}{47.7 \times 10^6}\right)^2} = 0.049$$

Armature reactance $= x_{pa} = 0.10$ (see example, Part I)

$$x_{p0} = x_{pa} + x_{pf} = 0.10 + 0.049 = 0.149$$

That is, the three-phase short-circuit current (*a-c.* component) would be

$$I_0 = \frac{1}{0.149} = 6.7 \text{ times normal}$$

Consider now the question of attenuation factors, α_f and α_a. The attenuation factor of a circuit is the ratio of the effective resistance to the (interlinkages per ampere), that is, the inductance. In the field attenuation factor, α_f, the inductance is not as Diamant gives[14] the exciting inductance of the machine, (using his notation)

$$\frac{\Phi N_f}{I_f 10^{-1}}$$

14. TRANS. A. I. E. E., Vol. 34, p. 2247.

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where Φ is the flux which the normal field excitation, I_f, would produce in the normal magnetic circuit, as given by the design sheet. It is rather

$$\frac{\Phi' N_f}{I_f 10^{-1}}$$

where Φ' is the flux, linking the field winding, which I_f would produce in the *leakage paths of both field and armature*. The correct attenuation factor of the field to be used in determining the rate of decrease of field flux on short circuit is the ratio of the effective resistance of the circuit to the (transient field interlinkages maintained through the leakage paths of both armature and field on short circuit, per ampere rise in the field circuit). Therefore the question is, what is the value of field interlinkages thus maintained per ampere *rise* in field current on short circuit. It is,

$$\frac{\Phi_t N_f}{I_{f0}} \quad \text{per pole}$$

or

$$\frac{q\,\Phi_t N_f}{I_{f0}} \quad \text{total}$$

That is,

$$\frac{q\,\Phi_t N_f}{10^8\, I_{f0}} \quad \text{henrys} \tag{15}$$

Where

Φ_t = transient flux = $\Phi - \Phi_s$

Φ = normal flux per pole

Φ_s = flux per pole, absorbed in armature self-induction on sustained short circuit

$$\Phi_s = \frac{F\Phi}{\dfrac{A}{x_{p_a}} + F_g}$$

x_{p_a} = armature reactance (x_{pa} = 0.2 means 20 per cent)

F_g = m.m.f. consumed in air gap by normal flux

N_f = turns per pole

q = number of poles

I_{f_0} = *d-c.* component of *rise* in field current at short circuit. This corresponds to the reaction of the *alternating component*

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in the armature, and for reasons previously stated is less than the armature m.m.f. as shown by equation (3).

$$I_{f_0} = \frac{F_a}{x_{p_0} N_f} = \frac{F}{x_{p_0} N_f} \left[\frac{\frac{A}{F}}{1 + \frac{F l L}{\Phi}} - x_{p_f} \right] \tag{16}$$

Hence the field inductance during transient is,

$$L_{f_0} = \frac{q\, x_{p_0}\, \Phi_t\, N_f^2}{10^8\, F} \frac{1}{\left[\frac{\frac{A}{F}}{1 + \frac{F l L}{\Phi}} - x_{p_f} \right]}$$

$$L_{f_0} = \frac{q\, x_{p_0} \Phi\, N_f^2}{10^8\, F} \frac{\left(1 - \frac{F}{\frac{A}{x_{p_a}} + F_g} \right)}{\left(\frac{\frac{A}{F}}{1 + \frac{F l L}{\Phi}} - x_{p_f} \right)} \text{ henrys} \tag{17}$$

The field attenuation factor is therefore

$$\alpha_f = \frac{r_f}{L_{f_0}} \tag{18}$$

where r_f = effective resistance of the field circuit. In laminated pole machines, this may be taken in approximate calculations as the ohmic resistance of the field circuit.

The inductance in the armature attenuation factor α_a, which determines the rate of decay of the *direct component*, is determined as follows: the interlinkages of the armature per pole per phase, maintained through the leakage paths of both the armature and the field, per effective ampere turn rise in the armature m.m.f. on short circuit, is

$$\frac{(K' N_q)\, \Phi}{\frac{A}{x_{p_0}}} \tag{19}$$

or, for all poles

$$\frac{q\, x_{p_0}\, \Phi\, (K' N_q)}{A} \tag{20}$$

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But A, for three-phase, is the product

$$\sqrt{2}\, I_n \times 1.5\, (K' N_q) \tag{21}$$

where K' is a constant depending upon the pitch and distribution of coils, and upon the flux distribution; I_n = normal rated current; and N_q = series turns per pole per phase. Hence, the inductance, that is the interlinkages with $(K' N_q)$, is

$$L_{ao} = \frac{1.5\, q\, x_{p_0}\, \Phi\, (K' N_q)^2}{10^8} \text{ henrys} \tag{22}$$

The factor 1.5 accounts for the mutual induction of phases. For single phase or two phase this factor would be unity. L_{ao} can be arrived at in another and more convenient way.
It is equal to

$$L_{ao} = \frac{x_0}{2\, \pi f} \tag{23}$$

Where x_0 = transient reactance in ohms
f = frequency in cycles per second

$$x_o = \frac{x_{p_0}\, E}{I_n}$$

Hence

$$L_{ao} = \frac{x_{p_0}\, E}{2\, \pi f\, I_n} \tag{24}$$

The armature attenuation factor is

$$\alpha_a = \frac{r_a}{L_{ao}} \tag{25}$$

where r_a = effective resistance per phase.

It may seem at first thought that since the sum of the inductances of both the field and armature are involved in the attenuation factors, the sum of resistance of both circuits also should enter. However, this is not the case. The armature and field transients involve magnetic energy storage under unstable conditions. In the case of the field transient, immediately after short circuit the energy stored in the field is the product of normal field flux times the large field m.m.f. which now maintains it. The flux can die down only if its energy is dissipated in the circuit with which it links, because, by the very way it is constituted, the energy can decrease only by a decrease in interlinkages, and this can not happen except by resistance

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loss in the linking circuit. From a slightly different viewpoint, suppose the field circuit has no resistance. Then the field flux and current would never die out, regardless of the armature resistance. The armature m.m.f. operates only to increase the reluctance[15] of the path through which the flux must pass on short circuit. The armature resistance operates only to decrease the initial rise in current, that is to decrease the amount of flux which on short circuit must be sent through the paths of increased reluctance, and therefore to increase the sustained value of flux, that is, to decrease the magnitude of the transient, but not its time. The same reasoning will apply to the armature transient.

The relative values of α_f and α_a can be illustrated by taking some rough average figures for salient-pole machines. The ratio, armature reactance to armature resistance is, say, 20; field reactance to field resistance, 80; field reactance to armature reactance, 0.5.

Hence

$$\text{Armature} - \frac{\text{resistance}}{\text{transient reactance}} = \frac{1}{30}$$

$$\alpha_a = \frac{2\pi f}{30}$$

$$\text{field} - \frac{\text{res stance}}{\text{transient reactance}} = \frac{1}{240}$$

$$\alpha_f = \frac{2\pi f}{240}$$

That is, the armature transient will die out eight times as fast as the field, in this case.

Diamant found from oscillographic tests that the field attenuation factor was larger during the first few cycles than during the rest of the transient. The authors have found this to be the case; and agree with Diamant that, until this difference, due probably to saturation and eddy currents, can be further studied, the lower value (which corresponds to our calculation) should be used. This is on the safe side, since by this assumption, at any time after short circuit the current will be lower than the calculated value.

15. An equivalent reluctance could theoretically be produced, with the same effect on the field transient, by a sudden increase, at open circuit, in full reluctance of the armature iron.

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Saturation of the leakage paths at short circuit and the consequent reduction of reactance may cause an increase in α_f, and probably does so in machines which have restricted leakage paths—turbine generators, for instance. In salient-pole machines, however, where tests show the short-circuit current to be practically proportional to the voltage at short circuit, it is probable that the increase is largely due to eddy currents. The increase in flux at the bottom of the field pole—*i. e.* next to the rotor rim—at sudden short circuit, as explained in preceding pages, operates in two ways to increase α_f. If the rotor rim is solid, as is usually the case, this change in flux is largely restricted by eddies in the solid rim. To whatever extent it is

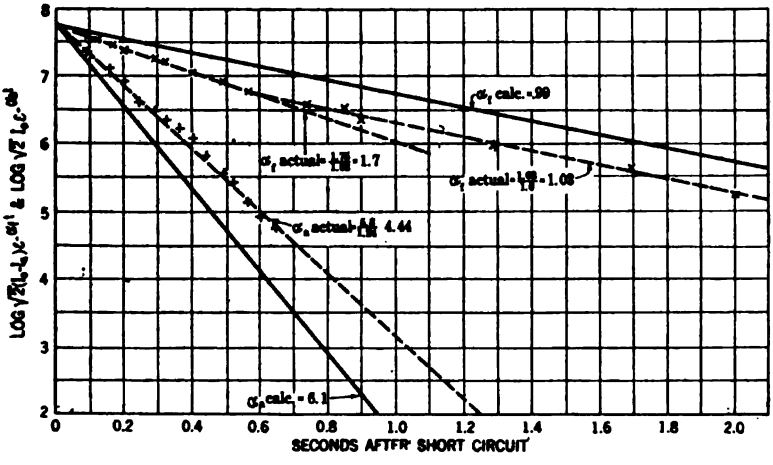

FIG. 33A—ATTENUATION FACTORS—6600 KV-A.—375 REV. PER MIN.—14000-VOLT ALTERNATOR

restricted, the effective field self-induction is reduced, which would increase α_f. But a much larger increase in α_f results from loss of energy due to eddies in the rim, increasing the effective resistance. Since the alternating flux in the rim is caused by the direct component of the armature current, *i. e.* the armature transient, the increased α_f would by this theory, exist only during the armature transient, which lasts about ¼ second. That is what actually happens, as shown in Fig. 33A. The initial value of α_f is 1.7 times the constant value which exists after the termination of the armature transient.

There is further evidence that this theory is correct. If the entire rotor magnetic circuit were laminated the change in flux

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at the bottom of the pole could occur without producing serious eddies, and therefore, without appreciable change in α_f. Actual tests on such a machine shows this to be the case. The results are shown in Fig. 33 B. There is no appreciable change in slope of the curve.

The question arises, how much will calculations of reactance be thrown out by saturation? From the data available it would seem that in salient-pole machines without amortisseur windings, saturation is not a very serious factor. Although at 180 deg. after a three-phase short circuit there must be two times normal flux through the leakage paths, these paths are widely distributed, and therefore very serious concentration is prevented. Yet there is

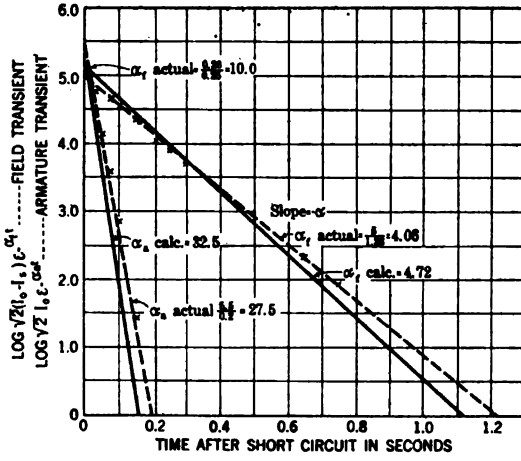

FIG. 33B—ATTENUATION FACTORS—125-KV-A.—900 REV. PER MIN.—60-CYCLE—2300-VOLT ALTERNATOR

some. The smaller the section of the principal leakage paths (the pole tip and teeth) the greater the saturation. It is still further increased by the action of an amortisseur winding, and by reason of the slots that contain it. But we believe it is safe to assume that in the case of most laminated, salient-pole machines, without amortisseur windings, the short-circuit current is practically proportional to the voltage at which short circuit occurs. When thin pole tips and shallow armature slots are used, the short-circuit current may be 10 to 20 per cent greater than indicated by calculations which neglect saturation. In the case of low resistance amortisseur windings, solid steel poles, or in turbine generators, where the leakage paths are much more restricted,

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saturation makes a greater difference. Hence, for this, as well as another reason given in the following paragraph, the calculation of such machines should be based upon the armature self-induction only. The matter of saturation of leakage paths is one which should be much further investigated.

Another matter of importance is the effect of amortisseur windings, short-circuiting collars around the pole, solid steel poles, etc. Obviously if the amortisseur winding were without resistance, the flux at short circuit could not change through it and consequently the effect of field self-induction would be nil, since the field m.m.f. would not rise. Moreover, the armature leakage paths would be more restricted. That is the transient reactance would be less than the armature self-induction. But, of course, amortisseur windings and collars are never perfect conductors. On the other hand, many of them are poor conductors; hence in the case of laminated field poles they permit a large part of the field self-induction to exert itself—the more, naturally the higher the resistance of the amortisseur winding. However, in single-phase generators, or such machines, in which heavy copper amortisseur windings are used, and also in the case of solid steel poles, the resulting increase in short-circuit current can not be neglected. For safety, therefore, it is best in such cases, as proposed above, to neglect the field self-induction in calculating the short-circuit current; that is, to assume that the transient reactance is equal to the armature reactance.

External, as well as internal, field reactance will decrease the short-circuit current. The rate at which the field current rises at short circuit will not be impeded by the external reactance, because the new magnetic interlinkages established in the external reactance represent a corresponding decrease in the interlinkages of the field winding itself, since the net total of the circuit can not change appreciably during the first cycle or so. But, by reason of this decrease in interlinkages of the field winding itself, or what is the same thing, the increase in interlinkages in the external reactance, there remains in the field less flux to be forced across the leakage paths in the machine; hence the short-circuit current will be less by just that amount.

With zero external field reactance the rise in field m.m.f. on short circuit is less than the armature m.m.f. in the ratio

$$G = \frac{1}{1+\frac{FlL}{\Phi}} \text{ by equations } (\mathbf{2}) \text{ and } (\mathbf{5})$$

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Or, in terms of total interlinkages in the machine,

$$G = \frac{q\,\Phi\,N_f}{q\,\Phi\,N_f + \frac{F}{N_f}\,L_f\,10^8} = \frac{1}{1 + \frac{F\,L_f\,10^8}{N_f^2\,\Phi\,q}}$$

where L_f = coefficient of self-induction of the field winding.

$$L_f = \frac{q\,l\,L\,N_f^2}{10^8} \text{ henrys}$$

F = no-load field m.m.f.
l = gross length of armature core (iron + ducts)
L = bracket quantity in equation (**33**) or (**33a**)
Φ = normal flux per pole (maxwells)
q = number of poles
N_f = turns per pole on field.

If an external reactance, x_{fz}, (coefficient of self-induction, $= L_{fz}$), is placed in the field circuit, the ratio becomes,

$$G = \frac{1}{1 + \frac{F\,L_f\,10^8}{N_f^2\,\Phi\,q} + \frac{F\,L_{fz}\,10^8}{N_f^2\,\Phi\,q}}$$

$$G = \frac{1}{1 + \frac{F\,10^8}{q\,N_f^2\,\Phi}\,(L_f + L_{fz})}$$

Let x_{pf}'' represent the per cent reactance of the field circuit, assuming the rise in field m.m.f. equals the armature m.m.f. on short circuit. Then

$$x_{pf}'' = \frac{\frac{A}{N_f}L_f\,10^8}{\Phi\,N_f} + \frac{\frac{A}{N_f}L_{fz}\,10^8}{\Phi\,N_f}$$

$$x_{pf}'' = \frac{A\,10^8}{N_f^2\,\Phi}\,(L_f + L_{fz})$$

The reactance of the field circuit is

$$x_{pfz} = G\,x''_{pf}$$

$$= \left[\frac{\frac{A\,10^8}{N_f^2\,\Phi}\,(L_f + L_{fz})}{1 + \frac{F\,10^8}{N_f^2\,q\,\Phi}\,(L_f + L_{fz})}\right]$$

$$x_{pfz} = \frac{A}{F}\left[\frac{1}{1 + \frac{1}{\frac{F\,10^8}{N_f^2\,\Phi\,q}(L_f + L_{fz})}}\right] \qquad \textbf{(25a)}$$

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The total transient reactance of the machine with external field reactance is

$$x_{p0} = x_{pa} + x_{pf_x}$$

where x_{pa} = armature reactance

(x_{p0} = 0.2 means 20 per cent reactance)

The principal objection[16] to the use of reactance in the field circuit is the high voltage which occurs across its terminals at short circuit. With an external reactance in the field circuit equal to the internal field reactance, this voltage may be of the order of 50,000 volts in the case of large generators.

Under load the short-circuit current is slightly higher than at no-load. The alternating component, which depends upon the flux per pole, is greater for short circuit under load than at no-load, by the increase in flux which is required under load to overcome the true impedance of the armature. Hence for the same load current, the alternating component of short-circuit current will be greater, the lower the power factor, until at zero power factor, it will be increased by the per cent armature reactance of the machine—10 per cent reactance, 10 per cent increase in the alternating component. At unity power factor, the 10 per cent being added in quadrature, would not cause appreciable increase. The direct component, depending upon the flux linked with the armature circuit, *i.e.* upon the terminal voltage, is the same for any load or power factor, providing there is no external reactance in the armature circuit. External reactance decreases the direct component because it decreases the net magnetic interlinkages of the armature circuit. With 10 per cent external reactance, zero power factor load, the direct component would be 0.9 as large as if the short circuit occurred at no load, the same terminal voltage. The reverse would be true of leading current. For unity power factor, the difference would be negligible, for reasons mentioned above.

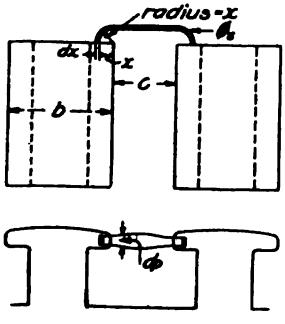

FIG. 34

Derivation of Formula For Calculating Field Self-Induction

The field leakage paths shown in Figs. 24A and 24B are approximated by the assumed paths shown in Figs. 28, 34 and 35A.

16. This point has been observed by K. Ito, *Journal Elec. Society* (Japanese), Dec. 1917, p. 891.

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With a m.m.f. equal to NI per pole the effective leakage lines, linking the entire field winding, per unit length of machine will be as follows: the flux element $d\,\phi_1$ is

$$d\,\phi_1 = 2 \times 0.4\,\pi \frac{x}{h}\,NI\,\frac{d\,x}{Kx}$$

where $$K = \frac{1}{\tan\theta}\text{ (approximately) } = \frac{2\,h}{m}$$

$$d\,\phi_1 = 0.4\,\pi\,NI\,\frac{m}{h^2}\,dx$$

Effective flux due to ϕ_1 linking entire winding is

$$\phi_{n1} = \int_0^{\frac{m^2}{4\,h}} \frac{x}{h}\,d\phi_1$$

$$\phi_{n1} = 0.039\,NI\left(\frac{m}{h}\right)^5 \tag{26}$$

$$d\,\phi_2 = 2 \times 0.4\,\pi\left(\frac{\frac{m^2}{4\,h} + y}{h}\right) 2\,N\,I\,\frac{dy}{\frac{2\,\pi}{3}\,\frac{m}{\sqrt{3}}}$$

$$d\,\phi_2 = 1.05\,\frac{NI}{h^2\,m}\,(m^2 + 4\,h\,y)\,d\,y$$

It is here assumed that the length of path is one third of the arc of a circle whose radius is $\frac{m}{\sqrt{3}}$, that is, the pole sides intercept $\frac{1}{3}$ of the arc.

Effective flux of ϕ_2 linking the entire field winding is

$$\phi_{n2} = \int_0^{h-\frac{m^2}{4h}} \left(\frac{\frac{m^2}{4\,h} + y}{h}\right) d\,\phi_2$$

$$= 0.27\,\frac{N\,I}{h^4\,m}\int_0^{h-\frac{m^2}{4\,h}} (m^2 + 4\,h\,y)^2\,d\,y$$

$$\phi_{n2} = 0.27\,N\,I\left[\frac{16}{3}\,\frac{h}{m} - \frac{1}{12}\left(\frac{m}{h}\right)^5\right] \tag{27}$$

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$$d\,\phi_3 = 2 \times 0.4\,\pi \times 2\,N\,I \frac{d\,z}{c + \pi\,z}$$

$$d\,\phi_3 = 5\,N\,I\,\frac{d\,z}{c + \pi\,z}$$

All of ϕ_3 is assumed to be linked with the winding. Hence

$$\phi_{n3} = \phi_3 = 5\,NI \int_0^a \frac{d\,z}{c + \pi\,z}$$

$$\phi_{n3} = 1.6\,N\,I \log\left(1 + \pi\,\frac{a}{c}\right) \tag{28}$$

In determining ϕ_4, it is necessary to find the portion of the pole tip from which ϕ_4 emanates, Fig. 28, and further to find what depth of path in the air between tips should be used. By plotting a large number of diagrams, such as Figs. 24A and 24B for different pole proportions, it was found that this average depth of path could be approximated by the expression

$$d_p = d_t - \frac{c}{4} \tag{29}$$

That is the factor may entirely disappear, which happens in the case of thin pole tips and low pole arc ratio. When $\frac{c}{4}$ is greater than d_t, ϕ_{n4} should be neglected.

$$\phi_{n4} = \phi_4 = 2 \times 0.4\,\pi \times 2\,N\,I\,\frac{d_p}{c}$$

$$\phi_{n4} = 5\,N\,I\,\frac{d_p}{c} \tag{30}$$

$$d\,\phi'_5 = 4 \times 0.4\,\pi \times 2\,N\,I\,\frac{d_p\,d\,x}{\pi\,x + c} \quad \text{(See Fig. 11)}$$

$$d\,\phi'_5 = 10\,N\,I\,d_p \int_0^{b/4} \frac{d\,x}{\pi\,x + c}$$

$d\,\phi'_5$ is integrated between 0 and $\frac{b}{4}$ because toward the middle of the pole the leakage from the end of pole shoe no longer goes from pole to pole, but instead bends upward and enters the stator iron, cutting the armature winding. The

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approximation is therefore made that the dividing line falls at $b/4$.

$$\phi_5' = 3.2\, N\, I\, d_p \log\left(1 + \frac{\pi}{4}\,\frac{b}{c}\right)$$

$$\phi_{n5} = \frac{\phi_5'}{l} = 3.2\, N\, I\, \frac{d_p}{l} \log\left(1 + \frac{\pi}{4}\,\frac{b}{c}\right) \tag{31}$$

where l = gross length of armature core (iron + ducts).

The leakage at the end of the pole is in paths practically straight and downward to the field spider rim as shown in Fig. 35A and 35B. At about three-fourths of the way up the pole the leakage bends up and enters the stator iron, part of it, certainly, cutting the armature winding. Of course, depending upon design, the division line may fall at one-half or may be at the top of the pole. But in the authors' judgment, three-fourths represents the average case. On this and the further assumption that the length of path at any height ν, Fig. 35, is 1.3 ν,

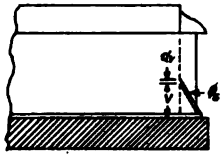

FIG. 35A

$$d\,\phi'_6 = 2 \times 0.4\,\pi\,\frac{\nu}{h}\,NI\,\frac{d\,d\nu}{1.3\,\nu}$$

where h = height of pole core

$$d\,\phi'_6 = 1.95\,\frac{d}{h}\,N\,I\,d\,\nu$$

$$\phi'_{n6} = \int_0^{\frac{3}{4}h} \frac{\nu}{h^2}\,d\,\phi_6'$$

$$= 1.95\,N\,I\,\frac{d}{h^2}\int_0^{\frac{3}{4}h} \nu\,d\,\nu$$

$$\phi_{n6} = \frac{\phi'_{n6}}{l} = 0.55\,N\,I\,\frac{d}{l} \tag{32}$$

While one or two rather arbitrary assumptions were made in deriving the above equations, they were made in the light of an experimental and graphical study of the leakage paths, and with the object of avoiding complicated expressions; and it is believed that the error in the final result will not be serious, one reason being that the assumptions involving the most uncertainties are in the smallest factors, such as ϕ_{n5} and ϕ_{n6}.

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Adding equations (26), (27), (28), (30), (31) and (32), the total self-induction of the field is

$$\phi_{ln} = \phi_{n1} + \phi_{n2} + \phi_{n3} + \phi_{n4} + \phi_{n5} + \phi_{n6}$$

For lengths in centimeters,

$$\phi_{ln} = N\,I\left[\,0.039\left(\frac{m}{h}\right)^5 + 0.27\left\{\frac{16}{3}\,\frac{h}{m} - \frac{1}{12}\left(\frac{m}{h}\right)^5\right\}\right.$$

$$+ 1.6\log\left(1 + \pi\,\frac{a}{c}\right) + 5\,\frac{d_p}{c}$$

$$\left. + 3.2\,\frac{d_p}{l}\log\left(1 + \frac{\pi}{4}\,\frac{b}{c}\right) + 0.55\frac{d}{l}\,\right]$$

$$\phi_{ln} = N\,I\left[\,1.42\,\frac{h}{m} + 0.16\left(\frac{m}{h}\right)^5 + 1.6\log\left(1 + \pi\,\frac{a}{c}\right)\right.$$

$$\left. + 5\,\frac{d_p}{c}\left\{1 + 0.62\,\frac{c}{l}\log\left(1 + \frac{\pi}{4}\,\frac{b}{c}\right)\right\} + 0.55\frac{d}{l}\right] \qquad (33)$$

If inches are used, the equation becomes

$$\phi_{ln} = N\,I\left[3.6\,\frac{h}{m} + 0.04\left(\frac{m}{h}\right)^5 + 4\log\left(1 + \pi\,\frac{a}{c}\right)\right.$$

$$\left. + 12.8\,\frac{d_p}{c}\left\{1 + 0.62\,\frac{c}{l}\log\left(1 + \frac{\pi\,b}{4\,c}\right)\right\} + 1.4\,\frac{d}{l}\right] \qquad (33a)$$

ϕ_{ln} is the effective leakage lines per pole, linking the entire field winding, per unit length of machine, produced by a m.m.f. of NI ampere turns.

The coefficient of self-induction of the field is therefore

$$L_f = \frac{q\,N_f^2\,l\,L}{10^8}\ \text{henrys} \qquad (34)$$

where q = number of poles
N_f = turns per pole
l = gross stacked length of core (iron and ducts)
L = bracket quantity in equation (33) or (33a)

For the value of leakage, ϕ'_{ln}, per unit length, corresponding to a m.m.f. on the field equal to normal armature reaction, A, substitute A for $N\,I$ in equation (33) and (33a).

That is

$$\phi'_{ln} = AL \qquad (35)$$

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Summary and Conclusions

(1) A reliable prediction of the armature self-inductive reactance of synchronous machines can be made by equations (**45**) to (**48**) inclusive, given in Part I. Moreover, working curves for three-phase, based on those equations and given in Figs. 20A 20B, etc., make the complete calculation from the design sheet a matter of four or five minutes. It is proposed that this reactance be called, for brevity, *armature reactance.*

(2) The variation of the effective armature m.m.f. or armature reaction, (under zero power factor conditions) with the no-load flux distribution has been determined. The armature reaction of three-phase machines, as calculated by the familiar formula based on sinusoidal distribution, is the basis of the new calculation. This result is modified by a single factor, K_ϕ given in Fig. 20, and developed in appendices A and B. The correct calculation of armature reaction is very important in its relation to the armature self-inductive reactance, since the latter is determined from test by the difference between the field and armature m.m.f.'s on sustained short circuit.

(3) As a corollary of (2), the effect of the harmonics in the no-load flux distribution upon the generated voltage is accounted for also by K_ϕ. See appendix A.

(4) A simplified method of calculating field excitation that has worked out well in conjunction with the reactance given by Figs. 20A, 20B, etc., is shown in appendix C.

(5) The reactance which determines the initial short-circuit current of most synchronous machines is not the armature self-induction only; it is rather the combined self-induction of both the armature and field circuits. The internal field self-induction (which, on short circuit is transient) is often 50 per cent of the armature self-induction; in which case, the calculated short-circuit current, based on the armature self-induction alone, would, of course, be 50 per cent too high.

(6) A method of calculating the transient field self-induction is given in Part II, equation (**6**). It is proposed that this reactance be called, for brevity, *field reactance.*

(7) Equation (**7**), Part II, gives a value, x_{p0} of the total per cent reactance, which determines the alternating component of short-circuit current. It is proposed that this reactance be called *transient reactance.*

(8) The attenuation factors α_f and α_a, which determine the

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rate of decrease of the field and armature transients, respectively are given in equations (**18**) and (**25**).

(9) External field reactance reduces the short-circuit current. Its effect can be calculated by equation (**25a**). The high voltage across the field collector rings (which may be 50,000 volts on large generators, assuming external field reactance equal to internal) precludes its use in most cases.

(10) Table IV confirms the following points, which are brought out in the theory given in Part II:

(a) The short-circuit current of synchronous machines can be calculated with practical accuracy.

(b) In the case of turbine generators with solid steel rotors, and also of salient-pole machines with low resistance amortisseur windings, the transient reactance should be taken equal to the armature reactance.

(c) In the case of high resistance amortisseur windings, the same value of transient reactance should be used as for laminated salient-pole machines without amortisseur windings. That is, the value given by equation (**7**) Part II.

(d) While strictly, the complete calculations apply only to salient, laminated pole machines without amortisseur windings, and the proposals in (b) and (c) are only approximations, nevertheless the results shown in Table IV justify the use of these approximations until the method is further extended.

(e) Saturation at short circuit does not practically affect value of short-circuit current of salient, laminated pole machines without amortisseur windings, unless the magnetic densities in the pole tips and teeth are high at normal voltage, as in the case of the 1500-kv-a. generator in Table IV. It does affect the short-circuit current of turbine generators, and, probably, of salient-pole machines with low resistance amortisseur windings.

(f) The short-circuit current is practically the same whether the short circuit on three-phase machines occurs between two terminals or three.

(g) A single-phase short circuit-terminal to neutral, produces a current 50 per cent greater than that of a three-phase short circuit.

In conclusion the authors wish to express their thanks to Dr. C. P. Steinmetz and W. J. Foster for their interest in reviewing the results of the paper, and to Messrs. E.J. Burnham, E. S. Henningsen, H. K. Humphrey, and J. J. Thalheimer for heir assistance.

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APPENDIX A

Effect of Harmonics in No-Load Flux Wave

The shape of the wave of flux distribution is determined largely by the shape of the pole face and the ratio of the pole arc to pole pitch. The armature teeth of course add ripples of high order, but usually of small magnitude. The latter are neglected in the following derivation because, even if they are of appreciable magnitude to cause trouble in telephone circuits, etc., they have little effect upon the effective armature m.m.f. or upon the value of terminal voltage. The third, fifth, seventh, and possibly the ninth, all of whose magnitude are determined by the design of the pole, are the terms which introduce errors into calculations based upon fundamental sine functions. To determine the effect of these harmonics, the following equations have been developed. They show that the effect of the harmonics upon the terminal voltage, for a given value of flux per pole, can be taken into account by a single factor, K_ϕ.

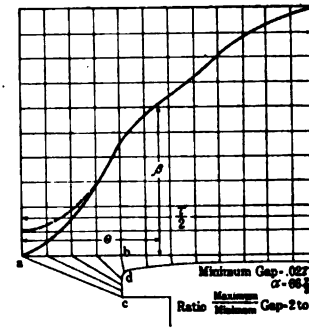

Fig. 36—Flux Distribution Curve

The flux waves of a number of representative machines were plotted by the following method. Refer to Fig. 36.

The flux at the middle of the pole was assumed to be 10, and the flux at any point toward the edge of the pole was taken inversely as the air gap. To obtain the flux beyond the edge of the pole, the distance *a b* was divided into the same number of spaces as the distance *c d* and lines drawn joining corresponding points on these lines. Assuming the flux to be inversely proportional to the length of air gap, the broken curve was derived. Since the effect of the adjacent poles would be to make the flux zero at the point midway between the poles, the solid line was drawn gradually deviating from the broken curve to pass through zero at the middle point.

The calculated flux waves were compared with test flux waves taken by exploring coils on three machines and found to agree very closely. The effect of saturation of the teeth in the middle of the pole was found to make a slight difference in the flux wave shapes on the same machines at different voltages.

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At crdinary tooth densities, however, the variation will make only one or two per cent difference in the value of K_ϕ, for values of voltage from one half to normal.

Flux waves were obtained by the method shown in Fig. 36 for ratios of pole arc to pole pitch of 50, 66⅔ and 75 per cent, for ratios of maximum to minimum air gap from unity to three to one, and for different minimum air gaps varying from one to four per cent of the pole pitch. The flux waves were then analyzed for the harmonics up to the eleventh, and the values of K_ϕ were calculated. These values of K_ϕ are given in Fig. 20.

The factor K_ϕ gives the ratio of the total flux in the irregular wave to the total flux in a sine wave which would give the same effective voltage at the terminals of the machine.

This is exactly true only when the terminal voltage has a form factor equal to that of a sine wave. This is approximately the case with practically all commercial machines as now built, as the distribution of armature winding, short pitch of armature coils and star connection of the machines all operate to eliminate the effect of harmonics in the flux wave.

Let Fig. 36 represent the flux distribution at no load.

$$\beta = (\beta_1 \sin \theta + \beta_3 \sin 3\,\theta + \beta_5 \sin 5\,\theta + \ldots.) \qquad (1)$$

Instantaneous voltage generated in unit length of one conductor is

$$e' = \frac{\nu\beta}{10^8} = \frac{\nu}{10^8}(\beta_1 \sin \theta + \beta_3 \sin 3\,\theta + \beta_5 \sin 5\,\theta + \ldots.) \qquad (2)$$

Where

ν = velocity of flux relative to conductor, in centimeters per second.

β = flux density, lines per sq. cm.

Effective value of voltage is

$$e_{eff.} = \sqrt{\frac{2}{\pi}\int_0^{\pi/2} e'^2 d\,\theta}$$

$$= \frac{\nu}{10^8}\sqrt{\frac{2}{\pi}\int_0^{\pi/2} \beta^2\, d\,\theta} \qquad (3)$$

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$$\beta^2 = (\beta_1 \sin\theta + \beta_3 \sin 3\theta + \beta_5 \sin 5\theta + \ldots)^2$$
$$= \beta_1^2 \sin^2\theta + \beta_3^2 \sin^2 3\theta + \beta_5^2 \sin^2 5\theta + \ldots$$
$$+ 2(\beta_1 \beta_3 \sin\theta \sin 3\theta + \beta_1 \beta_5 \sin\theta \sin 5\theta$$
$$+ \beta_3 \beta_5 \sin 3\theta \sin 5\theta + \ldots)$$

Since $2\int_0^{\pi/2} (\beta_1 \beta_3 \sin\theta \sin 3\theta + \beta_1 \beta_5 \sin\theta \sin 5\theta + \beta_3 \beta_5 \sin 3\theta \sin 5\theta + \ldots)$ is zero.

$$e_{eff.} = \frac{\nu}{10^8}\sqrt{\frac{2}{\pi}\int_0^{\pi/2}(\beta_1^2 \sin^2\theta + \beta_3^2 \sin^2 3\theta + \beta_5^2 \sin^2 5\theta + \ldots)\, d\theta}$$

$$e_{eff.} = \frac{\nu}{10^8\sqrt{2}}\sqrt{(\beta_1^2 + \beta_3^2 + \beta_5^2 + \ldots)} \tag{4}$$

$$e_{ff.} = \frac{\nu\beta_1}{10^8\sqrt{2}}\sqrt{1 + k_3^2 + k_5^2 + \ldots} \tag{5}$$

Where

$$k_3 = \beta_3 \div \beta_1$$
$$k_5 = \beta_5 \div \beta_1 \quad \text{etc.}$$
$$\nu = 2\tau f \tag{6}$$

where

τ = pole pitch

f = cycles per second.

$$e_{ff.} = \frac{2\tau f\, \beta_1}{10^8\sqrt{2}}\sqrt{1 + k_3^2 + k_5^2 + \ldots} \tag{7}$$

The effective voltage of a complete, full-pitch coil of N_q series turns, (that is, concentrated winding) is

$$E_{eff.} = 2\, N_q\, l\, e_{eff.} \tag{8}$$

where

l = gross stacked length of core (iron + ducts)

$$E_{eff.} = \frac{4\tau\, l\, f\, N_q\, \beta_1}{10^8\sqrt{2}}\sqrt{1 + k_3^2 + k_5^2 + \ldots} \tag{9}$$

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If the coil does not span the full pitch τ, the quantity under the radical is reduced. The voltage of the two coil sides are out of phase by angle $(1-p)\ \pi$, for fundamental and by angle

$$(1 - p)\ n\ \pi$$

for n *th* harmonic,
where,

$$p = \frac{\text{coil pitch}}{\text{pole pitch}}$$

Hence the reduction factor will be the cosine of half the angle, that is,

$$k_p = \cos\ (1-p)\ \frac{\pi}{2} \tag{10}$$

for fundamental and

$$k_{p_n} = \cos\ (1-p)\ \frac{n\ \pi}{2} \tag{11}$$

for n th harmonic.

[17]Adams has given curves for k_{pn}

If the N_q series turns per pole per phase are distributed in two or more slots, there is an additional reduction. This factor k_{d_n} is given in table prepared also by [17]Adams.

If the three-phase machine is connected Y, there is a still further reduction, but since the reduction for fundamental is same as for all harmonics, namely 0.866, except the 3rd and 9th which are entirely eliminated, it is not necessary to take this into account separately, as is done with k_{p_n} and k_{d_n}. [17]Adams gives also a table showing the product

$$k_{p_n}\ k_{d_n} = k_{r_n} \tag{12}$$

for different conditions.

Hence the effective voltage per phase (per leg) is

$$E_{eff.} = \frac{4\ \tau\ l\ f\ N_p\ \beta_1}{10^8\ \sqrt{2}}\ \sqrt{k_r^2 + k_{r3}^2 k_3^2 + k_{r5} k^2_5 + \ldots} \tag{13}$$

Where

N_p = series turns per phase.

To find value of β_1, refer to equation (1)

$$\beta_1 = \beta_1 \sin\ \theta + \beta_3 \sin 3\ \theta + \beta_5 \sin 5\ \theta + \ldots \tag{1}$$

17. TRANS. A. I. E. E., Vol 28, Part 2, p. 798, 799, 805.

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$$\beta_{ave.} = \frac{2}{\pi}\int_0^{\pi/2} \beta \, d\,\theta$$

$$\beta_{ave.} = \frac{2}{\pi}\left[\beta_1 + \frac{\beta_3}{3} + \frac{\beta_5}{5} + \ldots\right]$$

$$\beta_{ave.} = \frac{2}{\pi}\,\beta_1\left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots\right) \tag{14}$$

Total flux per pole is

$$\Phi = \tau\, l\, \beta_{ave.}$$

$$\Phi = \frac{2}{\pi}\,\tau\, l\, \beta_1\left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots\right) \tag{15}$$

Let

$$K_\phi = \left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots\right) \tag{16}$$

$$\Phi = \frac{2}{\pi}\,\tau\, l\, \beta_1\, K_\phi$$

$$\beta_1 = \frac{\Phi}{\frac{2}{\pi}\,\tau\, l\, K_\phi} \tag{17}$$

Substituting in (13)

$$E_{eff.} = \frac{4.44\, f\, N_p\, \Phi}{10^8\, K_\phi}\sqrt{k_r^2 + k_{r3}^2\, k_3^2 + k_{r5}^2\, k_5^2 + \ldots} \tag{18}$$

It is obvious from (18) that if fractional pitch is used and the coils are distributed, k_{rn} becomes practically negligible for the higher harmonics. If the machine is Y-connected, the third and its multiples are eliminated, leaving the value of the radical very near to

$$k_r$$

which is

$$k_p\, k_d$$

In most cases the radical may be taken as $k_r = k_p\, k_d$ with very little error in E_{eff}.

In concentrated windings, full pitch, Δ-connected, the radical cannot be neglected. It can be neglected with little error for machines with two or more slots per pole per phase, or frac-

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tional slots per pole per phase, Y-connected armature. In such cases,

$$E_{eff.} = \frac{4.44\, f\, N_p\, \Phi\, k_p\, k_d}{10^8\, K_\phi} \tag{19}$$

This means that the flux of the fundamental only

$$\Phi_1 = \frac{\Phi}{K_\phi}$$

is useful. That is, the useful flux is

$$\Phi_1 = \int_0^\pi \beta_1 \sin\theta\, d\,\theta \tag{20}$$

This value Φ_1 is what design calculations have usually been based on, *i.e.* calculations which assumed sine distributions of flux.

Therefore, such calculations should be modified only by the factor

$$K_\phi$$

Thus

$$\Phi = K_\phi\, \Phi_1$$

$$\Phi = \frac{E_{eff.} \times K_p \times K_d \times K_\phi \times 10^8}{4.44\, f\, N_p} \tag{21}$$

where

$E_{eff.}$ = effective voltage per phase. *i.e.* per leg

N_p = series turns per phase.

f = frequency.

$K_p = \frac{1}{k_p}$ = fractional pitch coefficient ordinarily used in design

$K_d = \frac{1}{k_d}$ = distributed coefficient ordinarily used in design, and given in Table II.

$K_\phi = \left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots\right)$, and is given in Fig. 20.

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Where

k_3 = percent third harmonic in flux wave
k_5 = " fifth " " " "
k_7 = " seventh " " " "
etc.

If the radical in equation (18) is not negligible, (21) becomes

$$\Phi = \frac{E_{ff.} \times K_\phi \times 10^8}{4.44 f N_p} \sqrt{k_r^2 + k_{r3}^2 k_3^2 + k_{r5}^2 k_5^2 + \ldots} \quad (22)$$

APPENDIX B

Armature Reaction

Much has been written about armature reaction, particularly in Europe and during the last few years[18]. Practically without exception the method of attacking the problem has been to obtain an expression for the wave of armature m.m.f. and an expression for the wave of field m.m.f. and combine the two for the resultant which determines the flux. But at this point difficulty is encountered, especially on salient-pole machines. One must know the reluctance at all points, as well as the m.m.f. before the flux can be obtained. On turbine generators where reluctance is practically uniform over the whole pole pitch, the flux can be assumed to be proportional to the m.m.f., and the problem works out well. However, such a method has not succeeded in solving, without questionable assumptions, the case of salient poles.

The method given in the following pages for solving the case of salient poles is based on the principle that *the effectiveness of an armature ampere turn at any instant and at zero power factor, as compared with a field ampere turn, in establishing flux in the mutual magnetic path, depends upon the percentage of the total useful no-load flux that the armature turn would enclose at that instant on open circuit.* The method was worked out during the investigation of armature self-induction covered in the main body of the paper. The authors were met in this investigation by the necessity of determining more accurately the relative strength of armature and field, since the armature self-induction is tested by taking the difference between these quantities on sustained short circuit. This, as in the case given in Appendix A, is one of the few instances where an apparently hopeless case has

18. See bibliography at end of Appendix B.

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worked out to the simplest result: the armature reaction is calculated by the familiar formula based on sine wave assumption, and this is modified by a single factor which may be taken directly from the curve shown in Fig. 20

It was found by methods described in Appendix A that the no-load flux wave could be very closely determined from the dimensions of the machine. The flux-wave shape was thus plotted for the variety of conditions within the ranges of normal design. These waves were expressed as a Fourier's series. With the equation of the wave given, the solution easily followed.

Let Fig. 37A represent the no load flux wave. If one armature ampere turn is placed symmetrically about the field magnetic circuit (*i.e.* directly opposite it, when $\theta = O$ in Fig. 37A), the flux wave will be a replica of that produced by one field ampere

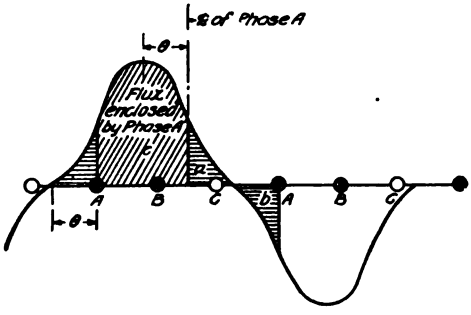

FIG. 37A

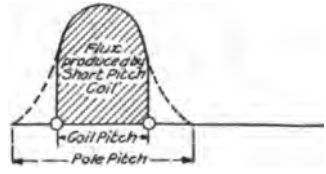

FIG. 37B

turn. If the coil does not span the entire pole pitch, then within the portion which is spanned the flux wave will be a replica of that same portion of a complete wave of a full-pitch turn, as in Fig. 37B.

On sustained short circuit, an armature current is maintained which is just sufficient to create an opposing m.m.f. acting on the mutual magnetic circuit, equal to that of the field; or in other words, sufficient to annihilate the mutual flux. Hence, as mentioned above, the measure of the effectiveness of any turn at any instant is the percentage of the total mutual flux it would enclose on open circuit.

Refer to Fig. 37A. A full-pitch turn, A, encloses all of the flux when in position $\theta = 0$. At any angle θ, observe that the flux enclosed by A is the same as if the two sides of the coil were moved toward the center of the flux wave by the same angle, θ;

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because the positive flux, a, is now neutralized by the negative flux b.

The effectiveness of the turns in phase A at the angle θ is

$$\lambda_\theta = \frac{\text{area } c}{\text{area total wave}} = \frac{c}{c + 2a} \tag{1}$$

As in Appendix A, the equation of the wave is

$$y = \sin\theta + k_3 \sin 3\theta + k_5 \sin 5\theta + \ldots. \tag{2}$$

$$a = \int_0^\theta y\, d\theta \tag{3}$$

From equations (14) and (16), Appendix A, it follows that

$$c + 2a = \pi \times \text{average ordinate}$$

$$= \pi \times 0.636 \left(1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots.\right)$$

$$c + 2a = 2K_\phi \tag{4}$$

From (3)

$$a = \int_0^\theta y\, d\theta = \int_0^\theta (\sin\theta + k_3 \sin 3\theta + k_5 \sin 5\theta + ..)\, d\theta$$

$$a = (1 - \cos\theta) + \frac{k_3}{3}(1 - \cos 3\theta)$$

$$+ \frac{k_5}{5}(1 - \cos 5\theta) + \ldots.$$

$$a = K_\phi - \left(\cos\theta + \frac{k_3}{3}\cos 3\theta + \frac{k_5}{5}\cos 5\theta + \ldots.\right) \tag{5}$$

From (4),

$$c = 2(K_\phi - a)$$

Hence, from (1)

$$\lambda_\theta = \frac{2(K_\phi - a)}{2K_\phi}$$

$$\lambda_\theta = \left(1 - \frac{a}{K_\phi}\right) \tag{6}$$

$$= 1 - \frac{K_\phi - \left(\cos\theta + \frac{k_3}{3}\cos 3\theta + \frac{k_5}{5}\cos 5\theta + \ldots.\right)}{K_\phi}$$

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$$\lambda_\theta = \frac{1}{K_\phi}\left(\cos\theta + \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}\cos 5\,\theta + \ldots \right) \tag{7}$$

For the case of a three-phase machine with 60-deg. phase belt, and concentrated winding, and assuming a sine wave armature current (which practically exists on sustained short circuit), the effective m.m.f. of phase A, Fig. 37A is

$$\lambda_\theta \sqrt{2}\, N_e\, I \cos\theta \tag{8}$$

of phase B,

$$\lambda_{(\theta+60)} \sqrt{2}\, N_e\, I \cos(\theta + 60) \tag{9}$$

of phase C,

$$\lambda_{(\theta+120)} N_e\, I \cos(\theta + 120) \tag{10}$$

where,

N_e = series armature turns per pole per phase.
I = r.m.s. amperes per turn.

$$\lambda_{(\theta+60)} = \frac{1}{K_\phi}\left[\cos(\theta + 60) + \frac{k_3}{3}\cos(3\,\theta + 180) + \frac{k_5}{5}\cos(5\,\theta + 300) + \ldots \right]$$

$$\lambda_{(\theta+60)} = \frac{1}{K_\phi}\left[(0.5\cos\theta - 0.866\sin\theta) - \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}(0.5\cos 5\,\theta + 0.866\sin 5\,\theta) + \frac{k_7}{7}(0.5\cos 7\,\theta - 0.866\sin 7\,\theta) \right] \tag{11}$$

This neglects harmonics above 7th.
By the same reasoning,

$$\lambda_{(\theta+120)} = \frac{1}{K_\phi}\left[(-\,0.5\cos\theta - 0.866\sin\theta) + \frac{k_3}{3}\cos 3\,\theta + \frac{k_5}{5}(-\,0.5\cos 5\,\theta + 0.866\sin 5\,\theta) + \frac{k_7}{7}(-\,0.5\cos 7\,\theta - 0.866\sin 7\,\theta) \right] \tag{12}$$

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The total m.m.f. per pole (*i.e.* the armature reaction) is

$$A = \lambda_\theta \sqrt{2}\, N_q\, I \cos\theta + \lambda_{(\theta+60)} \sqrt{2}\, N_q\, I\,(0.5\cos\theta - 0.866\sin\theta) - \lambda_{(\theta+120)} \sqrt{2}\, N_q\, I\,(0.5\cos\theta + 0.866\sin\theta)$$

$$A = \sqrt{2}\, N_q\, I\,[\lambda_\theta \cos\theta + \lambda_{(\theta+60)}(0.5\cos\theta - 0.866\sin\theta) - \lambda_{(\theta+120)}(0.5\cos\theta + 0.866\sin\theta)] \qquad (13)$$

Substituting equations (7), (11), and (12),

$$A = \sqrt{2}\, N_q\, I \Bigg[\frac{\cos\theta}{K_\phi} \left\{ \cos\theta + \frac{k_3}{3}\cos 3\theta + \frac{k_5}{5}\cos 5\theta + \frac{k_7}{7}\cos 7\theta \right\}$$

$$+ \frac{(0.5\cos\theta - 0.866\sin\theta)}{K_\phi} \left\{ (0.5\cos\theta - 0.866\sin\theta) - \frac{k_3}{3}\cos 3\theta + \frac{k_7}{7}(0.5\cos 7\theta - 0.866\sin 7\theta) \right\}$$

$$- \frac{(0.5\cos\theta + 0.866\sin\theta)}{k_\phi} \left\{ (-0.5\cos\theta - 0.866\sin\theta) + \frac{k_3}{3}\cos 3\theta + \frac{k_5}{5}(-0.5\cos 5\theta + 0.866\sin 5\theta) + \frac{k_7}{7}(-0.5\cos 7\theta - 0.866\sin 7\theta) \right\} \Bigg]$$

Simplifying, this becomes

$$A = \frac{1.5\sqrt{2}\, N_q\, I}{K_\phi} \left[1 + \cos\theta \left(\frac{k_5}{5}\cos 5\theta + \frac{k_7}{7}\cos 7\theta \right) + \sin\theta \left(-\frac{k_5}{5}\sin 5\theta - \frac{k_7}{7}\sin 7\theta \right) \right] \qquad (14)$$

If the armature turns are distributed over the phase belt in a number of coils instead of being concentrated in one coil, and if

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the coils do not span the full pole pitch, the bracket quantity of equation (14) is modified as follows:

1 becomes $k_p\ k_d$

k_5 and k_7 are multiplied by k_{rn}

Adams[19] has given tables for these reduction factors.

However, since the 5th and 7th are usually not large, and since, as shown by (14), their effect is decreased by the order of the harmonic, it may be safely assumed that the further decrease by k_{rn} renders negligible all of the harmonics, in cases where the winding is distributed in more than one slot per pole per phase.

Hence for such cases equation (14) becomes,

$$A = \frac{1.5\sqrt{2}\,N_t\,I}{K_\phi\ K_d\ K_p} = \frac{2.12\,N_t\,I}{K_\phi\ K_d\,K_p} \text{ (three-phase)} \qquad (15)$$

where

$$K_p = \frac{1}{k_p} = \frac{1}{\sin\,(p\ 90)}$$

$$K_d = \frac{1}{k_d} \text{ (See Table II, Part I, for value of } K_d.)$$

N_t = series turns per pole per phase

I = current per turn

K_ϕ = flux distribution coefficient given in Fig. 20.

APPENDIX C

Calculation of Excitation

The reactance as calculated by the method developed in this paper may be used in calculating excitation of alternating-current machines. The method used is an application of the generally accepted principles of excitation calculation, which have been developed by a number of writers in slightly different forms.

The diagram, Fig. 18, illustrates the factors used in the calculation for a generator with lagging current output.

I_n = normal armature current.

E = terminal voltage (is taken as unity for calculations on the per cent basis.)

E_i = in-phase component of terminal voltage

= $\cos\,\theta$

19. Trans. A. I. E. E., Vol. 28, Part 2, pp. 798, 799, 850.

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E_x = quadrature component of terminal voltage
= $\sin \theta$
$\cos \theta$ = power factor
r_{pa} = voltage required to overcome resistance drop in per cent of E. ($r_{p_a} = 0.01$ means 1 per cent)
x_{pa} = voltage required to overcome reactance drop in per cent of E. ($x_{p_a} = 0.2$ means 20 per cent)
E_1 = internal induced voltage under load in per cent of E.
= $(\cos \theta \pm r_{pa}) + j (\sin \theta \pm x_{pa})$

From the diagram it can be seen that $\cos \theta = \dfrac{E_i + r_{pa}}{E_1}$

Note that for a motor the sign of r_{pa} will be minus.

Also $\sin \theta = \dfrac{E_x + x_{p_a}}{E}$

This expression holds for the over-excited condition, that is, for a generator with lagging current or a motor with leading current, but the sign of x_{p_a} will be minus for the under-excited conditions.

A = ampere turns per pole required to overcome the effective armature reaction.

$$A = \frac{2.12\, N_q\, I_n}{K_p\, K_d\, K_\phi} \text{ for three phase}$$

$$= \frac{1.41\, N_q\, I_n}{K_p\, K_d\, K_\phi} \text{ for two phase}$$

N_q = series armature turns per pole per phase
K_p = fractional pitch coefficient
K_d = armature winding distribution coefficient.
K_ϕ = flux distribution coefficient.
Φ = no load flux per pole for normal voltage E.
Φ_1 = full-load flux per pole for internal induced voltage E_1
Φ_2 = flux for impedance drop.
F_1 = ampere turns from no-load saturation curve required to produce Φ_1.
F_0 = full-load ampere turns per pole.

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The m.m.f. diagram is all that is required for the calculation of the field current and this diagram taken from Fig. 18 for the over-excited condition is given in Fig. 19A, and for under-excited condition in 19B. The following relation may be derived from this diagram by inspection.

$$F_0 = (F_1 + A \sin \theta_1) + j\, A \cos \theta_1$$

For the under-excited condition the expression

$$F_0 = (F - A \sin \theta_1) + j\, A \cos \theta_1$$

The application of this theory can best be illustrated by an example. The formulas necessary for the calculations are given below

$$\cos \theta_1 = \frac{\cos \theta \pm r_{pa}}{E_1}$$

Sign is + for generator
Sign is − for motor

$$\sin \theta_1 = \frac{\sin \theta \pm x_{pa}}{E_1}$$

$$F_0 = (F_1 \pm A \sin \theta_1) + j\, A \cos \theta_1$$

Sign is + for generator lagging current or motor leading current.

Sign is − for generator leading current or motor lagging current.

$$E_1 = (\cos \theta \pm r_{pa}) + j\,(\sin \theta \pm X_{pa})$$

Example:

Generator—Lagging current.

Assume $r_{pa} = 0.01 = 1$ per cent
$x_{pa} = 0.15 = 15$ per cent
Power factor $\cos \theta = 0.80$
$\text{Sin } \theta = 0.60$
$F = 3800$ ampere turns (not used in calculations)
$F_1 = 4500$ ampere turns.
$A = 3000$ ampere turns.
Then $E_1 = (\cos \theta + r_{pa}) + j\,(\sin \theta + x_{pa})$
$= 0.81 + j\,0.75 = 1.10$

The value of F_1 is obtained from the no-load saturation curve

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(test or calculated) at 110 per cent normal terminal voltage and is 4500 ampere turns.

$$\sin \theta_1 = \frac{0.75}{1.10} = 0.68$$

$$\cos \theta_1 = \frac{0.81}{1.10} = 0.735$$

$$F_0 = (4500 + 0.68 \times 3000) + j\,0.735 \times 3000$$
$$= 6900 \text{ ampere turns.}$$

Motor—Lagging current.

Same data as above except $F_1 = 3400$ ampere turns.

$$E = (\cos \theta - r_{pa}) + j\,(\sin \theta - x_{pa})$$
$$= 0.79 + j\,0.45 = 0.91$$

F_1 corresponds to 91 per cent of normal voltage and is 3400 ampere turns.

$$\cos \theta_1 = \frac{0.79}{0.91} = 0.87$$

$$\sin \theta_1 = \frac{0.45}{0.91} = 0.495$$

$$F_0 = (3400 - 0.495 \times 3000) + j\,0.87 \times 3000$$
$$= 3230 \text{ ampere turns.}$$

The calculation of the above generator may be represented by the following simple calculation.

$$E \left\{ \begin{matrix} 0.81 \\ 0.75 \end{matrix} \right| = 1.10$$

$$F_0 \left\{ \begin{array}{l} \qquad\qquad\qquad \dfrac{3000}{4500} = \begin{matrix} A \\ F \end{matrix} \\ \dfrac{3000}{1.10} \times 0.75 = \left.\begin{matrix} 2040 \\ 6540 \end{matrix}\right| \\ \dfrac{3000}{1.10} \times 0.81 = 2210 \end{array} \right| = 6900 = F_0$$

Note that $\begin{matrix} 0.81 \\ 0.75 \end{matrix} \Big| = 1.10$ and $\left.\begin{matrix} 6540 \\ 2210 \end{matrix}\right| = 6900$

indicate combination of vectors at 90 deg.

This method of calculation of field current is an approximation, and does not allow for the increased field leakage under

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load. This is probably compensated for by including the added no-load saturation in the armature magnetic circuit which does not exist under load. The method has been found to give a very close check on test results on a very large number of commercial machines.

BIBLIOGRAPHY

PART I

Wechselstromtechnik IV, p. 8, E. Arnold.

Electrical Machine Design, p. 215, Alexander Gray.

Self-Starting Synchronous Motors, C. J. Fechheimer, TRANS. A.I.E.E. Vol. 31, p. 578.

Leakage Reactance, J. Rezelman, *The Electrician*, April 29, July 22, July 29, 1910.

PART II

Calculation of Sudden Short-Circuit Phenomena of Alternators, Diamant, TRANS. A.I.E.E. Vol. 34, p. 2237.

Short Circuit of Alternators, F. T. Hague, *Elec. Journal*, May 1916.

Alternator Short Circuits, C. M. Davis, *G. E. Review*, Aug. 1914.

Short-Circuiting of Large Elec. Generators and the Resulting Forces on Armature Windings, Miles Walker, Journal of I.E.E.—March 8, 1910.

Electrical Engineering, Berg & Upson, 1st edition, p. 301.

Turbo Generators, A. B. Field, TRANS. A.I.E.E. Vol. 31, Part 2, p. 1645.

The Transient Reactions of Alternators, W. A. Durgin and R. H. Whitehead, TRANS. A.I.E.E. Vol. 31, Part 2, p. 1657.

Electro-Magnetic Phenomena Following Sudden Short Circuit. Int. Elec. Congress, 1911, Vol. 2.

Mechanical Effects of Electrical Short Circuits, S. H. Weaver, *G. E. Review*, Nov. 1915.

Power Limiting Reactances, R. F. Schuchardt and E. O. Schweitzer, TRANS. A.I.EE. Vol. XXX, Part 2, p. 1143.

Wechselstromtechnik IV, p. 457, E. Arnold.

Electric Discharge, Waves and Impulses, p. 30, C. P. Steinmetz.

Generator Short-Circuit Waves, F. D. Newbury, *The Electric Journal* April 1914, p. 196.

Sudden Short-Circuit Current of Alternators, K. Ito, Journal of Elec. Society (Japanese).

Analysis of Short-Circuit Oscillograms, O.E. Shirley, *G. E. Review*, Feb. 1917.

APPENDICES A. AND B.

Wave Shape of Generators under Steady Short Circuit, A. E. Clayton, Journal of I.E.E. Vol. 54, 1916, p. 84.

Inherent Regulation of Synchronous A-C. Generators, Alfred Still, Journal of I.E.E. Vol. 53, 1915, p. 587.

Pressure Wave in Electrical Machinery, Smith and Boulding, Journal of I.E.E.. Vol. 53, 1915.

Decomposing Magnetic Fields into their Higher Harmonics, H. Weichsel, TRANS. A.I.E.E. Vol. 34, Part 2, p. 2721.

Wechselstromtechnik III, p. 175, E. Arnold.

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DISCUSSION ON "REACTANCE OF SYNCHRONOUS MACHINES AND ITS APPLICATIONS" (DOHERTY AND SHIRLEY), ATLANTIC CITY N. J., JUNE 28, 1918.

Carl J. Fechheimer: All who have been identified with the design of a-c. synchronous machinery have appreciated the great importance of internal reactance of the stator. All of us have grappled with the problem of estimating the value of the reactance, and probably there are no two who will agree either on a method of calculating its value or even on its magnitude after the machine is tested. Certainly, we cannot agree upon the value of reactance until we establish what reactance means, and are in accord upon its valuation from test data. Thus, as pointed out by the authors, the reactance determined from instantaneous short circuits may be different than that which obtains during steady operation.

We shall enumerate the most important ways by which the stator reactance can be measured.

(1) With the use of the open-circuit saturation and short-circuit curves, and a knowledge of the proportions of the machine, the ampere turns required to neutralize the armature reaction on sustained short circuit may be deducted from the total ampere turns on short circuit, thus leaving the ampere turns which are needed to drive the flux through the magnetic circuit. From this latter, and with the open-circuit saturation curve, the reactance can be estimated.

(2) From no-load saturation curve, a corrected curve which assumes full-load leakage may be drawn. From this latter curve, and with the use of the load saturation curve at nearly zero power factor, the drop may be broken up into armature reaction and armature reactance by assuming that these are constant along the curve.

(3) The rotor may be removed and single-phase current at normal frequency may be circulated in the stator windings between one terminal and neutral for three-phase star connection. Inasmuch as the resistance drop is usually negligible, we may take the reactance to be the volts from terminal to neutral, divided by the current.

For three-phase delta, circulate current in one leg: for two-phase, circulate current in one phase.

(4) Same as 3, except that instead of single-phase current between neutral and terminal, polyphase current may be circulated between terminals.

(5) Same as 3, except with the rotor in position. (This gives different values for different positions of the rotor.)

(6) Same as 4, except with the rotor in position. (This gives different values for different positions of the rotor.)

(7) A condenser may be connected across the terminals of the machine, the value of the capacity of which may be varied, so that approximate resonance can be obtained. It is then a simple matter to calculate the inductance.

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(8) The reactance may be estimated from oscillograms obtained on instantaneous short circuits. In that case, the peaks of the current should be projected back to the ordinates drawn at the instant of short circuit.

These eight methods of measurement give various values of reactance, although two or more methods may be found to check in some machines. What then shall we say is the correct method? The subject is well worthy of the attention of the Standardization Committee of the Institute. We shall attempt to discuss these methods briefly.

No. 1 (determination from short-circuit curve) has two decided objections: (a) It is difficult to estimate with sufficient accuracy the armature reaction, and a relatively small error in

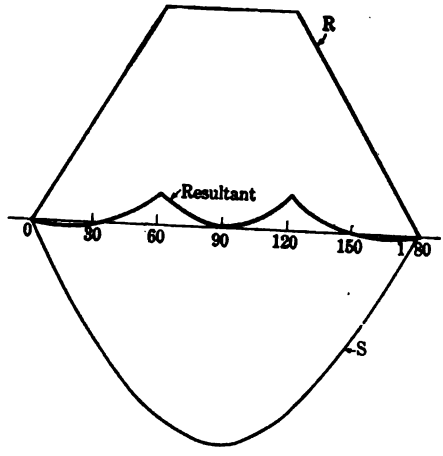

FIG. 1

this estimate will be magnified considerably in the value of the reactance; thus 5 per cent error in estimate of armature reaction may result in 40 per cent error in reactance. (b) The voltage which is induced in the individual conductors on sustained short circuit (and which is entirely consumed by impedance) is in general of badly distorted wave form.

For instance, we are showing in Fig. 1 the ideal field form of the magnetomotive force of the rotor of a turbo-generator *R*, and an assumed sine wave space distribution of the m. m. f. of the stator current *S* (ideal conditions). The resultant of these two gives the m. m. f. which drives the flux through the magnetic circuit. Although the higher harmonics which would result from the distortion are pretty well damped out in

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the current wave form, they still appear in the wave of induced e. m. f. in the individual conductors. It will therefore readily be seen that reactance computed in this way from tests is liable to be considerably in error. That the distortion is very much reduced at full load, zero power factor, is quite evident.

No. 2 (determination from no-load and zero power factor load curves) has the objection that it involves calculation of the full-load leakage no-load saturation curve, which is subject to error. The proportions of reaction and reactance as finally determined by this method, even if we were able to estimate the full-load leakage curve, are also liable to be in error. Furthermore, saturation in the magnetic circuit may cause the reactance to change in magnitude along the curve.

Nos. 3 to 6, inclusive, (pertaining to circulating current in windings) all give different results, but our observations lead us to believe that No. 3 gives results which are more nearly correct than 4, 5 or 6. It is certain that with the rotor in position the reactance measured is considerably higher than that which actually obtains. With three-phase currents, there is sufficient mutual inductance to give about 15 per cent higher inductance than with single-phase.

No. 7 has the disadvantage that a condenser has not lent itself readily to use in our test rooms. Furthermore, we have had too little experience with this method to compare it with others, but believe there are possibilities and we therefore suggest that it be further investigated.

No. 8 (pertaining to instantaneous short circuit) has the drawback that it subjects the machine to severe strains, and is in general expensive and too elaborate for general commercial use.

Discussing first the sustained reactance, it is our understanding that the authors of the paper used Method No. 1 (short-circuit, and no-load saturation curves) for checking their formula for sustained reactance against test. All of us will agree that an analytical determination of reactance is next to impossible, and we must have recourse to test data, and thereby obtain empirical constants; especially is this true of end connection leakage, and we think that it is essential for tooth-tip leakage also. Inasmuch as the authors used a test method which is subject to an error of the order of 50 per cent, we must be cautious in adopting their formula, and rely upon the accuracy of reactance estimated thereby. Aside from the error pointed out in determining reactance from test, we find in Table I, that 40 per cent of the machines show a difference between calculated and test values of more than 20 per cent, and the discrepancy in one case is 145 per cent, and in another 185 per cent.

We believe that the authors have allowed too much for increase due to mutual inductance with three-phase over single-phase. Tests which we have made removing the rotor and circulating current through windings, indicate only 16 per

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cent higher reactance three-phase (star-connection) than single-phase terminal to neutral.

Turning now to the reactance which obtains in instantaneous short circuit, we are in agreement with the authors that the leakage flux of the rotor plays a considerable part in reducing the rush of current, although we do not view the phenomenon from the same point of view. For instance, we believe that saturation of the rotor poles in conjunction with rotor leakage flux, may have considerable influence in reducing the pole leakage. For example, let us assume that the combined transient reactance is such that the stator current can rise to ten times normal r. m. s. value, and that the total ampere turns in the rotor also rise to ten times the no-load value. Then the difference in magnetic potential between poles, being ten times, the leakage flux will rise to ten times. If the no-load field leakage were 12 per cent, then with ten times field current, there will be $12 \times 10 = 120$ per cent leakage, so that, if the density in the poles at no load were 90,000 lines per square inch, the density would rise to $\left(\frac{1 + 1.2}{1 + 0.12}\right) 90{,}000 = 177{,}000$ lines per sq. in. That the density in the poles does not reach such high value will be quite evident from a glance at the B - H curve for the steel, and the result is that some of the flux which had previously been useful as generating e. m. f. in the stator conductors, "sinks" into leakage flux, so as to maintain substantially constant interlinkages with the conductors in the rotor (these may be field turns, rivets, or any paths for the currents). Saturation of the stator teeth may also play some part, for with the reduction in main flux, more leakage flux may flow, and this may, in some teeth, be of sufficient magnitude to reduce the percentage leakage. With smooth, solid rotors, such as those in turbo-generators, the phenomenon as regards pole saturation is quite different from that which obtains with definite poles, because most of the counter m. m. f. in the rotor is produced by eddy-currents near the rotor surface, and they are not, in turn, productive, of a great percentage increase in leakage on instantaneous short circuit. Although it is very difficult to check with the sustained short circuit and no-load and zero power factor saturation curves, it seems that the transient and sustained reactances in turbo-generators are not in very great disagreement, and we note that the authors are of the same opinion.

Other factors of not negligible importance as affecting the transient reactance might be cited. We shall mention only one: In salient-pole alternators, most of the spiders are solid, and as is well known, the flux (pulsates whether polyphase or single-phase short circuit) and eddy-currents are induced in the spider, which in turn influence the rotor leakage.

As we look over the authors' paper we note that they believe "saturation is not a very serious factor," and they have not

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considered eddy currents in the spider. We are very much of the opinion that such factors cannot be ignored if even reasonably approximate accurate results are desired.

Again, eddy-currents and saturation play their parts in estimating the attenuation factors—as both affect resistance and inductance. Owing to the fact that the saturation of the poles decreases with time, due to reduction in m. m. f. and flux, and therefore (1) the distribution of leakage fluxes from pole to pole are altered (assuming salient pole-construction) which in turn affects the interlinkages with the field winding, (thereby changing the coefficient of self-induction of the field circuit); and (2) the percentage of the total flux which enters the stator is automatically increased. Inasmuch as eddy currents are in parallel with the field currents, insofar as net m. m. f. is concerned, the resistance and inductance of their paths are necessarily coupled magnetically with that of the field circuit. It will, therefore, be seen that an analytical determination of the attenuation factors is well nigh impossible, and that these factors may not be constant, but change with time.

The whole problem involves so many difficulties and complications that we believe that we must rely upon empirical data. Our first problem is to find a way of measuring sustained reactance, and then modify the constants in an empirical equation to enable prediction of reactance within a maximum error of, say, 15 to 20 per cent. Then we can multiply this value of reactance by some factor or factors determined from test so as to enable agreement with transient reactance, the factors being of different magnitude for various types of machines. The determination of the field and armature attenuation factors which are probably not constants, must be determined from oscillograms of short circuits, and should be some function of the time.

In regard to the measurement of sustained reactance, we are planning to conduct some tests in line with method No. 7 (involving a condenser) and take oscillograms to determine the magnitude of the fundamental as well as of the harmonics. This method will not, we believe, be suitable for general application, especially not on large machines, because of difficulty in obtaining large condensers, but we hope thereby to establish some constants for modifying measured sustained reactance (say by circulating current in stator with rotor removed) which we can adopt as reliable figures.

I have noted the authors' reference to an error in the writer's paper, and wish to thank them for calling attention to it. I previously noted this mistake about four years ago, but have not called it to the attention of the Institute members.

I believe the authors deserve a great deal of credit for the great amount of work which they have done, for more work of this character would be helpful in cleaning up some of the problems with which we are confronted.

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V. Karapetoff: It seems to me that the terms reactance and inductance are used somewhat indiscriminately in this paper. After formula (3) in Part II. the authors say: "The effective field self-induction or field reactance." Reactance is a different concept from self-inductance, and in a paper which is intended for reference purposes it would seem rather unfortunate not to have these terms more definitely discriminated because one has to find out from the context whether the reactance or the inductance is meant. The same criticism applies to other places in the paper.

I should like to see the next administration of the Institute make a strong effort towards improving the "readibility" of the Institute papers. In a mathematical paper like the one under discussion a very careful list of symbols should be demanded of the author, and not left to the author's discretion. It is true that in this particular paper partial lists of symbols are found in two places, but they do not include, by any means, all the symbols, nor do they show in which units these symbols are expressed. I think that this is an important matter, and should not be left to the discretion of the author, but should be made uniform by definite rules of the Editing Committee of the Institute.

Now, as to the method of computation of inductance. A number of years ago our new honorary member, Mr. Oliver Heaviside, showed that inductance and magnetic permeance are almost identical concepts, that the inductance of a coil is equal to the permeance of the magnetic path, times the square of the number of turns. Since the number of turns is a numeric, the physical dimension of inductance is the same as that of magnetic permeance.

The expression for a magnetic permeance is similar to that of the electrical conductance. The permeance of a uniform path is equal to the permeabilty of the medium times the cross-section of the path, and inversely as the length of the path. If the path is not uniform a differential expression for permeance must be properly integrated to cover the whole field. What I am driving at is that computations of inductance should be performed in the same way as computations of the permeance or conductance of an irregular path, that is, by adding the permeances of the individual component paths. Most authors including these authors, compute inductances by taking ratios of magnetomotive forces to the fluxes. That is an unnecessary complication.

Suppose you had an irregular resistance to figure out, you would not assume certain e. m. fs. and currents, and divide one by the other, you would use a formula for an element of resistance, and properly make a summation of such terms for an irregular shape. Why, then, is it necessary in a paper on inductance always to use the ratio of ampere-turns to fluxe ? If you will glance at the formulas in the paper you will find every-

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where a ratio of flux to ampere-turns. Why not call this ratio the permeance, and deal with permeances entirely? This procedure has been followed consistently in the chapters on inductance in my book entitled "The Magnetic Circuit."

I notice from the list of references that the authors are more familiar with the English and German literature, and have apparently neglected the French contributions to the subject. While they give credit to Rezelman, through a translation in *The Electrician*, his greater work on leakage reactance in alternators in *La Lumiere Electrique* is not mentioned. Also there is no reference to the latest contributions by André Blondel on the armature reaction in synchronous machines. I believe the paper would gain if more use had been made of the work of the French and Belgian investigators on the subject.

I do not wish to detract from the value of the work done by the authors and the remarkable frankness with which the results are presented, including the test data. I fully appreciate the importance of such a paper for practical designers and students, and my only plea is for a better and clearer presentation of the subject. We are swamped with new reading matter, and yet a day still has only twenty-four hours, so that the skill in presentation almost determines the fate of a paper, that is, whether it will be used or not. If it is not properly presented so that it takes too long to get the meat out of it, many people will not take the trouble to refer to it.

F. D. Newbury: Before we can harmonize various methods of calculating reactance, we must agree upon a method of determining reactance by test; we must define the quantity we are attempting to calculate.

In the alternator, there are two fluxes that are set up by the armature current and the difficulty and confusion in this matter arises from the difficulty in separating one of these fluxes from the other. One produces armature *reaction* and the other produces armature *reactance*.

When the alternator is operated at synchronous speed, on short circuit, and rated current circulates through the armature, the value of field ampere turns is a measure of the m. m. f. necessary to produce both of these fluxes.

The flux of armature *reaction* is that part of the flux set up by the armature current that combines with the field winding flux to produce the effective or resultant exciting flux; the flux of armature *reactance* is that part of the armature flux that is more or less completely localized about the armature winding, and that, by its self-inducing effect, produces a voltage of self-induction in the armature winding that combines with the main induced voltage to produce the terminal voltage. Thus armature reaction may be conveniently treated as a flux phenomenon, as its action does not progress beyond the flux stage; following the same thought, the armature reactance may be considered as a voltage phenomenon. The effect of armature reaction

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on the total m. m. f. required for a given voltage and current condition is the same at zero voltage, normal voltage, or any other terminal voltage; the effect of armature reactance on the total m. m. f., on the contrary, varies considerably with the terminal voltage because it appears as a voltage and the m. m. f. to generate a certain additional voltage at low terminal voltages is very much less than the m. m. f. required to generate the same additional voltage at a high terminal voltage, because of magnetic saturation.

This distinction is important because it leads to a fairly satisfactory method of separating these two parts of the armature flux. I refer to the graphical method based on the no-load saturation curve and saturation curve at zero-power factor and full-load current. This method, I believe, was first called to the attention of the Institute some fifteen years ago by Mr. B. A. Behrend. In my experience, it gives consistent results for a wide range of machines. It has the advantage that it is obtained by test under flux and voltage conditions very nearly approaching those existing during normal operation. This cannot be said when the separation is based entirely on the short-circuit current at zero terminal voltage which is the method used by the authors. The authors calculate the armature reaction, which is the larger of the two parts, and subtract it from a test value, representing the total armature flux. This test method is in reality a calculated method, because an error of, say five per cent in the calculation of the armature reaction flux, may result in an error of 20 to 40 per cent in the reactance voltage. The graphical method referred to above also involves a calculated correction but a given error in this has a much smaller effect on the result than does the same percentage error in the calculated part in the authors' method.

Mr. Fechheimer referred, in his communication, to various methods of obtaining the reactance by test. Each of these methods involves different conditions, and no two methods really give the same quantity. The first difficulty in this problem, and the main difficulty, I feel, is that we have not agreed on a method of test for reactance. We have not agreed on the fundamental definition of the quantity we propose to calculate. If we can take that step, I think it is a fairly easy matter to obtain methods of calculating reactance that will check with such test results. It may not be possible to obtain a general formula such as the authors have tried to do, that will hold for all machines of all proportions, but it is not a difficult matter to make up a more or less empirical formula for a given line of machines which will check with test results.

As an example of results from the use of some of the methods cited by Mr. Fechheimer, I give the test results of a 4500-kv-a., 11,000-volts, single-phase, 25-cycles, 500 rev. per min. generator.

Tests were made at 4500 kv-a., single-phase, and at 7800 kv-a., three-phase:

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	At 4500-kv-a. 1-phase.	At 7800-kv-a. 3-phase.
From instantaneous short-circuit test, (by oscillograph)	13%	10%
From no-load and zero power factor saturation curves	13	11
From locked saturation test (rotor at rest)	15	12.5
From test without rotor in position	21	18

This machine had a low resistance damper winding on the rotor. From the locked saturation test, that is, with the rotor at rest, the reactance is somewhat higher than during synchronous operation, but still quite close to it. This is reasonable, because although the rotor is at rest, the low resistance damper winding practically wipes out that part of the flux corresponding to armature reaction in synchronous operation, so that there is left practically the same flux that, in synchronous operation, is responsible for armature reactance. The same agreement would not exist if the generator did not have the damper winding. A test without the rotor in position showed the reactance to be about 50 per cent greater, or 21 per cent. This, again, is reasonable, because the entire flux set up by the armature winding is effective in producing reactance voltage.

The second part of the paper, referring to sudden short-circuit phenomena, is extremely important, in a practical way. Reactance of alternator windings is of interest, quantitatively, only to designers, except as applied to sudden short-circuit conditions. In this matter, users generally are interested in the value of reactance since it largely determines the value of transient currents during short circuit.

The authors state that the total reactance effective during sudden short circuit is considerably greater than armature reactance, this difference amounting in some cases to 50 per cent of the armature reactance. The authors' method of obtaining armature reactance from tests is so open to question that this generalization is not warranted. In my own experience the armature reactance during synchronous operation (obtained from no-load and full-load saturation curves) is not materially different in value from the total reactance effective during sudden short circuit. This does not mean that the two reactances are identical; it only means that in the total reactance (effective during short circuit) the increase in reactance due to the field has been neutralized by some other action. What this complete action is, we don't know. The fact remains that in short-circuit tests made on generators varying from 4000 to 20,000 kv-a. in rating, with a wide range in speed, and with laminated and solid rotors, with and without damper windings, the tested

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sustained armature reactance has practically the same value as the total reactance obtained from short-circuit oscillograms. Here again, the difference in test results and conclusions may be due to differences and errors in determining the true armature reactance and agreement can hardly be expected until this question of test method is settled.

W. F. Dawson: Mr. Newbury raised the point of the discrepancies between the figures shown by Messrs. Doherty and Shirley in their method, and his own method, and I am moved to ask if he has considered fully the question of end coil support, the nature of the material that supports the end coil, particularly in turbo-alternators, and the material that clamps these coils? I know personally there is a slightly different method right within the General Electric Company's works. The Schenectady engineers employ a different material for coil support, and rather more of it that I do myself, and I believe the Westinghouse support is different again, and the method of clamping the coils is different. There certainly must be a difference in reactance, whether it is transient reactance or the steady reactance, that effects the excitation, due to these several different methods of support. I simply offer this as a suggestion which may explain the discrepancy.

V. Karapetoff: I should like to ask the authors regarding Fig. 25A. The orthogonal rulings shown there indicate magnetic lines of force and equipotential surfaces. This ruling extends into the space occupied by the coils. As far as I am aware, the magnetic flux has no potential in the space occupied by the current, and I do not quite see how equipotential surfaces could consistently be continued into the space occupied by the coils.

In the simplest case of a circular conductor the magnetic lines of force are concentric circles surrounding the conductor, and the flux inside the conductor also consists of concentric circles. The equipotential surfaces outside the conductor are radial planes, but these planes do not extend inside the conductor, because there the difference of magnetic potential between two points depends upon the path selected. I therefore feel that it might be better to omit the orthogonal ruling or at least the equipotential lines, within the space occupied by the coils, so as to avoid a possible criticism on this point.

Charles L. Fortescue: Is not the uncertainty of this method due, as Mr. Newbury says, to its empirical nature? If we consider the dynamic equations of a system like this, we find that we have a set of three or more linear differential equations of the first order. The solution of such a system of equations depends upon another differential equation, which is of the third or fourth order according to the number of equations. That would mean that the actual transient solution would involve three or more arbitrary constants and, instead of one reactance three or four different reactances would enter into the transient

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solution in a manner depending on the conditions of the initial short-circuit. It seems to me therefore that it would be very difficult, to express the results in terms of a single compound reactance. We would find possibly that the value of the effective reactance would depend upon at what point on the voltage wave the short circuit took place and that the solution for other conditions, would require different values for the effective reactance, or, putting it another way, one would have to have a different effective reactance for each short circuit. The problem is quite complicated, and as I said before, it is possible that the empirical nature of this reactance accounts for the fact that it does not check up under all conditions.

H. R. Summerhayes: This question of reactance is exceedingly important, not only from the standpoint of the designing engineer to ascertain what the machine will do but for the engineer working on the system who wants to know what amount of energy is released when a short circuit occurs. His calculations on the rupturing capacity of switches, the strength of transformers the installation of protective reactance and setting of relays all depend on the short-circuit values of the generator for different time intervals. In order to properly design the system we must know the characteristics of the generator and this work of the authors is a pains taking attempt to outline methods of obtaining these data.

I hope that the paper will lead to a discussion among the engineers and manufacturing companies which will, if it does not bring to standardization all methods of calculation, at least cause them to come to some agreement on the nomenclature, so that, when the reactance of synchronous machines is discussed engineers will talk the same language and will know what kind of reactance is meant and what factors are included. This will go a long way toward the correct determination of the actual and comparative values for different machines.

N. S. Diamant: In order to make the discussion of this paper as brief and as definite as possible let me start by making the following statements, even at the risk of seeming to be a little dogmatic:

A. The writers do not in any way contradict or disprove any theoretical or experimental results given in my 1915 paper on sudden short circuits.

B. It is very gratifying that they corroborate a number of points brought out in the paper referred to and even those which they think they contradict.

C. The writers are laboring under the serious and entirely false impression that *they* for the first time are calling attention to a fundamental fact which is the a, b, c, of sudden short circuits or of any transient phenomenon for that matter. It was clearly explained by Boucherot in 1911; it was referred to by Berg and experimentally illustrated in my paper.

D. The writers are using in their paper, as published at pres-

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ent, a terminology entirely different from that of other writers and mine, and for some reason which I cannot quite understand, they have failed to ascribe to the terms self-inductance the proper or at least a possible meaning and thus were led first to believe that they disagree with me in certain points, when they actually do not, and secondly were led to the false impression stated under C.

E. The writers make several valuable contributions: (1) They give an expression for calculating the armature leakage reactance. (2) An expression for calculating the leakage reactance of the field. This so far as I am aware is new, although the leakage of the field in "per cent" is derived and used by designers. (3) They have shown how unpractical and inefficient it is to use terms in a loose and unscientific fashion. (4) They have shown that the bibliography they give at the end of their paper, (to which it would be well to add Boucherot's memoirs and Douglas' paper) if properly studied and used, would have shortened their paper considerably and made it more useful. However, the mental attitude of the writers towards the work of others does not seem to be critical and aggressive enough if one is to judge from their statement—"A bibliography which may be of historical interest is included".

Now I shall proceed to prove the above statements, one by one. I regret that with reference to C and D I shall have to introduce here some elementary text-book material, as follows:

Definitions of Four Kinds of Inductances or Reactances

A straight thin wire carrying ten amperes and having Φ lines set up around it will have a self-inductance L equal to Φ abhenries. In case of a solenoid, etc. exactly the same definition holds, except that the calculation of L becomes more complicated. (See Bulletin Bureau of Standards, Vol. 8, No. 1, where formulas and tables are given for the calculation of L. For experimental measurement of L see Laws, Eric Gerard and other works on electrical measurements.) Now instead of a single isolated coil or system as above we have a second coil, mutually inductively related to the first, we introduce a second concept—that of the mutual inductance M. M is defined in any text in physics or electrical engineering as $\Phi_{12c}\, n_2$; where, Φ_{12c} = flux per abampere (flowing in the primary) which is common to both coils or links both coils, and n_2 = number of turns in secondary. Similarly if we reverse the role of the coils and send 10 amperes through coil No. 2, instead of No. 1 we get $M = \Phi_{21c}\, n_1$ ($= \Phi_{12c}\, n_2$ if μ is constant). For calculation and measurement of M see references given for L.

The above concepts of self- and mutual inductance are fundamental and absolutely necessary both to the physicist and engineer. However, the former seems to find these sufficient while the engineer is forced to introduce two more concepts, namely that of "leakage inductance" of primary, or secondary

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or of armature or of field etc. and that of "total leakage inductance reduced to" primary or to secondary or to armature, as the case may be. Let us denote these by α and λ respectively. Then L = leakage inductance of coil No. 1 = $n_1 \Phi_{1l}$ where Φ_{1l} is the leakage flux of coil No. 1, *i. e.* the flux that interlinks with coil No. 1 only. Thus if Φ_1 = total flux set up by 10 amperes flowing through coil No. 1 = $(\Phi_{12c} + \Phi_{1l})$, we have,

$L_1 = n_1 \Phi_1$ = self-inductance of coil No. 1
$\alpha_1 = n_1 \Phi_{1l}$ = leakage inductance of coil No. 1
$M = n_2 \Phi_{12c}$ = mutual inductance of coil No. 1

Similarly if we reverse the role of the two coils we get,

$L_2 = n_2 \Phi_2$ = self-inductance of coil No. 2
$\alpha_2 = n_2 \Phi_{2l}$ = leakage inductance of coil No. 2
$M = n_2 \Phi_{21c}$ = mutual inductance of coil No. 2

From the above it follows directly that,

$$M^2 = (L_1 - \alpha_1)(L_2 - \alpha_2)$$

and if we imagine the leakage of No. 2 to be zero and charge all the leakage to coil No. 1 we get

$$M^2 = (L_1 - \lambda_1) L_2$$

or $\lambda_1 = \left(L_1 - \frac{M^2}{L_2}\right)$ = total leakage inductance reduced to coil No. 1.

Similarly for λ_2.

Simple Application of Above. Consider the case of a single coil or in general the case where the mutually inductive effect of coil No. 2 is nil. Then, if we impress an alternating e. m. f. of frequency f across coil No. 1, I_1 = current in coil No. 1 $= E_1/Z_1$ where $Z_1^2 = r_1^2 + 2\pi f L_1^2$. This of course refers to permanent conditions. Under transient conditions, with either alternating or direct current we will find that the rise or decay of current depends upon α_1 = attenuation factor of coil No. 1 $= r_1/L_1$.

Next let us take the case where $M \pm O$, *i. e.* there is a closed coil No. 2, mutually inductively related to No. 1. If we impress the same e. m. f. as before, we shall find that I_1 = current in coil No. 1 $= E_1/Z_1$ where now Z_1^2 is equal to $(r_1^2 + 2\pi f \lambda_1^2)$ and not $(r_1^2 + 2\pi f L_1^2)$ and the latter may be 5 or 10 or 20 times as large as the former. Furthermore α_1 = attenuation factor of coil No. 1 $= r_1/\lambda_1$ and not r_1/L_1 as before. Thus as we all know, from our study of a choke coil and transformer, it makes all the difference in the world whether there is a secondary coil affecting the primary coil or not. At this point let me call attention to: First, that the matter is simplified like this, if the secondary resistance is negligible; and secondly, that though I have used the general terms coil No. 1 and No. 2 the above apply to any system be it primary and secondary or field and armature or rotor and stator, etc.

In my papers I have used the above terminology and I propose to do the same in this discussion, and for sake of consistency

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in quoting from Doherty's and Shirley's paper I shall call the *leakage inductance*, leakage inductance and not self-inductance. It is true that with our typical engineering looseness and crude superficiality some authors use the two terms as synonymous, just like the writers do; but I find it extremely difficult to explain or understand why the writers failed or refused to ascribe to the term self-inductance, as is used in my paper, its usual meaning of self-inductance, at least as a possible interpretation. Instead no matter how clearly I expressed myself they have taken self-inductance to mean leakage inductance, or something akin to it.

With reference to the reactance which affects sudden short circuits, they state that "the authors will show that this reactance includes the field leakage inductance (writers use term, self-induction) as well as that of the armature." Unfortunately for the authors this has been shown by Boucherot in 1911, who discussed the matter, in a very much more thorough and masterly way than the authors have done. It may be well to call attention here to the fact that my discussion of L, α, M and λ is not quite as thorough as Boucherot's who gives seven and one-half pages to this question, from a purely theoretical point of view. Those interested further on this may consult his memoir. However, even if the writers did not consult Boucherot, it is difficult to understand why they overlooked the following statements in my paper in 1915 TRANS., page 2244: ". . . x_f is not the self-inductive reactance of the field spools but the total leakage reactance of the field and armature. Similarly α_a is a complicated quantity . . . The complex nature of these quantities has been recognized by several investigators such as Miles Walker, and Berg who without attempting to define them rigorously (from a mathematical point of view as Boucherot has done) has called them the equivalent resistance and reactance. . . Others, however, have apparently failed to understand the real meaning of these and have attempted either to simplify them too much . . . or complicate them unnecessarily. . ." Following these remarks I described experimental methods of determining α_a and α_f from the rise of direct current in the armature and field respectively. I showed that the attenuation factor α_f obtained with the secondary or armature open-circuited and at rest is quite different from the attenuation obtained with the armature dead short-short-circuited and the machine running at full speed. I went a step further and showed that α_f obtained from the rise of direct current in the field with the armature running at full speed and short-cricuited through a high resistance is practically the same as α_f obtained with the armature open-circuited and at rest. Thus I illustrated fully the effect of mutual inductance which may be negligible even with the secondary closed. Now it is clear that in the 1st and 3rd cases, above $\alpha_f = r_f/L_f$; while in the 2nd case $\alpha_f = r_f/\lambda_f$. I called attention to these facts at the top and bottom of page 2246 TRANS. 1915.

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To quote from page 2246 this is what I said: in this case we have "the important difference that x_f is no longer the true self-inductive reactance but the total leakage reactance reduced to the field circuit" I fail to see how one could have expressed the facts more clearly. Again on page 2247 it is stated that "for this particular machine, . . . the ratio of the self inductive reactance to the total leakage reactance reduced to the field is about three." Immediately following this, I state that ordinary methods of calculating x_f from equation (**14**):

$$\frac{N_f \Phi_f}{I_f 10^{-1}} \text{ abhenry, or more accurately}$$

$$\frac{N_f \Phi_f}{I_f 10^{-1}} \text{ (1 + per cent leakage of poles)} \qquad (14)$$

"utterly fail to give correct results" since "it is the attenuation factor involving the leakage reactance that enters the problem." All the above quotations occur within four pages of each other. However in spite of all this, the writers give the foregoing equation and triumphantly state that in the field attenuation factor, the inductance is not as Diamant gives the exciting inductance of the machine . . . but rather $\Phi^1 \; N_f \,/\, I_1 \; 10^{-1}$ where Φ^1 is the flux, linking the field winding, which I_f would produce in the leakage paths of both field and armature." It may be well for the writers to compare this and other long clumsy expressions which they use—see for example (**5**) in their statement of conclusions, etc.—to the standard brief and clear statement: $\Phi^1 \; N_f \,/\, I_f \, 10^{-1} =$ total leakage inductance reduced to the field.

On page 1250 the writers make the following statement: "The authors would make it clear at this point that they are not in agreement with conception given in Diamant's paper . . ." I trust that it is clear to any one by this time that the authors really do not disagree with any of my conceptions—at least I agree with theirs although I am inclined to think that they are not practical and scientific enough either in expressing their ideas or using the proper terms. I shall take up these points again later, but right here, let me illustrate the truth of my statements: They begin by stating clearly that they do not agree with the *conception* given in my paper and to prove this they quote from my paper that the alternating component "is equal to the maximum phase voltage divided by the armature impedance, with good approximation." Now I fail to see why the writers use such poor logic. In engineering problems we frame a more or less ideal theory and base our conceptions on it; but in practical calculations we are invariably forced to make approximations and the good engineer is the one who can make good approximations. Thus my *approximations* do not necessarily affect my *conceptions*.

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However, if we go a step further we shall find the extremely gratifying fact that the writers are entirely in agreement with my "good approximation" which I found to be true for 1 or 2 machines and theoretically sound; while, they have found them to be true from extensive tests on a great many machines. Within one-half page from where they say that they do not agree with me, they state that in case of machines with solid steel rotors or alternators equipped with low resistance amortisseur windings "the armature leakage inductance is a reasonably accurate measure of the sudden short-circuit current." Furthermore they not only thus corroborate my approximation but "propose its use for at least the present." See also (b) and (c) page 1281. In connection with this the writers deserve credit for determing that this approximation is allowable for machines with low resistance amortisseur windings only, as was the case with the machine I used, as stated in my paper.

In 1915 I called attention to the fact that single-phase sudden short circuits are quite complicated and that in case of a short circuit between neutral and line the armature current, for a given set of conditions, is about 140 or 150 per cent higher than in case of three-phase or line to line short circuit. Further I urged care in such cases and state clearly whether by single-phase short circuit reference is made to a short circuit between line and line, or line and neutral.

It is very gratifying that from extensive tests the writers corroborate both of my results, namely, that for a given set of conditions, three-phase or line to line short circuits give about the same rush of current; but short circuit between line and neutral is more serious. They offer an explanation for the latter on the basis that with three-phase currents flowing in the armature the leakage inductance is increased 50 per cent on account of the mutual inductance between phases. (See their paper bottom page 1262 and top of 1263.) Reprints of the paper were available so late that I have been unable to check this explanation. However, I would ask the authors to explain on the same basis why there seems to be practically no difference between a three-phase and line to line sudden short circuit.

I called attention in 1915 to the fact that α_f, when determined, either directly from oscillographic records according to methods developed on pages 2260-2264 or when obtained experimentally from the rise of direct current in the field with the armature dead short-circuited and running at full speed, comes out larger for the first cycle or so of the transient than for subsequent ones. I gave an explanation of this on page 2262 and it is gratifying that the writers not only corroborate this fact but agree with me that until we know more about the subject it is well to use the lower and safer value.

On page 1255 they state that "it may not have been so clear how and to what extent the field leakage inductance affects the short-circuit current." As I have already shown the writers

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are simply laboring under the wrong impression, in making a statement like this and I regret to say that their attempt, to show that the quantity to be used is the total leakage inductance reduced either to field or armature is rather crude and superficial compared to the explanation of Boucherot. They devote several pages to this (pages 1251 to 1259) and therefore I may be justified in calling attention to the following. On page 1251 they give the important law that "an electric circuit, without resistance, must so long as it is closed contain the same number of magnetic interlinkages. Now it is really unnecessary to make the ideal assumption of a circuit without resistance." Let me state the law, which as they say *is* very important, in a clear general way as follows. Let the flux in the field, or any electric circuit in general, change from Φ_1 to Φ_2 during the transient interval t; then the e. m. f. induced by this change will be $\frac{\Phi_1 - \Phi_2}{t}$. This transient e. m. f. will produce a quantity of electricity $Q = i \times t$ where i is a decaying direct current. As I pointed out in a recent article in the *Electrical World* this is a well known fundamental law used sometimes in electrical measurements (see *Electrical World*, May 18, 1918 page 1030, foot note 15). Let us apply this principle to the alternator. During sudden short circuit the flux in the field dies down from Φ_f to Φ_{fsec}. This produces a decaying direct current in the field which in turn produces the decaying alternating component of the armature current. Again if for any given armature phase there be a change of flux from Φ_a to Φ_{asec}; this will produce a decaying direct current in the armature etc.

For further details the reader may consult Boucherot or the 1915 paper of mine referred to by the writers or the recent articles in the *Electrical World* of May 18 and June 1. Before leaving this subject which has been well thrashed out in the above references let me call attention to the fact that the writers should be careful not to confuse the explanation of sudden short circuits based on the flux per phase or m. m. f. per phase and another possible explanation based on the total polyphase flux or m. m. f. of the armature circuit. The latter explanation is given in a short article, illustrated by oscillograms, in the *Electrical World*, Dec. 29, 1917, page 1251 and it is not necessary to go into it here.

Another point that I must mention in this connection is this. The statement of the writers on page 1275 that "the alternating component, . . . depends upon the flux per pole, . . . " is entirely wrong. So is the next statement, that the "direct component depends upon the flux linked with the armature circuit." As I have just explained, and as required by the fundamental law given on page 1251 by the writers, and which I have endeavored to generalize, the alternating and direct components depend upon the *difference* between normal and

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sustained short-circuit fluxes ($\Phi_f - \Phi_{fssc}$) and ($\Phi_a - \Phi_{assc}$) and not upon the flux per pole or the armature circuit as the writers state. This is a very fundamental point and there is little use for the writers to take several pages to explain things in a crude fashion, compared to Boucherot's explanation or the one given by me in the references already mentioned, and then come back and state that the direct component depends upon the flux linked with the armature circuit. No, first it depends *not* upon the *flux* but the *difference* in flux at the instant of the sudden short circuit and at the end of same; secondly, it depends upon the difference in flux at the start and end of sudden short circuit linking with the phase we are considering and not linking with the armature circuit.

Furthermore it seems to me that their statement that the direct component depends upon the terminal voltage is wrong. I think, it should be internal voltage due to the resultant flux.

There are other points in connection with appendix A and the factor $K\phi$, etc., that I would like to discuss, but as I have taken enough time I shall leave those to others. However, as I am anxious to see some good come out of this discussion, I will make the following definite proposals.

(1) That the qualifying adjectives sudden and sustained or permanent, etc., be used to qualify the kind of short circuit implied whenever there is room for ambiguity. Otherwise the shorter term short-circuit be used.

(2) That the terms leakage inductance and leakage reactance be exclusively used in referring to these quantities.

(3) That the term self-inductance or self-inductive reactance never be used to mean leakage inductance or leakage reactance. This is unscientific and impractical, since no term is left to be used to refer to "self-induction." Furthermore the word leakage is 50 per cent shorter than self-inductive and tells us the whole story.

(4) The term self-inductance be reserved to refer to what is sometimes called coefficient of self-induction. This is the universal term used consistently by physicists, writers on electrical measurements etc. etc.

(5) In conformity with transformer practise the sum of leakage reactances of two systems, be referred to total leakage inductance reduced to primary, or armature etc, as the case may be.

(6) If desired, the term inductance or reactance be referred to any of the above when there is no danger of ambiguity.

(7) According to the above it is the only common sense and practical scientific procedure not to introduce any terms like "transient reactance," "exciting reactance" etc. The x_{pa} of the author's is simply the leakage reactance of the armature; the quantity that one would use in calculating regulation etc. x'_{pf} is simply the leakage reactance of the field. x_{pf} (see equation (4) page 1259) is simply the leakage reactance of the field reduced to the armature, and it is extremely interesting to note the way the

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writers express this on page 1259. They say, "the effective field self-induction or field reactance, x_{pf}, expressed as per cent reactance in the same terms as the per cent armature self-inductive reactance." Why not say x_{pf} is the "leakage reactance of the field reduced to the armature?" This is very much more simple and familiar, since we have the same thing in connection with the transformer. The writers, however, do not stop here, they take the sum of the leakage reactance of the armature and leakage reactance of the field $= x_{pa} + x_{pf} = x_o$ and they give to this a new impressive, misleading name of "transient reactance." I say misleading because I wonder how many engineers suspected that this wonderful "transient reactance" is nothing but the "total leakage reactance reduced to the armature."

It is well to note here that the writers in introducing this short term did not even attempt to explain its real meaning or give any reasons for introducing such a term. I am afraid they are thus preparing the ground for further unnecessary complications and misunderstandings; these are very common and annoying in practise and there are cases in which they cost quite a good deal of money and trouble. The point I wish to make is that the loose unscientific terminology of the writers or any engineer is not an academic question but an extremely practical one.

Thus why not call x_o, the total leakage reactance reduced to armature—because that is what it is. The length of the term will not cause half as much trouble as misunderstandings in discussions, letters, contracts and specifications etc., etc. Furthermore if it is necessary to have a short term why not call it "equivalent reactance" this I believe is used in transformer practise in referring to $(x_{\text{primary}} + \alpha^2 x_{\text{sec.}})$ and we ought to try and keep our electrical engineering theory, practise, terminology etc. as a single unit rather than narrow so much that those who specialize in alternators, transformers, electrical measurements etc. etc. should develop theories and terminologies without due regard to the needs of their fellow workers. Personally I care very little what the outcome of the above may be, but I am inclined to think that it is to the best interests of the profession to standardize things in some such way as given above and not introduce any unnecessary weird sounding impressive terms.

In conclusion I would like to ask the opinion of the authors as to the results of a paper which was presented at the Pittsburgh meeting of the A. I. E. E. by Douglas. This paper, represented what seemed to be fairly exhaustive work carried on at Cornell on the question of reluctance and leakage reactance of machines. It gave flux distribution figures similar to 24a, 24b, etc. and checked these experimentally. It stated clearly that present day formulas for calculation of field leakage, etc. are very inaccurate. I hope this paper was of more than "historical interest" and I am interested to know what the writers think about it.

C. M. Laffoon (communicated after adjournment): Additional refinements in the methods of calculating the reactance of syn-

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chronous machines, as suggested by the authors, are important on account of the inherent relation of the reactance to the still unsettled questions of field excitation, voltage regulation, and generator protection under short-circuit conditions. Before such refinements, however, can be applied, and the results generally accepted without serious confusion, the following two fundamental conditions should be universally agreed upon and adopted;

(a) A concise interpretation of what is meant by, and included in the term armature reactance under different stable and transient conditions; and,

(b) A standardized reliable test method for checking the calculated values of armature reactance and the determination of constants of reactance formulas.

In extending the refinements to the calculation of the reactance of polyphase synchronous machines, the authors are justified in raising the question whether or not a portion of the armature reactance flux of each phase interlinks part of the conductors of the other phases. Obviously, this is true to a limited extent, with regard to the end connections, in which the reactance is produced by the total flux interlinking this part of the winding; but for the leakage flux, which crosses the slots and leaves the tooth-

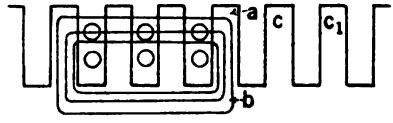

FIG. 2

tips, it is not so apparent that there is mutual reactance effect between the different phases. On polyphase synchronous machines the magnitude of the mutual inductance effect, if any exists, of the different leakage reactance fluxes, can be determined analytically only by actually laying out the flux diagrams produced by all of the electric circuits acting simultaneously (taking into consideration the difference in permeability and the damping currents in the iron portions of the magnetic circuit) and combining the different fluxes in the proper space relations at different instants of time. This is an extremely difficult task to accomplish on account of the varying saturation in the iron, and irregularity or indefiniteness of the flux paths. At best it can only be very roughly approximated, even when wide sweeping simplifying assumptions are made. Since the authors apparently have not followed this procedure in arriving at their conclusions, the writer feels that they have neither established nor justified their major premise that the leakage reactance in three-phase synchronous machines is increased approximately 50 per cent, due to a mutual leakage inductance effect, while in two-phase machines it is negligible.

If the slot leakage flux is assumed, to act independent of other flux interlinking the coils, it is evident from Fig. 2 that the flux

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produced by the current in a given phase belt will not cross the slots of the other phases from a to c, c_1, etc., unless there is an appreciable m. m. f. drop from a to b. Then, on the basis of the usual assumptions, wherein the saturation of the teeth is totally neglected, there would be no m. m. f. drop from a to b, and hence no flux from a to c, c_1, etc. But in actuality there must be more or less of an m. m. f. drop from a to b, due to the other flux, which is present, and hence some mutual inductive effect must exist. However, the increase in slot reactance, due to this effect, would in all probability be more than compensated for by considering the total m. m. f. of each phase belt acting across its own slots.

The extent to which the tooth-tip leakage flux produces a mutual inductance effect upon the adjacent phases depends upon the path that it is constrained to take. If the leakage paths were confined to an air circuit, the density of the tooth-tip leakage flux would diminish very rapidly, (assuming as before that this flux is free to act independent of all other fluxes) as the distance from the magnetic center of the phase group around the armature per-

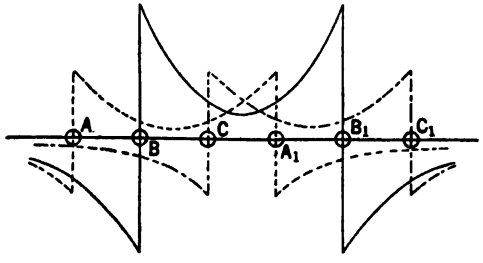

FIG. 3

iphery increases, and consequently the mutual leakage flux would be small, if not inappreciable. However, when iron is in the presence of the teeth, the path of the tooth-tip flux is dependent on the action of the damping currents in the iron. It is the writer's opinion that the authors have idealized the laminated salient-pole rotor, and have not given sufficient consideration to the damping action of the eddy currents which unquestionably circulate in the lamination of the pole faces, rivets, bolts, etc. The cross-section of the tooth-tip flux path must be more or less restricted and confined to the air gap on account of the damping action of these currents. This in turn tends to minimize the mutual effect of this part of the leakage flux. The authors have not made it clear in their presentation how the armature current can force leakage flux across the air gap from the pole face to the teeth of other phases when the same m. m. f. has been used on the same path as a counter m. m. f. to the field m. m. f.

In the case of the end connections, where there is no question concerning the existence of a mutual inductance effect, the flux density diagram produced by each phase will be as in Fig. 3. if the following radical simplifying assumptions are made.

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(a) The adjacent iron parts of the machine do not affect the flux interlinking the end connections;

(b) The end connections are rectangular in shape and very long;

(c) The machine has only one slot per phase per pole; and

(d) The equivalent conductors are circular in section.

Under these conditions, it can readily be shown, by combining the respective flux forms, that the mutual inductance effect could at the outset amount to only a few per cent. Moreover, in actual machines the percentage of mutual inductance effect will be still further reduced on account of the fact that in most machines with V-shaped end connections, half of the end connectors of each phase are at approximately right angles to one-half of the end connectors of the other phases, and the mutual flux interlinking the end turns is thereby very appreciably reduced.

At all events, it is certainly evident from the above that the space distribution of the leakage flux of each phase is not sinusoidal and, consequently, in three-phase machines there is not a simple rotating field of leakage flux, as in the case of the work flux of an induction motor, and the mutual inductance effect in no wise amounts to 50 per cent. These conclusions are practically confirmed by test results, when single-phase and polyphase currents are circulated in the stator with the rotor in and removed, as indicated in Mr. Newbury's discussion.

The argument, advanced by the authors, that the mutual inductance effect must be 50 per cent because certain formulas which gave good results on three-phase machines gave results approximately 50 per cent too high on two-phase machines, is not necessarily conclusive, for the fault might have rested elsewhere in the formula. The writer has used certain formulas that did not consider any mutual leakage inductance effect, but which checked test values of reactance equally well on both two and three-phase machines.

Even if it is assumed that the damping currents are negligible, and the leakage flux is free to enter or leave the pole face, the authors' method of calculating the tooth-tip leakage flux is not fundamentally sound, for it assumes that all of the tooth-tip flux passes from teeth to pole face, independent of the ratio of the slot width to the air-gap radial length. It is evident that if the slot is wide, compared to the air gap, as shown in Fig. 4 a most of the tooth-tip leakage flux would cross the gap and then return to the next tooth. However, if the slot width is narrow, compared to air-gap width, as shown in Fig. 4b, the most of the tooth-tip leakage flux will not enter the pole face at all.* Since in actual machines the ratio of slot width to air gap varies through fairly wide limits, the method of calculating tooth-tip leakage must consider the total flux that leaves the tooth tip. That is, it should include the flux that crosses the equipotential

*This argument applies particularly to machines which have few slots per pole per phase.

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section *ab-bc* (Fig. 5). The part of the flux that enters *ab* can be found by using Carter's pole fringing constant for different ratios of slot width to air gap; the part that crosses *bc* is more difficult to determine. In determining this flux, the author has apparently used Carter's coefficient for slots and vent ducts, which is based on the assumption that the tips of the teeth are at the same magnetic potential and that there is no tendency for the flux to pass from tooth to tooth. This is evidently incorrect, for it does not include the flux crossing section *bc*. Table I shows the values of reactance as calculated by the authors' formula, and the formulas used by the writer for machines with

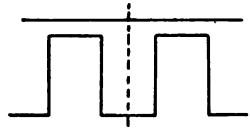

FIG. 4A

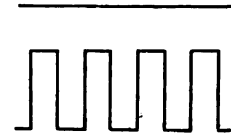

FIG. 4B

different ratios ot slot width to air gap. These results indicate that in the case of machines having small gap and wide slots, the calculated values of reactance obtained by the two formulas are in reasonably close agreement, but as the ratio of slot width to air gap decreases, and a smaller percentage of the tooth-tip flux crosses the gap to the pole face, the calculated values of reactance by the two formulas diverge very rapidly.

In this connection, the writer also questions the advisability of using curves which are based on the ratio of the maximum to the minimum air gap, to determine constants for the tooth-tip

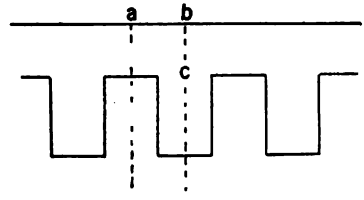

FIG. 5

leakage formula. In the first place, such curves could only be applicable to machines having a given pole-face shape, for the maximum and minimum gaps have little or no relation to the variation of the air gap in the intervening space. In the second place, the maximum gap is not a definite quantity and is subject to considerable variation in interpretation as to where it should be measured. At least, this was the writer's experience in attempting to calculate the reactances of several machines by the authors' formulas. The most natural and logical place to measure the maximum air gap would be along the path that the

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no-load flux is assumed to take. It is true that in laying out the no-load flux form, it does not make a great deal of difference how the lines are assumed to pass, so long as a consistent policy is followed. But in the determination of the authors' constants, it makes an appreciable difference, for a small per cent increase in the maximum air gap may mean quite a large variation in the ratio of maximum to minimum gap.

Much of the confusion and difference of opinion concerning the reactance of synchronous machines is due, as has been previously suggested, to the fact that there is no standard and accepted method for determining it experimentally. The test method used by the author is in reality a calculated method, and, as such, is subject to serious limitations, for the armature reaction forms the major portion of the measured field ampere-turns, and a small percentage error in its calculation causes a large percentage error in the test value of the reactance.

In the writer's opinion, the most satisfactory experimental method, at present, of determining the reactance is from the no-load saturation and full-load zero power factor saturation test curves, used in connection with the calculated curve, which shows the increase of ampere turns on the no-load saturation curve due to the full-load leakage. This method also more nearly approaches the actual load conditions than any of the others. It is true that it is subject to limitations in that (a) the increase due to full-load leakage curve is calculated and subject to error, and (b) the reactance is probably not constant for all points on the saturation curve. The latter is not felt to be a serious disturbing limitation, for it is probably of the same order of magnitude as the errors in ordinary commercial tests. Granting the same degree of accuracy in the calculated terms by the two methods, the error in the test reactance obtained from the above method would be considerably less than that obtained by the method used by the authors, (as has been pointed out in Mr. Newbury's discussion). However, in most cases of salient-pole machines, in which the saturation of the pole core and yoke is not carried too high and the percentage field leakage flux is not too large, the increase in the no-load saturation curve, due to full-load leakage, can be calculated with a reasonable degree of accuracy. However, the method of obtaining it is apt to be rather long and tedious, and require painstaking effort. The accuracy of the calculated curve can be verified to a certain extent by checking the no-load and full-load zero power factor saturation test curves by the same method.

In columns 5, 6, and 7 of Table I are tabulated the per cent reactance as obtained by the authors' formula, the formula used by the writer, and by the above mentioned experimental method for a number of two and three-phase laminated salient-pole synchronous machines of quite a wide range in rating. This comparison indicates that there is a very close agreement between the results obtained by the method used by the writer

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and the test method for both two and three-phase machines, whereas, as was previously stated, the values obtained by the authors' method check the test results reasonably close for machines which have a large ratio of slot width to air gap, but for machines with a small ratio of slot width to air gap, it gives results from 30 to 50 per cent too low.

It is becoming more generally recognized by different engineers that the maximum value of the initial current delivered by an a-c. generator under short circuit is limited by a reactance more complex than the armature reactance under normal balanced load conditions. For, under short-circuit conditions, the armature reaction pulsates in magnitude, and shifts in position with respect to the field poles, and alternating currents are induced

TABLE I.

Machine	Kv-a.	Phase	Slot width/Air gap	Per cent reactance		Test
				†D. & S.	‡*L*	
1	500	3	4.94	20.6	22.7	23.1
2	175	2	4.67	..	21.5	22.75
3	1132	2	3.56	..	18.5	19.0
*4	1250	3	3.4	9.6	10.9	..
5	875	3	2.78	13.6	16.7	16.5
*6	700	3	2.4	13.6	22.85	..
7	600	3	2.34	10.2	13.2	..
8	1132	2	1.92	..	15.0	14.1
9	1132	2	1.92	..	17.7	18.1
10	1250	3	1.62	11.7	15.7	..
11	3750	3	1.6	9.36	13.63	13.25
*12	560	3	1.067	17.6	26.0	..
13	12500	3	0.94	13.2	20.2	19.3

*The design data on machines No. 4, 6 & 12 were submitted to writer by Messrs. Doherty & Shirley.

†The values of reactance in column 5 were obtained by Doherty & Shirley formula.

‡The values of reactance in column 6 were obtained by formula used by writer.

in the rotor and field circuits. The magnetic leakage of these circuits, consequently, produces a self-induction which must (after multiplying by the proper ratio of transformation) be added to the armature self-induction in order to give the short-circuit or transient reactance, just as in a transformer.

The authors, in treating this problem, apparently have assumed that the armature component of the transient reactance is the same as the armature reactance under normal load conditions, and have derived their reactance formula to meet this condition. It would be more logical to consider the armature reactance under stable and transient conditions as different phenomena, and treat each case separately.

It has been the writer's experience that the calculated value of the armature reactance does not happen to differ widely from the value of the transient reactance determined from short-

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circuit oscillograms. Table II shows the calculated value of the armature reactance and the transient reactance as determined from oscillographic records for several different types of synchronous machines. There does not seem to be any very definite relation between the stator reactance under load and the transient reactance for different classes of machines, and it is our opinion that it is more or less of a coincidence. However, the coincidence can be partially explained on the basis as suggested above, that the armature reactance under short circuit is not the same as under load; that is, the value of the armature reactance under short circuit is decreased by an amount approximately equal to the added equivalent field reactance. This reduction of the armature reactance may be due to (a) the saturation of the iron part of the magnetic circuit of the leakage paths, and (b) the

TABLE II.

No.	Machine.	Kv-a.	Phase.	Number phases short-circuited.	Ohms reactance	
					Calculated. Syn. operation.	From osc. trans. conditions.
1	Turbo-Alt.	15800	3	1	1.622	1.572
2	" "	"	"	3	1.622	1.695
3	" "	15000	"	"	1.09	1.11
4	" "	20000	"	1	1.96	1.87
5	" "	15000	"	3	1.82	1.61
6	Syn. Cond.	4500	"	"	3.84	3.87
7	" "	"	"	1	3.28	3.95
8	Alt. Gen.	12500	"	3	1.22	1.25
9	" "	"	"	"	1.22	1.28
10	Syn. Cond.	15000	"	"	1.066	1.064

NOTE: Machines 1 to 5, inclusive, are of solid cylindrical rotor type; the remainder are of the laminated salient pole type; 6 7 & 10 are equipped with damping windings.

reduction in cross-section of the leakage flux paths by the damping currents in the field poles and adjacent iron parts. The writer is also of the opinion that this damping action is not confined to solid types of rotors, but exists to an appreciable extent in laminated salient-pole machines. However, definite quantitative conclusions, regarding the effect of saturation and the damping currents on the armature reactance under short-circuit conditions, cannot be given until more complete experimental data are available.

R. E. Doherty: If there is a single characterizing feature of all this discussion, it is disagreement. The authors naturally expected much disagreement on a subject of this nature, and of course were not disappointed on that score. They were disappointed, however, at the nature of the criticism. While some of it contains constructive suggestions, much of it is purely destruc-

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tive. It says the method, and therefore the calculated result, is wrong; yet it does not show how to make it right, nor does it give adequate reasons for saying the method is wrong. For instance, Messrs. Newbury, Fechheimer and Laffoon, who, as designers, are in possession of data from which to form opinion, apparently all agree that the method of calculation is fundamentally wrong because they believe serious factors are omitted; that even if it were correct, the method employed to arrive at the "test" value is wrong, because, they believe armature reaction can not be calculated with sufficient accuracy: and therefore agreement of "test" and calculated results could not be of much significance. But not a word is said about the theoretical soundness of the method, given in Appendix B of the paper, for more accurate calculation of armature reaction, taking into account the influence of harmonics in the flux wave. This influence has always been the cause of the greatest uncertainty regarding armature reaction calculations. Neither do they show any way of including the factors which they believe we have omitted. They are therefore urged by the weight of that acknowledgment to suggest, in the way of an alternative, the old friend—the expedient of having different formulas and different empirical constants for different lines of machines. That is, they are apparently agreed that this thing which we are trying to do can not be done. Mr. Fortescue subscribes heartily to that viewpoint, giving reasons in mathematical terms which to those even moderately versed in mathematics, can not possibly have much significance, and which to others seem to be evidence of a more thorough knowledge of mathematics than of alternators, inasmuch as he fails to consider that "in a system like this" the integration constants for his different differential equations may turn out to be the same constants for all, and that he may as well have had one equation. This is not the first instance where a voice has been raised that "it can not be done", when at that very moment it was being done. It may not be possible to obtain a single reactance formula, which can be depended upon in practical application to synchronous machines of all ranges in speed, voltage and capacity now being manufactured, but as evidence that it might be possible, we submit the fact that one of the large manufacturing companies has actually used such a formula for about two years, and the predictions based on those calculations have proved to be dependable. And that formula is the one given in the paper.

That sums up in a general way the larger and more serious objections to the paper. There are, of course, several others which, although not of a nature to determine whether the results of the paper shall stand or fall, are yet very important. These, as well as the individual points included in the discussion referred to above, will now be taken up in order.

Prof. Karapetoff offers a number of criticisms. First, he contends that "field self induction" and "field reactance" are

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different concepts, and therefore the authors can not properly say "field self-induction or field reactance." If he had read the next line of the same sentence he would have seen that these terms are expressed as a ratio, *i. e.*, percentage, and therefore are numerically identical. This result is what the sentence which he criticizes, refers to.

He believes next that the Institute should demand of the author a list of symbols, instead of allowing him to define them in the context, and thereby increase the "readability." That may be worth while, but there are obvious advantages and disadvantages on both sides, which have been discussed so often we shall not enter it here.

Referring to Fig. 25 A, he asks why flux and equipotential lines are drawn through the field conductor, since he understands that the flux has no potential in space occupied by current. The paper says: "In Fig. 25 B the paths as indicated by the iron filings can not be traced to the pole body on account of the field winding. But it is clear from the direction at the edge of the field winding that they will terminate approximately as shown in Fig. 25 A." Further than that, we can not answer.

Prof. Karapetoff's other criticism is that the authors are guilty, with many other authors, of expressing inductance as a ratio of flux to ampere turns, instead of taking simply the permeance. He then calls attention to the fact that the permeance idea is followed consistently in his book on the "Magnetic Circuit." Our supposition is that because most of us were started off in college from the "interlinkages per ampere" viewpoint, we are inclined to fall back upon that viewpoint both in writing and in our effort to understand what is written. In order, therefore, not to decrease the "readability" of the paper, the authors were less interested in revolutionizing peoples' viewpoint to confirm to Prof. Karapetoff's book than in helping readers understand the problem in hand by using terms familiar to them.

Messrs. Newbury, Laffoon and Fechheimer have raised a fundamentally important question, what is "the quantity we are attempting to calculate." Also how are we to test for that quantity in the actual machine. What the authors consider as the "quantity," must be clear to any one who has studied the paper. They were not aware that there was so much uncertainty regarding what particular reactance should be considered in various circumstances of operation. Since such uncertainty and also the question of testing do exist, these should be cleared away to permit some approach to agreement.

As a starting point toward settling what armature reactance means, take Mr. Newbury's lucid explanation regarding armature reactance and armature reaction. We will take up transient reactance later. With that portion of his discussion (1st four paragraphs) covering the explanation, we are in entire agreement, as may be obvious from reading the paper. Figs. 27 A and 27 B show our conception of the fluxes he describes. There can

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be no question, therefore, about what leakage flux is involved when we say *armature reactance*. Fechheimer's discussion breaks the accord at this point, by injecting the old bogy of harmonics which produce distortion and upset all calculations; then he closes by admitting: "that the distortion is very much reduced at full load, zero power factor, is quite evident." Now in order to compose this particular difficulty, let's say we mean the armature reactance which exists under zero power factor condition—*i. e.*, armature and field m. m. f's. in direct opposition. Then the complication of distortion is eliminated, and we can start once more. If it happens, as actually does and as explained in the paper, that this same value may be applied in regulation calculation, etc. under watt load conditions also, that is another matter which has no direct bearing on this present problem of establishing a bench mark to work from. Permit us to say with much emphasis: that is the reactance which is called *armature reactance* in the paper, and which equation (**47**) in Part 1 is intended to calculate. It is the calculated value listed in Table I.

Before taking up the difficulties involved in testing this value consider further Mr. Fechheimer's discussion regarding the influence of the harmonics which remain, even at zero power factor. It is pointed out in the paper that the salient-pole construction causes variation in reluctance of the leakage path, producing principally a third harmonic, which, with other third harmonics, disappears in the terminal voltage of Y-connected machines for well known reasons. There are other higher harmonics naturally but how much influence do they exert in the reactance or impedance, *i. e.*, in determining, in conjunction with the armature reaction, the current which flows under sustained short circuit, where, as already mentioned, the field and armature m.m.f's. are in direct opposition. The armature reactance under this condition, remember, is the reactance we are concerned with. That current, as Fechheimer acknowledges, is substantially the fundamental, free from harmonics. A glance at any oscillogram of this current will confirm that. Whether these harmonics are damped out by eddies alone, as he suggests, or are partly suppressed by the higher reactance to the higher harmonics, matters little. The simple fact is they are not there. If we want to know what determines the current, which is the fundamental, it must be obvious therefore that the fundamental of the reactive voltage is the factor we are after. A fifth harmonic, or any other higher harmonic, may be in the reactive flux or voltage, as he says, but what if it is? If we were studying core loss, that fact might concern us, but not here.

Take up now the several methods of testing reactance which Mr. Fechheimer mentions. This is important, because much of the disagreement centers on the test method. On the same machine, Mr. Newbury obtains from test by Mr. Fechheimer's No. 2 method, which he adopts, a value of armature reactance

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different from the value the authors obtain by No. 1 method, which they adopt. Don't misunderstand. We are both after the same thing. The question is, which method of obtaining it by test is right. As we see it, the principle in each method is sound, and by his explanation referred to above, Mr. Newbury agrees. To draw the issue clearly, Messrs. Newbury and Fechheimer think that our respective "test" determinations of armature reactance on the same machine do not check, because they believe we are unable to calculate armature reaction accurately. Hence the difference between that wrong armature reaction and the test value of field m. m. f. (synchronous impedance *A T*) gives the wrong reactance. He agrees, however, that that difference, properly determined, gives *armature reactance*. Our only answer is that we have given in Appendix B a new method of accurately calculating armature reaction, and it seems that they should have felt obliged to show why that method is not right. To simply say it is inaccurate is not at all convincing. Not a word was said about the method in Appendix B; not a word as to *why* armature reaction can not be calculated. We are therefore not in position to know and discuss what is in their minds.

In applying No. 2 method, which Mr. Newbury recommends, one meets a number of difficulties. It involves, first a graphical solution in which a cut-and-try triangle is made to fit in at all points between two curves. One of those curves is calculated, and involves estimates of pole leakage and also estimates of how saturation is changed by having, under load, more flux in the pole body, but practically no more flux in the armature and a varying amount of increase in the teeth. Hence that calculated curve must rest considerably upon judgment. It is important to note that the same objection which was raised to No. 1 method regarding the large error resulting in the difference of two large quantities, from a small error in one of the large ones, applies equally to method No. 2; as does also the objection that one of the large quantities is calculated. It is our opinion, therefore, that method No. 1 involving a simple subtraction of ampere turns armature (in which the amperes are tested, the actual turns are known, and the only calculated factor is "effective" turns) from ampere turns field (in which the amperes are tested and the known turns are 100 per cent effective) is much more reliable than No. 2 method with its difficulties mentioned above; and that the discrepancy, mentioned by Messrs. Newbury and Laffoon, between their test determination and ours is due to their errors in applying No. 2 method. It may be asked: if the "test" value of armature reactance determined in error by No. 2 method is applied in excitation and regulation calculations, why will not these calculations also be in error. Simply because the steps by which the reactance was determined from load excitation curves are retraced in load excitation calculations. This pursuit and follow of one's self about a circle does not apply to the

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use of No. 1 method. Therefore, assuming that armature reaction calculations are even approximately correct, then it would seem that to obtain a value from test with which to compare the calculated value, No. 1 method is far superior. It is acknowledged that no method yet proposed, which is based on commercial tests of saturation curves and synchronous impedance curves, can be depended upon, for reasons already mentioned, to give in every case the armature reactance within, say, 10 or 15 per cent. This may mean only 2 or 3 per cent error in the tested curves. If that error is more serious, as sometimes happens, then the reactance error goes skyward. The error in reactance value incident to test error applies alike to both methods referred to. Hence, although accurate tests of course eliminate this point, it is yet true that where, as in our case, the experience available involves such tests, this fact must be weighed in the interpretation of the data. This has important bearing upon the data in Table I, which I shall refer to later.

Considering further Mr. Fechheimer's list of methods, No. 3 and No. 4 give nothing but the impedance of the stator under those test conditions, *i. e.*, with rotor out. It is absurd to take this as the *armature reactance*, since obviously the latter would be different in the same stator with rotors of different pole proportions. Methods 5 and 6, give, as did 3 and 4, the impedance of the machine under the conditions of the test, *i. e.*, with stationary rotor, where any given phase group has a fixed relation to the pole. If there happens to be a secondary winding, *i. e.*, a squirrel-cage, solid-rotor or short-circuiting collars on the poles, the transformer impedance is involved, that is none of these is the *armature reactance* which we have been discussing. This seems too obvious for comment. Method 7 may give some interesting results of the synchronous reactance in Mr. Fechheimer's investigation. However, it is probably as accurate and infinitely easier to read the synchronous reactance directly from the saturation and synchronous impedance curves.

Method 8 gives the *transient reactance* accurately.

Of the eight methods, therefore, either 1 or 2 (if 2 is correctly applied) should be used for obtaining *armature reactance*. Method 8 should of course, be taken for *transient reactance*.

Referring to Table I, Mr. Fechheimer urges caution in adopting the formula, because 40 per cent of the machines show a difference between calculated and test values of more than 20 per cent, and points out two others in which the difference is 145 per cent and 185 per cent. Please bear in mind these two facts: In that table of 138 machines, all of the bad ones, as well as all of the good ones are there, *i. e.*, all the cards are on the table. Secondly, the "test" values, with few exceptions, were determined from commercial tests of saturation and synchronous impedance, in which circumstance, something would be wrong, as already intimated, if all machines checked within 10 or 15 per cent. Many of those tests are all right; others may be off by 2 or 3

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per cent giving 10 or 15 per cent in reactance; others, more. It would be nice to have 138 different, accurate tests to compare with, but they are not available. We are obliged to accept the commercial test records for the mass of test data. Where tests are known to be accurate, no radical disagreement in results occurs: But we submit that knowing the probable error that *may* result from test, and noting the consistency of the majority of the cases of widely different machine characteristics, in falling in line with calculations, the extreme cases mentioned, as well as a variation of 20 per cent in 40 per cent of the cases, are to be expected, and can not be taken, as Mr. Fechheimer suggests, as evidence that the formula is wrong. If the entire table is studied with these points in mind, I think it must be granted that the agreement is most encouraging. If the table contained only eight or ten machines of about the same characteristics, then it could not be offered as evidence that the formula is reliable. But that is not the case. There are 138 of them. We might add that during the seven months which have passed since the paper was written, additional data, including many accurate tests, have been obtained, all confirming the reliability of the formula.

Referring to transient reactance, Mr. Fechheimer submits an ingenius example to show that saturation prevents the field leakage from having much influence in limiting the initial short-circuit current, and says also that eddy currents in the spider should be considered as to their influence on transient reactance and attenuation factors. Now that example showing the tremendous increase in pole density at short circuit is most interesting. We agree that saturation would certainly be a "serious factor" in such a case. But Mr. Fechheimer is confused. It has apparently escaped him that the flux in the pole at short circuit is roughly what it was before short circuit. He adds the 10 times 12 per cent leakage, to the 100 per cent normal flux, giving a total of 220 per cent. He forgets that the field leakage can not possibly be more than a fraction of the original 100 per cent. As a matter of fact, the only difference after short circuit is that *part* of the 100 per cent which before short circuit crossed the air gap into the armature, now crosses the field leakage paths. The rest goes through the armature leakage paths. So you see he has accounted for 220 per cent which, if it were not for saturation, would cross the field leakage paths, and has not yet mentioned the armature paths, which, by the charitable assumption that the field leakage is 50 per cent as great as the armature leakage, and that the ratio of effective field interlinkages to the product of field turns times flux is about $^2/_3$, the armature paths would absorb $^2/_3$ times 440 per cent equals about 300 per cent more. That is, 5.2 times normal flux, which of course is absurd, since it means manifold increase in field interlinkages. To have remembered that these interlinkages can not change significantly at short circuit would surely have prevented such a weird example. A study of Part 2 of the paper will clear up this point.

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A glance at Table II may save discussion about the influence of saturation. Short-circuit tests are shown at different voltages. It will be noted that this evidence is in accord with conclusion (**10e**) given at the end of Part 2.

Regarding the effect of eddies in a solid rotor rim upon transient reactance and upon the attenuation factors, this phase of the problem was considered, as mentioned briefly in the last paragraph of the abstract at the beginning of the paper. Specific data regarding this effect was not ready in time for the original copy of the paper, but has been included in the TRANSACTIONS.

In speaking of transient reactance Mr. Newbury states that in his experience "the armature reactance during synchronous operation (obtained from no-load and full-load saturation curves) is not materially different in value from the total reactance effective during sudden short circuit." He grants that they may not be the same reactances, and explains that their numerical equality may be due to the fact that whatever increase may occur in reactance due to the field leakage, is offset by "some other action." From what we have said in previous paragraphs, our explanation of that numerical coincidence is due less to "some other action" in the machine than to the fact that Mr. Newbury draws in the calculated modification of no-load saturation curve too near the no-load curve. It is a hopeful augury that he agrees in his last sentence that it "may be due to difference and errors in determining the true armature reactance and agreement can hardly be expected until this question of test method is settled."

It is very important to note at this point that from an exchange with Mr. Newbury of designs and test data for the purpose of removing all question regarding application of the formulas and interpretation of tests, calculations by equation (**47**) Part 1 and equations (**6**) and (**7**) Part 2 checked the tests on his machines, as shown in a following table, in connection with Mr. Laffoon's discussion. That is, the formulas apply alike to machines of different manufacture, when the calculated result is referred to a "test" value which the formula is intended to calculate, and not to some other "test" value.

Mr. Dawson suggests that the difference between the results given in the verbal discussion by Mr. Newbury, and those in the paper might be explained by different practises in end-coil support. I think we have made clear where the discrepancy lies. I remember a case where the same machine was tested first with steel-ring end-coil support, and then with brass support. The difference in reactance could not be detected. This was a moderate speed, about 1000-kv-a. machine, and the steel ring was about one inch (25.4 mm.) square.

Mr. Laffoon's comments center more upon the details of theory underlying the reactance formulas. But he prefaces those comments with some general objections. His reference to

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definition and "standardized reliable test", is repetition of Messrs. Newbury and Fechheimer's discussion of the same points, which has been fully answered.

He rather more fully amplifies their viewpoint regarding the extremely complicated nature of the problem. He believes that the problem is too intricate to treat analytically; that necessary flux diagrams, effect of eddy currents, change in permeability, indefiniteness of the flux paths and a few other things render the calculation hopeless. To take account of all these factors in an equation would be well nigh hopeless, we acknowledge. But if we understand the problem correctly, there is an important distinction to be drawn between a method involving a possible error of 10 per cent or so which for the practical purpose for which the reactance is used, is negligible, and a method required to include all factors to the extent of reducing the possible error to, say, 1 per cent. It is somewhat like the distinction between the method of a house carpenter who works to inches, and the factory mechanic who works to thousandths of an inch. In one the relative error may be many times larger than in the other; but, practically, they are of the same order. Our point is from our understanding of the problem in hand, and from our experience, we believe that practically all of those annoying and confusing factors which Mr. Laffoon refers to, are trifling and may be neglected in obtaining a practical estimate of reactance from the design. Where, for instance a factor can not be neglected, as harmonics in armature reaction, we have taken it into account. But to worry about most of the things he mentions is unwarranted, for reasons given in previous comments.

Regarding slot leakage and his Fig. 2, he reasons that there will be slight mutual action between phases due to saturation of teeth, then decides that there is some other effect that compensates for it. That is, agrees with the authors that there isn't any mutual effect of slot leakage.

Now as to the mutual induction of the end connections and tooth-tip leakage. Mr. Laffoon shows why in the case of the end connections the mutual effect could amount to "only a few per cent:" then shows that it would have to be less than that. Perhaps the shortest answer to his logic is that if exploring coils are put on the end connections following around the V-ends of a phase group, and then from an exterior source, the same current is put through the armature, first single phase and then three phase, the exploring coil voltage will be 50 to 60 per cent more in the latter case than in the former. This ratio for end leakage is the same whether the rotor is in or out. The following Table III gives data from such a test on a 125-kv-a. machine, four slots per pole per phase, and also on coils assembled in a wooden armature dummy.

He believes that the tooth-tip leakage is more complicated still, and also that the mutual induction between phases due to this leakage must be very small. Why? Because damping

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currents in the laminations, rivets, etc., etc., prevent the tooth-tip flux from entering the pole face. This restricts this leakage to the path along the air gap. This of course would not only prevent mutual induction with other phases, but also cut down radically the leakage itself. You see he is still allowing harmonics to cause undue confusion. Certainly there are no damping currents to the fundamental which we are after, and which being polyphase is stationary with respect to poles. Instead, the problem is rendered more simple that the harmonics are partially

TABLE III
EXPLORING COIL TEST FOR END LEAKAGE ON 125 KV-A. ALTERNATOR

Test	* Rotor	Phase	† Exp. Coils in series	End Shield	Amps. in Phase group	‡ Exp. coil voltage	Column 7 corrected for 32 amps.	Ratio taken of tests	Ratio
1	Pos. 1	3	4	on	31.2	0.983	1.02	1 & 3	1.69
2	Pos. 2	3	4	on	33.6	1.09	1.04		
3	Pos. 1	1	4	on	31.4	0.595	0.605	2 & 4	1.52
4	Pos. 2	1	4	on	31.4	0.673	0.685		
5	Out	3	4	on	32.4	1.03	1.02	5 & 6	1.6
6	Out	1	4	on	31.5	0.627	0.637		
7	Out	3	4	off	31.3	1.04	1.07		1.67
8	Out	1	4	off	31.4	0.627	0.64.		
Similar Test on Armature Dummy									
9		3	2		130	2.99		9 & 10	1.49
10		1	2		130	2.01			

*Position 1, phase group bearing exploring coils, symmetrically surrounding the pole. Position 2, phase group directly over pole face.

† Exploring coils were arranged as shown in Fig. 7, except there were four instead of two slots per pole per phase.

‡ Maximum of fundamental. Fig. 6 shows typical oscillogram of exploring coil voltage.

or totally damped out. We trust that the puzzling features he mentions will clear up if these points are kept in mind.

He adduces Mr. Newbury's test with rotor removed showing 16 per cent increase three-phase over single phase reactance to show that that increase can not be of the order of 50 per cent. At the time the problem was being studied, the authors of course made such tests, and agree that under that condition (rotor out) the ratio is about 1.16. But as we have emphasized before, we are not interested in what it is with the rotor out. What we are after is the reactance with the *rotor in and running in synchronism at zero power factor*. Under which circumstance (and relegating

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this damping action of the pole rivets etc., as mentioned above) the iron rotor permits mutual interlinkages by the fundamental with other phases. This idea is fully developed in the paper. In this respect, there is other possible evidence, available to those concerned, that has bearing on this point. Theoretical considerations indicated that this mutual induction factor, three-phase to single phase (terminal to neutral), obtains in the transient as well as the armature reactance. This led to equation (13) Part 2. While at the time the paper was printed, no tests were available, a test has since been made which confirms that point of view. It might be instructive for those interested to try this out. The results of that test which was made on a 1000-kv-a., 750 rev. per min., 50-cycle generator are as follows: Short circuit, three-phase, no load gave 1900, 1800, 1900 amperes (a-c. component) on three different tests. Short circuit, one phase, terminal to neutral, no load and same voltage, gave 2950 and 3060 amperes (a-c.

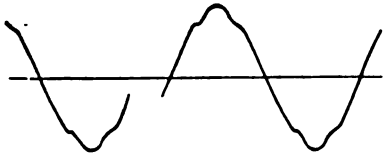

FIG. 6—OSCILLOGRAM OF EXPLORING COIL VOLTAGE OF END CONNECTION LEAKAGE ON 125 KV-A. ALTERNATOR

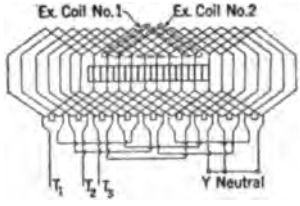

FIG. 7—EXPLORING COIL TEST FOR END LEAKAGE—ARMATURE COILS ASSEMBLED IN DUMMY INDICATED BY HEAVY LINES AT CENTER OF FIGURE

component) on two different tests. Average ratio three phase to single phase is 1.6.

Mr. Laffoon believes we have made an error in the calculation of tooth-tip leakage by neglecting the important component of this leakage which goes along the air gap and does not enter the pole face. Now there may be a number of viewpoints from which the authors have not studied this problem, and there may be possibilities of significant factors which they have failed to search out and weigh, but may we not, in the face of this avalanche of criticism, properly give ourselves credit for having been pretty careful and painstaking to do these things. The question just mentioned about tooth-tip leakage was analyzed and the factor for including this "along-the-air-gap" component was put in the equation; but it was found to be practically negligible, and was omitted. We take it that Mr. Laffoon has not properly made such an analysis, or he would not have brought the question up. The space to discuss this criticism may be worth while, because it is a characteristic type. At first thought, the matter seemed to us very much the same as it still does to Mr. Laffoon. That

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is why we worked it out. It seems obvious that with slot width equal to air gap on a one slot per pole per phase, the leakage from the face of one tooth to the face of next tooth, will not all go across the gap to the pole face and return; part will go along the air gap. With two slots per phase group, the path across to the pole face has not changed, but the path of the component along the air gap is doubled. For three slots per group the first path between outside teeth is still the same, the path along the gap is trebled. The middle slot has its own leakage, but as explained by Gray, this contributes only one ninth to the total leakage of the group. And so on; the component along the air gap growing progressively less as the slots per phase group are increased. Then why does not that component have to be considered for one or two slots per pole per phase. Because no one, I think, ever saw an air gap equal to the slot width on such a machine. If one did, one saw a monstrosity. Assuming tooth and slot width equal, and three slots per pole, would mean an air gap equal to one-sixth of the pole pitch. A normal air gap is, say, one to 3 per cent of the pole pitch, possibly four per cent. Take four per cent, for instance. If the slot width is the same, it means eight per cent per tooth pitch, *i. e.* about 12 teeth per pole, or more. If two per cent gap had been taken, it would have meant 24 slots per pole. The more the teeth the less, by the square, do the middle ones count, and the middle slot is all that would be practically affected by this component we are accused of having omitted. Moreover, if there were such a machine where this factor was a significant portion of the total tooth-tip leakage, then the total tooth-tip leakage would be of practically negligible magnitude compared to end leakage and slot leakage; and the error would be of no practical significance in the total reactance. Those are reasons why we omitted that factor. We did not overlook it.

Regarding the application of Carter's Fringing Coefficient to tooth-tip leakage. Take any alternator and lay out the distribution of tooth-tip leakage over the pole. Determine the coefficient to account for the spread of flux. Then look up the value on Carter's curve. It is practically the same, as one might expect, except possibly in some monstrous example as Mr. Laffoon suggests. Even then, for reasons mentioned, the error, is harmless.

The title for the curves based on the ratio maximum to minimum air gap, which assume a constant pole arc radius for any given pole, has been modified to meet his objection regarding the difficulty of applying them. They can not properly be applied to a pole involving different arc radii.

Turn now to Mr. Laffoon's Table I. Before commenting on the comparison it gives, we would refer to the following table, giving the rest of the data which Mr. Laffoon should have felt obliged to include, but did not. We have already referred to an interchange of design and test data with Mr. Newbury for

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the purpose of eliminating the possibility of wrong application of formula and of different interpretations from tests. Mr. Laffoon's Table I is supposed to show the results of that interchange. In our Table IV only machine numbers 4, 6, 11, 12 and 13 appear, since we have not data on the others. But sufficient data is in Table IV for interesting comparison with Mr. Laffoon's Table I. These are the main points of interest:

1. Our calculation of *armature reactance* checks reasonably with the test value (No. 1 method), which the formula is intended to calculate, on Mr. Laffoon's machines, as well as on ours. Mr. Laffoon's Table I does not show the "test" value which our formula calculates, but something else.

2. His calculated and test values are, he avers, either *armature*

TABLE IV
PER CENT REACTANCE

Machine	Kv-a.	Phase	* Slot width air gap	Calc. D. & S.		Test †		Calc.	Test ‡
				Arm.	Transient	Arm.	Transient	L	L
D. & S. 4	1250	3	3.4	9.6	13.6	7.8	...	10.9	...
D. & S. 6	700	3	2.4	13.6	19.1	13.0	...	22.85	...
L 11	3750	3	1.6	10.2	14.5	12.9	...	13.63	13.25
D. & S.12	560	3	1.06	17.6	23.0	18.2	...	26.0	...
L 13	12500	3	0.94	13.2	19.0	14.2	20.8	20.2	19.3

* Minimum air gap is used in this ratio.

† *Armature reactance* from saturation and synchronous impedance curves. Method 1 in Mr. Fechheimer's discussion. *Transient reactance* from oscillograph records of short circuit.

‡Mr. Laffoon's application of test Method 2, as described in Mr. Fechheimer's discussion.

or *transient* reactance, since they are the same. Note that those values agree with our calculated *transient* reactance. Fortunately, on one of the machines he submits, the 12,500-kv-a., results from short-circuit tests are available. In this case, we calculate 13.2 per cent armature, and 19 per cent transient. Test was 14.2 per cent armature, and 20.8 per cent transient reactance. His calculated and "test" values are 20.2 and 19.3 per cent respectively.

3. Our calculation of armature reactance on machine 11, is 10.2 per cent instead of 9.36 per cent given in Laffoon's Table I.

Therefore the alleged agreement of results at high ratio of slot width to air gap, and a gradual taper to radical disagreement at low ratio, due to our omission of the "along-the-air-gap" component of tooth-tip leakage, vaporizes with the rest of the facts and logic which have been submitted to show that the results of

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the paper are entirely wrong. Note in Table IV that, instead, the agreement happens to be better at low ratio (see *D* and *S* 12 and *L* 13) than at high rates. The results in these tables simply show again what has been brought out in all this discussion, that, in the first place, the soundness or the theory underlying the method given in the paper has not been disproved; and also that the difficulty Messrs. Newbury, Fechheimer and Laffoon have experienced in adjusting their viewpoints rests on the fact that they have been working on the unsound principle that the transient reactance and armature reactance are the same. And by the treacherous possibility in No. 2 test method of taking in excitation calculation any value one pleases for armature reactance, (so long as that value is applied in the same way it was derived from the zero power factor excitation curve) they have lead themselves to believe that the above principle is right. A different value of reactance obtained by No. 2 method simply means a change in judgment regarding where the calculated saturation curve should be placed. The coincidence of the value of armature and transient reactance probably had its origin in the view that they should be the same, and the full-load leakage correction of the saturation curve adjusted accordingly.

Mr. Diamant's comments center principally on three points: 1. The author's statement of disagreement with certain results of his own paper. 2. Priority claims. 3. Terminology.

Since Mr. Diamant states that "the authors really do not disagree with any of my conceptions, at least I agree with theirs . . . .", the first point would seem at once to be settled. We disagreed because we misunderstood his meaning. We misunderstood not by a lack of diligence in trying to understand, but because he used terms familiar to us, but to which he attached a meaning entirely different from that almost universally understood, and which is uniformly defined in leading text books. Take one glaring instance: by armature self-inductive reactance, he means synchronous reactance. Referring to Fig. 27B in the paper, his meaning of "self-inductive reactance" involves all the flux shown. What is generally meant by self-inductive reactance, involves only that portion of the total flux shown that does not link with the field winding. With the almost generally accepted definitions in one's mind Diamant's statement "the ratio of the true self-inductive reactance to the total leakage reactance reduced to the field is about three," is perplexing indeed. Being familiar with leading texts that define these terms and with the consequent general understanding, and yet not having stated his intention to use a different meaning, he could hardly have expected his words to be understood.

But after definition has been cleared up, one point remains. We obviously can not agree to the general statement in Diamant's paper that the initial short-circuit current is determined "by the armature impedance with good approximation." Although true in some cases, as tacitly granted in our paper,

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"this assumption would lead to serious error in many cases." We gather from Mr. Diamant's interesting comments confined to logic, that, after all, he agrees.

Just a word about our reference to his paper. In taking up this problem of reactance and short circuits, we naturally used as a starting point the most recent work on the subject, and that, of course, was Diamant's paper. He had covered the matter historically, which to us, was a considerable factor in the value of his paper, and had offered interesting and valuable test data, and also some extensions in mathematics, one of which was used in our paper, Fig. 21. As stated, there were in his work points with which we did not agree. Hence it was necessary and proper that that disagreement should be stated, in order to clearly proceed to further development; surely not in order that we might "triumphantly state" that some one was wrong. We regret that the latter interpretation was made by Mr. Diamant, because it has done much to spoil what otherwise might have been a masterful criticism.

We are accused of having attempted to offer, for the first time, what Boucherot proposed in 1912. The first paragraph of Part 2 states that Diamant's paper sums up the theories underlying short circuits as developed by Berg and by Boucherot. These are in complicated mathematical terms from which the average engineer finds difficulty in obtaining a physical conception of the phenomena. Moreover, to apply their equations it is necessary to know the reactance. In order to calculate the reactance which limits the initial short-circuit current, it is necessary to establish a physical conception of what happens to the magnetic fields under that condition. It is just as necessary to establish that conception in the reader's mind, if he is to understand clearly what is being calculated. And to the contempt of those who would say everything mathematically, we have described the phenomena "in terms as free as possible from mathematics." It is an interesting circumstance, therefore, that we are now obliged to acknowledge that we did not discover the principles underlying that exposition; nor did Boucherot, or Diamant. Faraday and Maxwell deserve some credit for them. They are stated in any good text on physics. However, there are one or two points which were brought out which we believe were not generally appreciated. To mention one, the increase in flux density at the bottom of the pole at short circuit. This is not only an interesting result, but one that has important bearing upon the variation in attenuation factors, as explained in the paper. This phenomenon is not all obvious from previous literature.

Before referring to terminology, we will answer one or two other points. We are asked to explain why a single-phase short circuit between terminals of a three-phase machine produces the same current as a three-phase short circuit, especially since a terminal to neutral, single-phase short circuit produces 50 per

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cent greater current than the three-phase. If it is assumed that the two phases involved may be considered as different phases, as in three-phase, and with the three-phase value of reactance per phase, then the short-circuit current would be

$$I = \frac{1.73\,E}{2\,X} = 0.86\,\frac{E}{X} = 0.86\,I_3$$

that is 86 per cent of three-phase value. That assumption, however, involves the three-phase mutual induction between phases, which is strictly true only at one instant, when the current is a maximum. Then the pole iron is in same position relative to the armature m. m. f. as at one particular moment under three-phase conditions. If that relative position were maintained, then the current would be as given above, *i. e.*, $0.86\,I_3$. However, as the pole moves on, the relation of the iron path to the leakage flux is changed to decrease slightly the fundamental of the leakage flux, and as tests show, this decrease is about 15 per cent.

Mr. Diamant says the following statements in the paper are wrong: "the alternating component depends upon the flux per pole" and "the direct component depends upon the flux linked with the armature circuit." He says those components depend rather upon *the difference between the mutual and sustained flux.* The above quotations obviously follow if one accepts the principle of constant interlinkages in a closed circuit without resistance, and grants what is the generally accepted view that for a practical study of the phenomena, the resistance is negligible for the first cycle or so. But even if one dissents from that view, we will take the magnitude of these components as determined by projection of wave of crests to zero time. But however obvious those statements may seem to those who look at these matters from a viewpoint of facts, it is acknowledged that where one, instead, reasons from somebody's equations, say Boucherot's, instead of from the background of facts Boucherot had in mind, it is possible for one to arrive at a wrong, sometimes dangerous, conclusion. Consider what happens. At short circuit the entire useful flux is absorbed in the armature and field leakage paths in proportion to their respective reactances. There is of course a very small per cent of it taken up in $R\,I$ drop, but this is negligible. As the flux dies down, the relative proportion of it in the armature leakage path at any time is progressively greater, that in the field path, less, until at the sustained value, it is all absorbed in the armature leakage (except of course the original pole leakage due to original exciting field current). Now the practically universal practise of all authors in expressing that fact mathematically (and it is a mathematical convenience not free from inconsistency) is to assume that the sustained value existed all the time, and that the transient condition involves a change from initial flux and current to sustained flux and current. Diamant's opinion probably rests on such

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a *formula*, otherwise it is difficult to understand the basis for his opinion that the a-c. component depends upon the difference between initial and sustained flux. But, as intimated above, such a formula does not explain certain facts. If the sustained value is assumed to have existed from the first instant, then it means that by the formula there is a greater percentage of flux taken in the armature paths at the initial moment than actually is taken. As an instance, assume a ratio of armature to field strength equals two. Also assume ten per cent armature, five per cent field leakage. That means 6.66 times normal current at short circuit. Actually at the first moment the division of the *total* flux, which is 110 per cent, is 66.6 per cent in armature, (33.3 per cent + 10 per cent) in field leakage path. At the sustained condition, the current is practically two times normal, that is, the flux division is 20 per cent in armature leakage, 10 per cent in field. By the well known formula referred to, assuming that the sustained values existed from the start, the initial division would be as follows: the sustained value accounts for armature 20 per cent, field 10 per cent. That leaves 80 per cent of the useful no-load flux to be absorbed in armature and field in the ratio of 10 to 5, *i. e.*, 53.3 per cent armature, 26.6 per cent field. That is a total of (20 per cent + 53.3 per cent) in the armature and (26.6 per cent + 10 per cent) in the field. That is, this equation says 73.3 per cent is in the armature leakage, 36.6 per cent in the field leakage; whereas, in fact, the division is 66.6 per cent in armature, 43.3 per cent in the field. Now obviously, by the well known equation referred to, this question would not come up if the total reactance is assumed to be in the armature. In other words, that equation involves the fundamental assumption that the total reactance is in the armature. If, therefore, one thinks in terms of that equation, one may go astray.

Another viewpoint, which seems more rational, yet perhaps less convenient, is that the sustained value of flux is zero at the initial instant, and by an exponential, reaches the sustained value; and at the same time the total initial flux decreases to zero by the same exponential. That is the flux at any time is

$$\phi = \Phi \epsilon^{-\alpha_f t} + \Phi_s \left(1 - \epsilon^{-\alpha_f t}\right)$$

where Φ = initial flux
Φ_s = sustained flux
α_f = field attenuation factor based on the total leakage of armature and field.

The flux which is caught in the armature circuit starts at that value and decreases to zero by

$$\phi_a = \Phi_a \epsilon^{-\alpha_a t}$$

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The equation of short-circuit current crests would be, using notation in paper, and assuming wave completely offset,

$$i = \sqrt{2}\left\{I_0\,\epsilon^{-\alpha_f t} + I_s\left(1 - \epsilon^{-\alpha_f t}\right) + I_0\,\epsilon^{-\alpha_a t}\right\}$$

These forms are consistent with the facts. They say that the division of flux in the above example is initially 66.6 per cent armature and 43.3 per cent, as facts require; and at sustained value, 20 per cent armature and 10 per cent field leakage. And at any time during the transient, the proper division is obtained from the equation.

While the other form may be used in practical calculation for practical results (and hence the familiar instead of the new forms were used in the paper) it yet becomes a serious difference if generalization is to be made from the formula to the extent of Diamant's statement that the initial a-c. component depends upon the difference between the initial and sustained flux. Think. If at the instant the short circuit occurs, the field terminals were also short-circuited, the initial armature and field current would necessarily be the same as if those terminals had not been short-circuited. In the former case, however, there would be no sustained value. The flux and currents would die to zero. So the difference between initial and sustained flux could not affect the initial current, either the a-c. or d-c. components; unless it is assumed always that the sustained value of flux is zero, and that is assuming the armature leakage to be zero.

Regarding terminology, we are critized for introducing the "new impressive, misleading name of transient reactance." It has escaped his notice that in previous A. I. E. E. papers and in other literature, this term has been proposed; and in the present paper at the beginning of Part 2, the origin of the term is recorded: Durgin and Whitehead in, 1912; F. D. Newbury in 1914. It may sound to Mr. Diamant as unscientific; but sometimes scientific terms are so long that commercial engineers will not use them, and it is necessary to have a shorter, if less accurate term. How many busy engineers could be persuaded to say, "I want 'the total leakage reactance reduced to armature terms.'" While we reaffirm our acceptance of the term, "transient reactance," as practical and descriptive, and also believe the terminology used in leading tests such as Steinmetz, Berg and Lawrence are scientifically sound, we believe that there should be some terminology which would mean the same to everybody, and avoid such confusion as is experienced in reading Diamant's 1915 paper. His discussion devotes considerable space to this matter. His terms are sound and consistent. Whether it is too late to ask the engineering world to change entirely to adopt Diamant's suggestions, or instead to adopt officially what is already generally used, is questionable.

We acknowledge the omission of Douglas' paper in the bibliography. It properly should have been included.

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PROTECTION FROM FLASHING FOR DIRECT-CURRENT APPARATUS

BY J. J. LINEBAUGH AND J. L. BURNHAM

ABSTRACT OF PAPER

The equipment developed for the protection of direct-current apparatus as described in this paper is applicable to all direct-current apparatus and all methods of operation. Special means of protection for use only with particular apparatus or conditions of operation have not been mentioned. The principal steps in the experimental development of high-speed circuit breakers and flash barriers are briefly given.

The protection afforded by the high-speed breaker or barriers is sufficient for most apparatus and service, but *complete* protection for *any* direct-current apparatus and service requires both the high-speed breaker and flash barriers. Attention is directed to the importance of arranging the connections to the brush rigging so that the magnetic action on the arc will be a minimum, and properly directed, so the flash will do the least damage.

THE problem of protection from flashing has for many years confronted engineers who build and operate direct-current machines. Numerous schemes and suggestions have been put forward which it was hoped would overcome the tendency to flashover on extra heavy overloads or short circuits. Some time ago it was felt that the subject of prevention and protection from flashing had not received the study and investigation justified by the trouble experienced and it was decided to make a comprehensive study of the entire subject.

Some form of barrier has been the most common protection suggested, and different forms have been tried with a slight degree of success on some machines and absolute failure on others. It was the opinion of many engineers that barriers could not be designed to take care of a short circuit and that their value was doubtful. However, a special form of barrier, which gives the required protection, will be described later.

It was realized that the means for prevention of flashing at the commutator and brushes of direct-current machines must operate to remove the cause very quickly. The use of some form of high-speed device, which would open the circuit or insert re-

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sistance before the short-circuit current could reach a value which would cause flashing, seemed the most logical way to solve the problem, although it was appreciated that the action of the device must be much more rapid than any commercial circuit-opening device previously produced. An investigation was conducted along these lines and two distinct types of high-speed breakers developed, which will be described separately.

A flash at the commutator starts from excessive sparking. Sparking is produced by the breaking of current in the coils short-circuited by the brush as each segment of the commutator passes from under the brush. As the coil is inductive, the spark or arc tends to hold and, if the arc is of sufficient volume, the vapor produced thereby forms a low resistance path between segments and from brush to brush or to frame; through which a large current may pass. See Figs. 1 and 2.

Sparking may be prevented by providing a magnetic field of proper strength and distribution to influence the coils during reversal of their current as they pass through short circuit by the brushes. To provide the correct commutating field for all conditions of load has been the object of designers but success has been only partial. At high loads, saturation of magnetic circuits and distorting influence prevent attainment of the desired field, and for sudden changes in load the changes in field cannot be properly synchronized. It is more difficult to avoid sparking with rapidly varying loads than with gradually changing or steady load, but if a sudden load which would cause flashing is of short enough duration, the arcing at brushes may not produce enough conducting vapor to establish an arc supported by the main voltage. *The value of load that causes flashing when applied suddenly (short circuit) is a function of the time required to throw it off.* The quicker the circuit is opened the higher the value of current that will not cause arcing.

With the ordinary circuit breaker which begins to open in about 0.15 second, there is a certain maximum load which cannot be exceeded for each commutating machine without causing flashing. If feeders have sufficient resistance to limit the short-circuit current to this critical value, flashing will occur only on the rare occasion of a short circuit in a feeder itself. See Fig. 3. It has been the standard practise of nearly all manufacturers to recommend tapping the feeders, especially railway feeders, at a sufficient distance from the substation to insure enough resistance in the circuit to limit current in case of short circuit near the station.

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Inductance may be added to the circuit to retard the rate of increase of current on short circuit to such an extent that the ordinary breaker will have time to trip before the current in the machine reaches a value that would cause flashing. The amount of inductance required to delay the rise in current sufficiently, however, introduces other disadvantages which make its use undesirable. When the current is interrupted, the increase in voltage from inductive "kick" is difficult for circuit breakers to handle and introduces the possibility of applying dangerous voltage stresses to the apparatus.

Reactors have been tried in a few instances with some success but it has always been a mooted question whether the resistance of the reactor did not give as much or more protection than the inductance of the coil, and if this is the case resistance only would be much cheaper to install. A coil to give the delay required is usually very large and expensive and occupies much valuable space, giving a total cost out of proportion to the cost of the machines protected or the protection obtained.

With special high-speed circuit-opening devices operating in about 0.005 second, the more sensitive machines, such as 60-cycle synchronous converters for railway voltages, may be short-circuited without flashing over, even though the maximum current is of higher value than would cause flashing with suddenly applied load and ordinary circuit-breaker protection.

The speed at which a circuit breaker must operate to prevent flashing depends on the amount of load thrown on the machine but, under worst conditions, our tests seem to confirm that it must be *quicker* than one-half cycle of the machine to be protected. The time of operation of the breaker would be measured between the time that the current reaches the flashing value to the time that the current is again reduced to the same value after the breaker opens. If the arc formed between two segments is not blown out as they pass from one set of brushes to the next and all following segments have similar arcs formed between them, the arc would completely bridge between positive and negative brushes in one-half cycle, which would complete the flashover. Complete flashover might also occur from gases being blown by windage, magnetically, or by expansion, to increase or decrease the half cycle time.

The time of operation of circuit breakers as given herein is measured from the beginning of short circuit to the instant the breaker begins to reduce the current rise.

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Investigation covering these several schemes of protection was made, which it is believed will be of interest and will be described briefly with oscillograms, reproductions from photographs, etc., showing behavior under different loads and short-circuit conditions.

All short-circuit tests were made by connecting positive and negative terminals with a 500,000 circular mil cable; the only equipment in the circuit being the necessary current shunt for the oscillograph, a contactor to close the circuit, and a circuit breaker for overload protection, in addition to the protective device being investigated. Power for the 300-kilowatt, 25-cycle and 500-kilowatt, 60-cycle, 600-volt synchronous converters, used in fuse, barrier, reactor, and high-speed circuit breaker tests, was supplied from a 6000-kilowatt frequency changer set only a few feet from the test, so that there was very little drop in the voltage of the generator or from resistance, and the oil switch was set so that it did not trip out.

High-Speed Circuit Breaker

At the time this development was started it was felt that if a circuit breaker could be designed to operate *within* the time required for a commutator bar to pass from one brush to another; that is, within one half cycle, protection would be afforded against practically any short circuit. Designs were therefore begun on a circuit breaker which would open within 0.007 second, which would cover most commercial machines; *i.e.*, for 60 cycles and lower frequency.

High-speed breakers had been suggested and attempts made to produce such devices previous to this time but, as far as the wr ters know, had never been made to obtain as high speed as the discussion shows would be necessary.

Different types of construction were studied and samples of several preliminary models constructed without obtaining the speed desired. One of the most promising types of construction considered consisted of a knurled fly wheel operating continuously with a knurled cam, so designed and located that a current relay would insert a wedge between the wheel and the cam and trip a breaker attached to the cam by suitable toggle mechanism. This preliminary sample indicated that 0.035 second was the best speed that could be attained.

It was then decided to concentrate all energies on a circuit breaker using the well known principle of a latch, heavy spring

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FIG. 1

Flashing at brushes on 1000-kw., 1500-volt generator forming part of 2000-kw., 3000-volt motor-generator set at five times load, showing different stages of arc formation.

FIG. 2 [LINEBAUGH AND BURNHAM]

High-speed photograph of flashing on 300-kw., 600-volt, 25-cycle synchronous converter with short circuit on 0.015 ohms additional in the external circuit and standard circuit breaker.

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FIG. 3

High speed photograph of short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter with standard circuit breaker.

FIG. 4 [LINEBAUGH AND BURNHAM]

3000-ampere, 3600-volt, direct-current high-speed circuit breaker.

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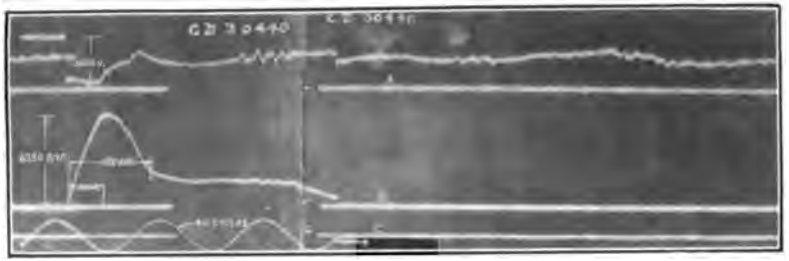

FIG. 5

Direct-current short circuit on 2000-kw., 3000-volt, motor-generator set with high-speed circuit breaker and standard 3000-volt switchboard type circuit breaker.

FIG. 6

2000-kw., 3000-volt, direct-current synchronous motor-generator set before assembly of flash-barriers.

[LINEBAUGH AND BURNHAM]

FIG. 7

Type of flash barrier installed on 2000-kw., 3000-volt, synchronous motor-generator set used in connection with high-speed circuit breaker.

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FIG. 9
High-speed air cooled fuse holder with magnetic blow-out used in test.

FIG. 10
High-speed oil-cooled fuse holder, used in test.

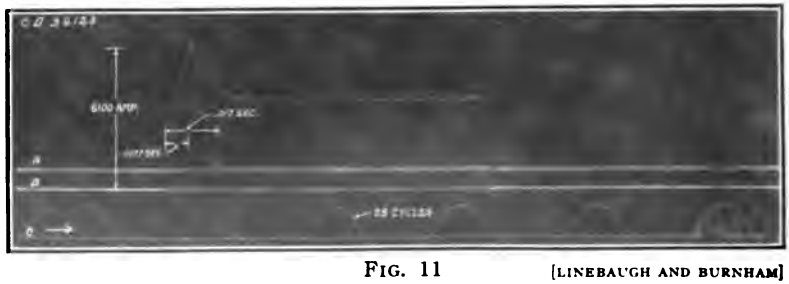

FIG. 11 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by air-cooled high-speed fuse. Curve A, voltage across fuse; Curve *B*, line current; Curve *C*, collector-ring voltage.

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and series tripping coil, and the high-speed breaker shown in Fig. 4 was finally built.

The problem was to obtain very quick tripping, rapid acceleration of contacts and a sufficient number of ampere turns in the magnetic blowout to insure rapid breaking of the arc. Previous ideas of design had to be abandoned when working for such high speed when a loss of 0.001 second meant a very serious increase in time of operation.

It was found that a series blowout coil had to be used, as sufficient time could not be allowed for the building up of a field after the contacts opened as is ordinarily done in circuit-breaker design, and the strength of this coil must be many times that usually used to rupture the circuit by giving the quick start and acceleration to the arc necessary for the speed desired. The breaker in question has a total of about 150,000 ampere turns at the maximum current obtained.

The moving parts must all be as light as possible, consistent with the great strength required, so that they can be started, accelerated and stopped in a very short space of time and distance. Even with this type of construction, it was found necessary to use somewhat high spring pressure; the spring being compressed to about 8000 pounds when the breaker was closed and ready for tripping.

A very special latch with very small tripping movement was designed somewhat similiar to the hair trigger on a rifle, in connection with a special high-speed tripping coil so that about 0.001 inch movement of the plunger would trip the breaker. It will assist in appreciating the speed attained when it is noted that the breaker must be arranged so that it will not trip under ordinary load condition and must be set above the tripping point of the regular substation breaker so that it will act while the current is increasing from say three and one-half times load to eight times load; current rising at the rate of about 1,000,000 amperes per second. Fig. 5 gives a very good idea of speed and limiting of current, from which it will be seen that the breaker starts to insert resistance in about 0.008 second and the load on the machine is reduced well below the flashing value in 0.020 second after the short circuit was applied.

A breaker was tested very exhaustively in connection with a 2000-kilowatt 3000-volt direct-current synchronous motor-generator set shown in Fig. 6, built for the Chicago, Milwaukee & St. Paul electrification, and found to give complete protection

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from damage or burning on short circuit when equipped with barriers shown in Fig. 7.

In connection with the test, it was found that even the speed of 0.008 second obtained would not completely protect machines from flashing on the most severe short circuit, and barriers shown were designed and installed. Tests referred to with high-speed breakers were taken with these barriers, which will be described later.

It is evident that it is preferable in case of short circuit to insert resistance by a high-speed breaker to quickly limit the current to some conservative value and then open the circuit. This type of protection has been adopted as standard. All tests, investigations, et cetera, were based on this theory, although some tests were taken by opening the circuit. It was found that there was a greater tendency for machine to flash if the circuit was opened completely at one time or if too high resistance was inserted, reducing the load to too low a value. For the sake of convenience and comparison, all tests were made throwing short circuit on the machine without load.

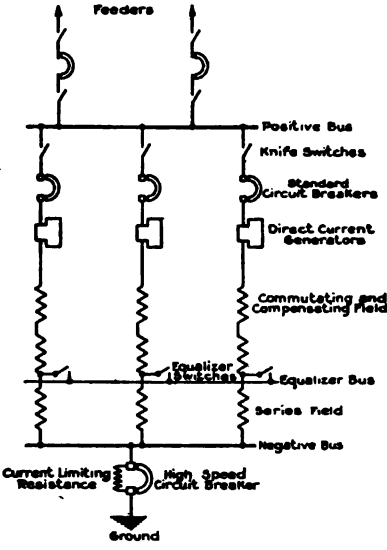

FIG. 8

Diagram of direct-current connections for substation equipped with three motor-generator sets protected by one high-speed circuit breaker connected across limiting resistance

Some of these breakers have been in service since early in 1917 in the substations of the Chicago, Milwaukee & St. Paul Railroad and have amply justified the faith of the railroad company and the designers, as they protect the apparatus from all short circuits experienced, although all feeders are tapped directly to the overhead trolley system immediately at the substation.

One of these breakers is installed in each of the substations connected between the negative bus of the station and the ground or return circuit, as this location gives maximum protection, and one breaker can be used for each machine or one for the entire substation, as shown in Fig. 8.

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High-Speed Fuse

It is evident that if a fuse could be developed that would melt at a very small increment of current above normal rating, it might be possible to obtain a speed which would limit the current on a short circuit along the same line as the high-speed circuit breaker just described.

A careful study of all available metals was made by Mr. P. E. Hosegood, who suggested using a silver fuse, and a number of silver fuses of different shapes were tried in the special fuse holders shown in Figs. 9 and 10. The oscillograph record, taken with air break fuse and magnetic blow-out, shown in Fig. 11, indicates that a very high speed is obtained, giving excellent protection and duplicating almost exactly the speed of the high-speed circuit breaker. It was found that a short circuit could be thrown on the 300-kilowatt, 25-cycle, 600-volt synchronous converter without flashing over and with very little sparking at the brushes. The oil-immersed fuse holder without magnetic blow-out gave practically the same result (Fig. 12), the operation being slightly better as far as speed was concerned but the mechanical difficulties of replacing the fuse, etc., being greater.

Reactors

Oscillograph records of short circuit on the 300-kilowatt, 25-cycle, 600-volt synchronous converter show an average initial current rise of about 1,300,000 amperes per second. To protect by reactance, the amount required would depend on the rate of circuit-breaker action. With coils made of 1000 feet of 500,000-circular mil cable, wound on cable reels having an inductance of approximately 0.02 henry in circuit, this particular machine could be short-circuited without flashing when protected by a breaker opening in about 0.15 seconds.

An examination of records, Figs. 13 and 14, will show the severe duty on the circuit breaker and increase in voltage on the apparatus.

It was suggested that shunting the reactor by resistance might reduce duty on the circuit breaker. The coils were shunted by 14 and by 100 ohms and it was impossible to determine from either observation or oscillograph any effect due to the resistance.

The effect of an iron core in a reactor having an inductance of 0.00105 henry is shown in Fig. 15, from which it will be noted that the iron saturated at about 1000 amperes in about 0.007

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seconds, after which the current rises abruptly, being limited only by the inductance of the coil as if there were no iron in its magnetic circuit. The delay of about 0.007 seconds due to the prescnce of iron in the eoil, is far less than the time required for the usual breakers, now in use, to open. The weight of this reactor was 7 per cent of the weight of the synchronous converter and would have to be many times larger to give protection with an ordinary breaker.

Second Form of High-Speed Circuit Breaker

Mr. J. F. Tritle has more recently suggested a design for a high-speed circuit breaker which is simple and substantial in construction. This device was built as shown in Fig. 16 and test indicated that the speed was even faster than the large breaker previously described, as will be seen by comparing oscillograms, Fig. 5, on the large breaker, and Fig. 17 taken with the later breaker. This device is essentially a contactor having a laminated structure with electric holding coil and series bucking coil so that it opens when the load current reaches a value sufficient to offset the ampere turns of the holding coil. Tests on the 300-kilowatt, 25-cycle synchronous converter with this device showed that a short circuit could be thrown on the machine without any tendency of the machine to flash over, and the only sparking obtained extended not over one-half inch from the brushes. Similar tests, Fig. 18, on the 60-cycle, 500-kilowatt synchronous converter showed more sparking and, although it protected the machine at times on short circuit, there were other times when the machine flashed over. When the machine was equipped with barriers, dead short circuit could be thrown on with impunity, there being no tendency to flash over and scarcely sufficient sparking to be noticeable.

This later type of high-speed breaker is a part of the more recent equipment being furnished the Chicago, Milwaukee & St. Paul Railway.

Barriers

The barriers shown in Fig. 7 in connection with the description of the high-speed circuit breaker were developed to delay time of flashover, so that the breaker would give complete protection. Such satisfactory and promising results were obtained without the breaker that it was decided to continue investigation to ascertain if it would be possible to devise barriers

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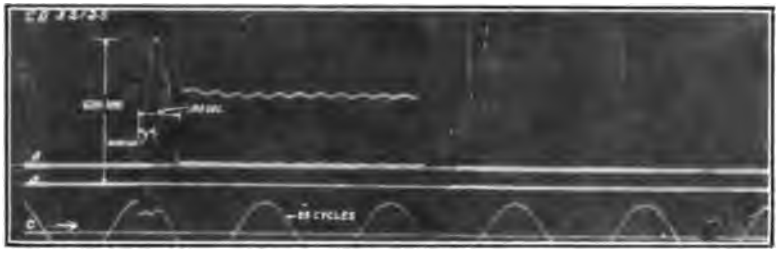

FIG. 12

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by oil-cooled high-speed fuse. Curve *A*, voltage across fuse; Curve *B*, line current; Curve *C*, collector-ring voltage.

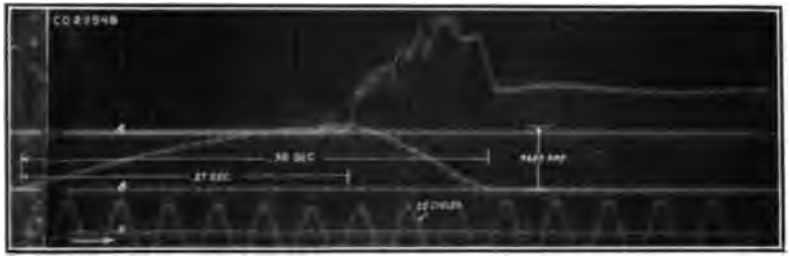

FIG. 13

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by air-core reactor in direct-current circuit and standard circuit breaker. Curve *A*, voltage across circuit breaker; Curve *B*, line current; Curve *C*, collector-ring voltage.

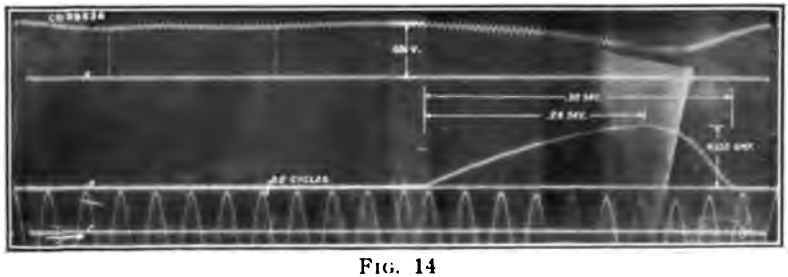

FIG. 14

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by air-core reactor in direct-current circuit and standard circuit breaker; Curve-*A*, voltage across armature; Curve *B*, line current; Curve *C*, collector-ring voltage.

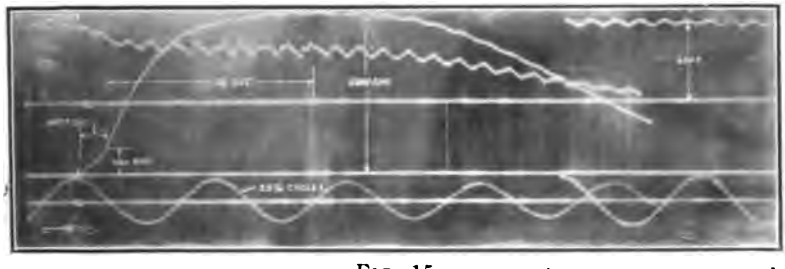

FIG. 15 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter, protected by iron-core reactor in direct-current circuit and standard circuit breaker. Curve *A*, voltage across the armature; Curve *B*, line current; Curve *C*, collector-ring voltage.

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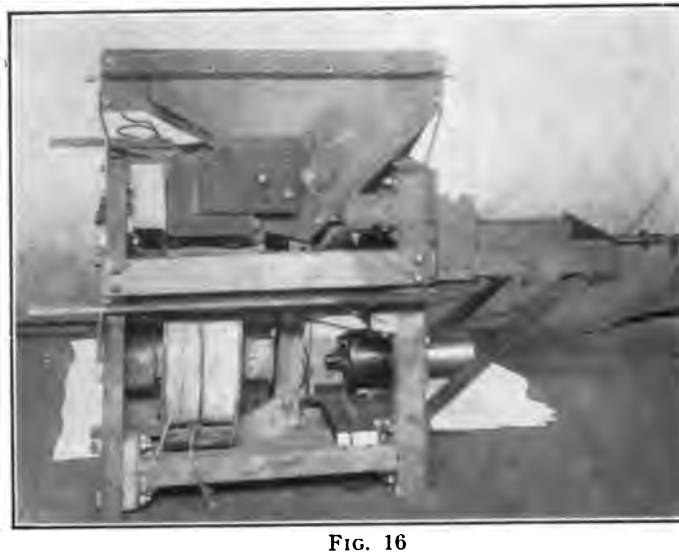

FIG. 16

Second form of high-speed circuit breaker, capacity 1500 amperes, 600 volts.

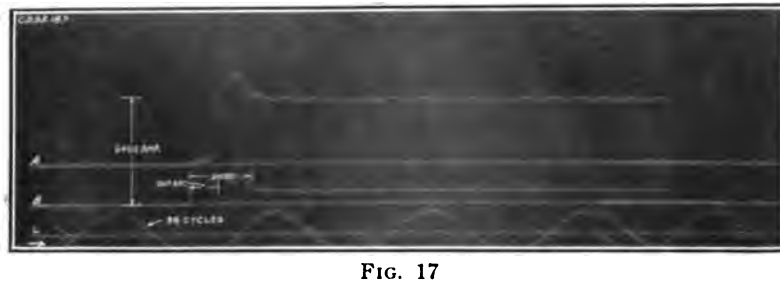

FIG. 17

Short circuit on 300-kw., 600-volt, 25-cycle synchronous converter protected by second form of high-speed circuit breaker. Curve *A*, voltage across circuit breaker; Curve *B*, line current; Curve *C*, collector-ring voltage.

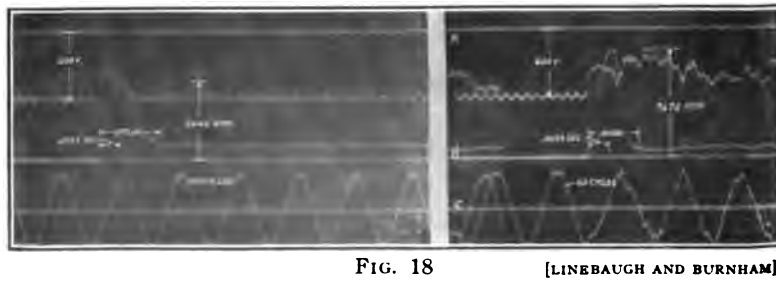

FIG. 18 [LINEBAUGH AND BURNHAM]

Short circuit on 500-kw., 600-volt, 60-cycle, synchronous converter protected by second form of high-speed circuit breaker.

Left hand curve — Load of 0.03 ohms

Right hand curve — Short circuit.

Curve *A*, armature volts
" *B*, line current
" *C*, collector-ring voltage.

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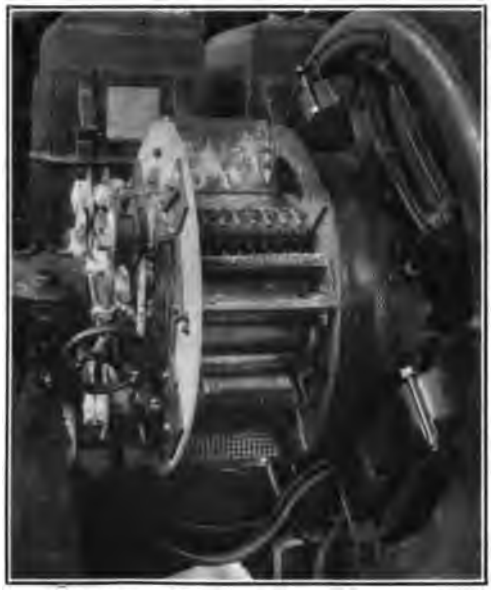

FIG. 19

Final development of flash barriers on 300-kw., 25-cycle, 600-volt synchronous converter.

FIG. 20 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 25-cycle, 600-volt, synchronous converter protected by flash barriers and standard circuit breaker.

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FIG. 21

Short circuit on 300-kw., 25-cycle, 600-volt synchronous converter protected by flash barriers and standard circuit breaker.

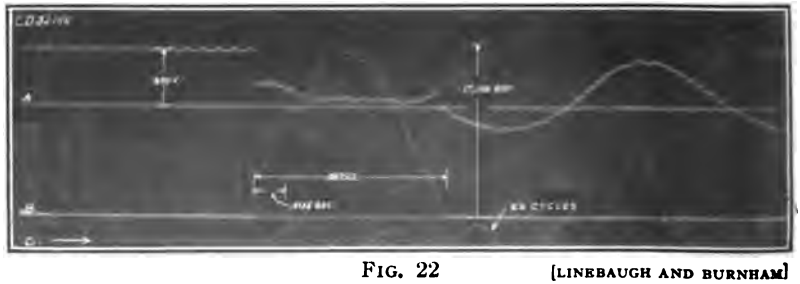

FIG. 22 [LINEBAUGH AND BURNHAM]

Short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter equipped with flash barriers and standard circuit breaker. Curve *A*, armature volts; Curve *B*, line current; Curve *C*, collector-ring voltage.

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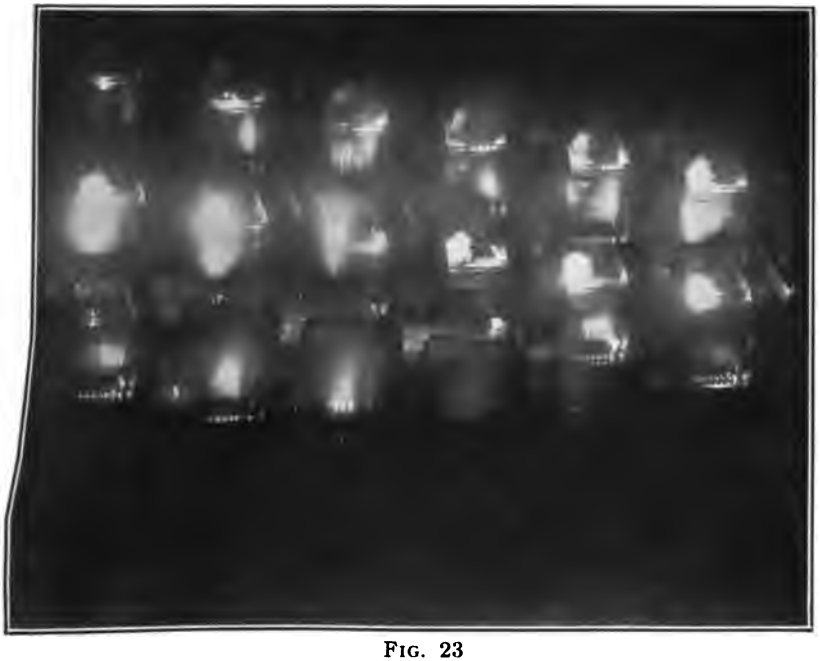

FIG. 23

High-speed photograph of short circuit on 300-kw., 600-volt 25-cycle synchronous converter protected by flash barriers and standard breaker.

FIG. 24 [LINEBAUGH AND BURNHAM]

High-speed photograph of short circuit on 300-kw., 600-volt, 25-cycle, synchronous converter protected by flash barriers and standard circuit breakers.

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FIG. 25

High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle synchronous converter without protection.

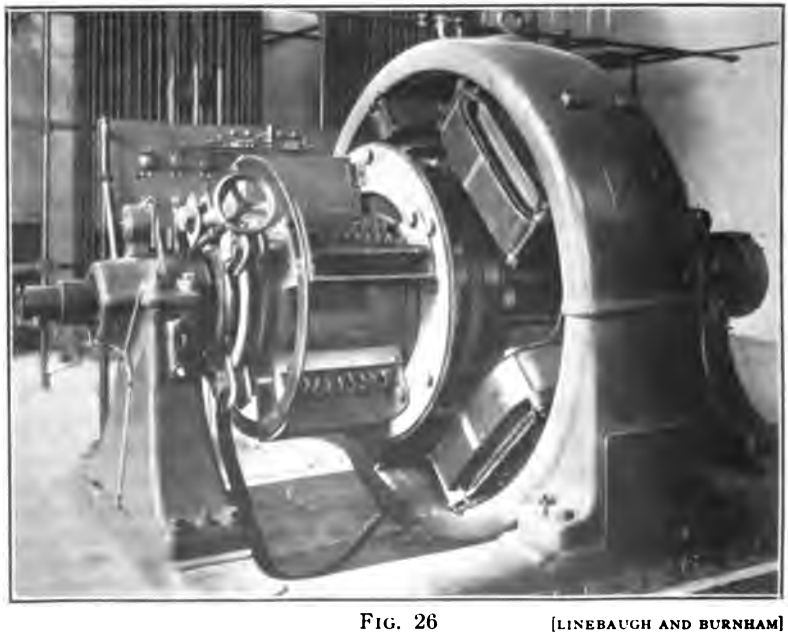

FIG. 26 [LINEBAUGH AND BURNHAM]

500-kw., 25-cycle, 600-volt, synchronous converter, installed in automatic substation, equipped with commercial form of flash barrier.

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FIG. 27

Flash barrier with front removed to show location and construction of arc scoop and wire-mesh arc coolers.

FIG. 28 [LINEBAUGH AND BURNHAM]

High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle, synchronous converter with flash barriers and standard circuit breaker with preliminary arrangement of brush rigging. Arc at outer end of commutator.

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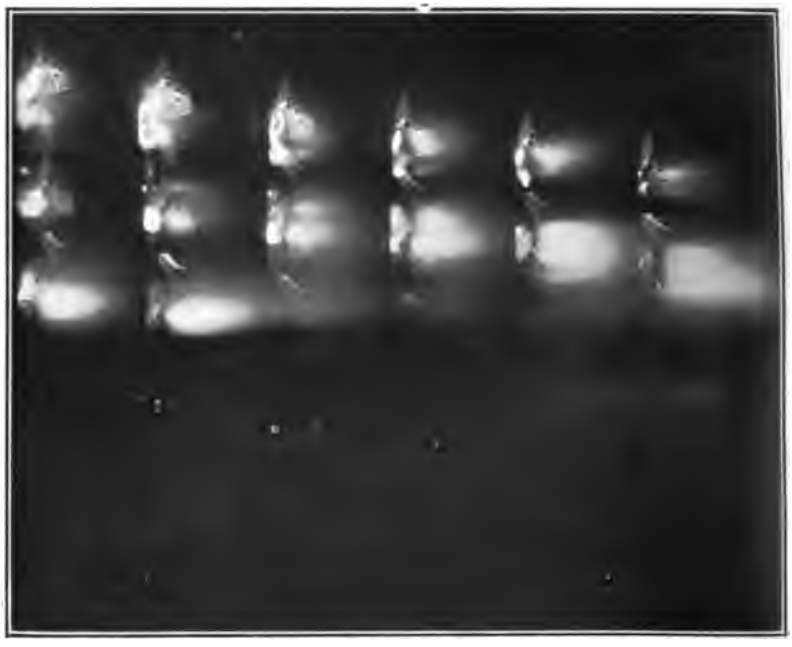

FIG. 29

High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle synchronous converter with flash barriers and standard circuit breaker with preliminary arrangement of brush rigging. Arc at outer end of brush rigging.

FIG. 30 [LINEBAUGH AND BURNHAM]

High-speed photograph of short circuit on 500-kw., 600-volt, 60-cycle synchronous converter protected by flash barriers and standard circuit breaker after arrangement of brush rigging has been changed. Uniform distribution of flashing.

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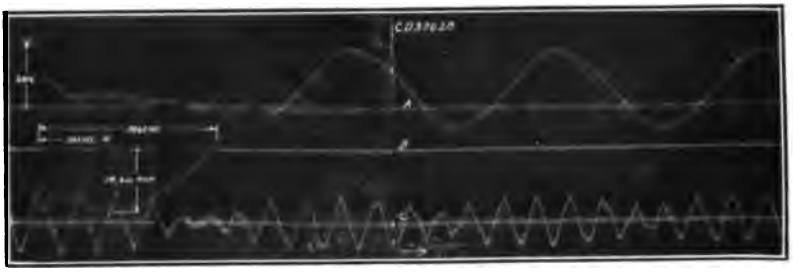

FIG. 31

Short circuit on 500-kw., 600-volt, 60-cycle, synchronous converter protected by flash barriers and standard circuit breaker after arrangement of brush rigging had been changed.

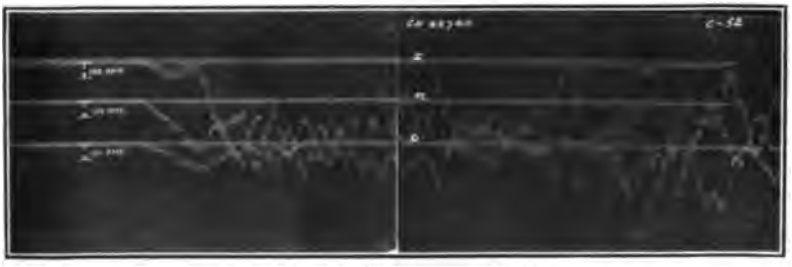

FIG. 32

Short circuit on 50-kw., 600-volt generator. Curve *O*, current in outside brush; Curve *M* current in middle brush; Curve *I*, current in inside brush.

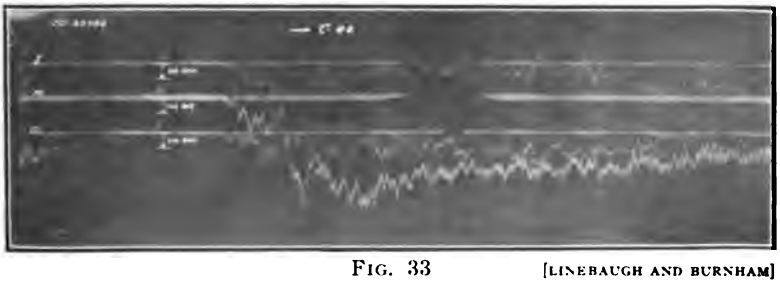

FIG. 33 [LINEBAUGH AND BURNHAM]

Short circuit on 50-kw., 600-volt generator. Curve *O*, current in outside brush; Curve *M*, current in middle brush; Curve *I*, current in inside brush.

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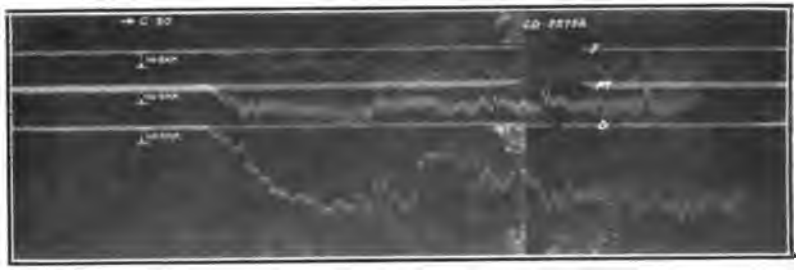

FIG. 34

Short circuit on 50-kw., 600-volt generator. Curve *O*, current in outside brush; Curve *M*, current in middle brush; Curve *I*, current in inside brush.

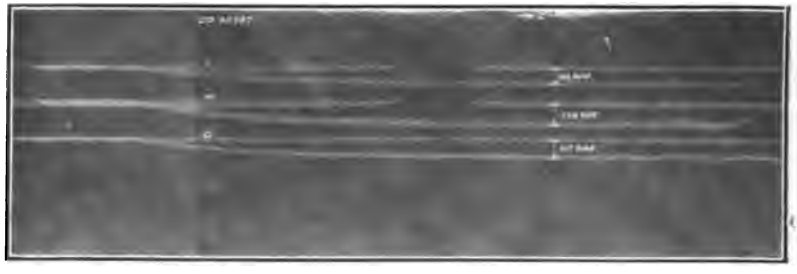

FIG. 35 [LINEBAUGH AND BURNHAM]

Current passed through 50-kw., 600 volt generator from an external source.

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that would take care of all short circuits experienced in actual service.

Under certain conditions it might be desirable to supplement rather than replace, appliances already installed or to protect from disturbances other than direct-current load which cause flashing. For instance, a synchronous converter could not be protected by a high-speed direct-current circuit breaker if flashing is caused by a.c. phase displacement. For this reason additional protection, such as barriers, to dissipate the arc when started was also needed.

Many different forms of barriers were tried on the 300-kilowatt, 25-cycle, 600-volt synchronous converter, previously mentioned. With increasing success as improvements were made to meet failures, the barriers shown in Fig. 19 were evolved. These barriers gave complete protection from flashover or damage on short circuit. Fig. 20 shows machine on short circuit giving a good idea of flashing and protection afforded, while Fig. 21 shows clearly the small amount of flash which extends beyond the barrier.

About 65 short circuits were thrown on the 300-kilowatt, 600-volt, 25-cycle machine without burning of brushes, brush connections or rigging, or damages of any kind to commutator or machine. Oscillogram, Fig. 22, shows a record of current reaching 34 times full load and gives a good idea of the protection afforded. Many of these short circuits were applied at very short intervals, even as close as one minute apart, without failure to hold and extinguish the arc when the breaker opened the circuit.

Figs. 23 and 24 are very interesting high-speed pictures of the same short circuit analyzed by means of a special high-speed camera devised by Lieut. Chester Lichtenberg, and the successful high-speed pictures we are able to show in this paper are mainly due to his efforts. This camera made it possible to obtain as high as 24 complete pictures of one short circuit, while the best results it was possible to obtain with a moving picture camera were two under-exposed and therefore indistinct pictures.

A little explanation is necessary to read these photographs as, due to the construction of the camera, the lower right hand picture is the first picture of the short circuit; the next picture being the one immediately to the left, and so on to the end of the plate; the first picture at the right of the the next row being

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the next picture in the same order and until the end of the plate and the number of rows of pictures. These pictures show very clearly the growth of the arc, disposition on commutator and dissipation of the arc as the regular breaker opens. These permanent records eliminated the personal factors of memory and observation and showed the way for changes to give improvements in barriers. Fig. 25 illustrates very clearly what happens if the machine is short-circuited without protection.

The general arrangement of a successful barrier, Fig. 26, is shown herewith.

A close fitting box of fire-proof insulating material surrounds each set of brushes and is located so as to give a small clearance between the box and the commutator.

On the side of the box towards which the commutator rotates after leaving the brush is fastened a V-shaped "scoop", Fig. 27, of fire-proof insulating material, preferably having good heat conductivity, pointing toward the brush and having small running clearance from the commutator.

Radially above the scoop, about one inch apart, are two metal screens, one coarse and one fine mesh, through which the arc is successively forced and cooled.

It was found that a moderate amount of material is required to give the necessary thermal capacity to prevent an arc from passing beyond a screen of this kind. The scoop running very close to the commutator with narrow edge and small clearance picks up the arc from the commutator and deflects it into the arc coolers which, from their construction, allow free passage of all gases generated by the arc. The cooling and condensing of the arc reduces the gas pressure so that shields at the end of the commutator, to prevent the arc being thrown from the end of the commutator and communicated to pillow block and frame, are permissible. It will be noted from the illustrations that the commutators extend beyond the end of the barrier as it was found that the arc must be prevented from being communicated to the end of the bars.

Investigation was then transferred to a 500-kilowatt, 60-cycle, 600-volt synchronous converter and barriers of similar type, but without continuous end shields, were tried.

Tests showed that these barriers did not give protection on short circuit although they prevented machine from flashing over on very high overload. The high-speed camera record in-

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dicated that the arc was being thrown to the outer end of the commutator for some reason, causing such high gas pressure at the outer end of the commutator that the arc was blown under the barrier and the machine flashed over. Figs. 28 and 29.

The differences in performance were ascribed to differences of magnetic fields acting on the arc.

To demonstrate the effect of the magnetic field, various arrangements of connections of brush rigging were made, each to produce a different field where the arcing occurs. The results indicate that it is possible to arrange the brush rigging and connection to make a barrier, as described above, effective on practically all commutating machines and to prevent complete flashover. Figs. 30 and 31 show the effects of change in connection on arc distribution, giving the uniform distribution most favorable to good barrier performance.

Other tests were made to record the simultaneous short-circuit current in the outer, middle and inner brushes by the oscillograph. The records in Figs. 32, 33 and 34 show typical variations of current distribution produced by different connections to brushes. The distribution of current is principally dependent on the magnetic field surrounding the brushes where the arc is formed. To show that differences of impedance have very little influence, record Fig. 35 was taken with current supplied from an exterior source with no flashing. It will be seen that current is practically the same in all brushes. With some connections the deflection of the arc can be plainly seen to follow the well-known relation, as given in Fig. 36, but with the more complicated connections the difficulty of determining resultant field from many sources makes it difficult to determine the direction of deflection of the arc except by experiment.

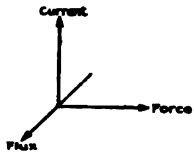

FIG. 36

Direct-current machines for use in * automatic substations are being equipped with these barriers and short-circuit tests at the substations have been taken, indicating that they will take care of any short circuit experienced in actual service. These barriers are in operation and short-circuit tests were taken on a 500-kilowatt, 600-volt, 25-cycle synchronous converter of the Des Moines Electric Railway, Des Moines, Iowa, a 500 kilowatt, 600-volt, 60-cycle synchronous converter of the Coumbus Electric Railway & Light Company, Columbus, Ohio, and a 500-kilowatt,

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30-cycle, 1200-volt synchronous converter at Montieth Junction, Michigan, and other installations are now in service.

The investigations and tests indicate that if any commutating machine is equipped with barriers and the last high-speed circuit breaker described, complete protection will be given against external short circuits of all kinds so that interruption to service will not be of any greater duration than necessary for closing the circuit breaker and main switch as in ordinary overload operation.

*See paper by Taylor and Allen, A.I.E.E. TRANSACTIONS Vol. 34, 1915, page 1801.

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DISCUSSION ON "PROTECTION FROM FLASHING FOR D-C. APPARATUS" (LINEBAUGH AND BURNHAM), ATLANTIC CITY, N. J., JUNE 28, 1918

Charles L. Fortescue: I would like to congratulate the authors on the fine results they have been able to obtain with a mechanically-operated high-speed breaker. I can assure you, gentlemen, that to attain a speed of 0.005 of a second on a mechanically operated breaker is no small achievement. The very nature of the problem and the materials available set a limitation to the possible speed obtainable.

I am very much interested in the subject, because I have been associated with various men of our company in a similar line of development. In 1911, in association with the late Mr. William Cooper, the problem of preventing flashing of railway motors came to my attention. On one of the elevated lines in New York City, they were having some trouble with flashing of the motors, due to short circuits and it was proposed to prevent it by operating the breakers quickly. Mr. Cooper conceived the idea that we could obtain a quick operating relay, which would operate the valve in the air-operated breakers, depending upon the magnetic energy released due to the short circuit. With this idea, we built a relay-transformer. The transformer had a small air gap in it, and we depended on the discharge of the magnetic energy stored in this gap to operate the relay. With this little device we were able to obtain such satisfactory results that we managed to overcome our troubles on that particular road. We obtained, as I recall, speeds of from 0.025 to 0.030 of a second, and that encouraged us to go a step further, and we decided to design a direct-operated circuit breaker of the carbon type. The second design was a carbon breaker with a magnetic blowout, and it had a rather powerful spring, somewhere around 300 or 400 lb. This breaker proved quite satisfactory. We were able to get speeds down to 0.015 of a second with the latch operated by the transformer device.

Later on I conceived the idea of using a condenser instead of a transformer for storing the energy required to operate the trip, in other words, we placed the condensers in series with the trip coil across the direct-current terminals. When a short circuit occurs, there is an initial fall of potential, and this discharges the condenser through the trip coil and operates the trip magnet. The frequency of the discharge current will depend on the capacity of the condenser and the self-inductance of the magnet. This arrangement was very satisfactory, in fact, with a circuit-breaker of very ordinary design, nothing very special about it, except it had a rather stronger spring than usual, we were able to obtain a speed of 0.011 of a second.

I would like to take exception to a few points in the authors' discussion of the question of reactors used for retarding the growth of current. The problem, as I see it, is not that the

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choke coil is expensive, it is a balance between the cost of the choke coil and the high-speed breaker. As it increases in speed the breaker becomes more and more expensive, and as I said before, it is a big problem to produce a mechanical device which will always operate satisfactorily at high speed. There is a chance of breakage and crystallization of parts, the springs are tremendously powerful, in some cases several tons, and the working parts are subject to heavy jarring action, so that the cost and maintenance of a device like that is necessarily very high. By using a choke coil, it is possible to use a lower speed and less costly device. It has been estimated that the economic speed was somewhere around 0.009 of a second.

A description of the early tests referred to in this discussion was published some time around 1913 in "*Electric Traction*" by Mr. Funk. Later on, 1914, Mr. Yardley presented a paper on the "Use of Reactors with Synchronous Converters," in which he showed a set of films that were taken with converters operated with one of these breakers. We also have films showing them successfully operated with 1500-volt direct-current machines.

I might mention that in these mechanically-operated breakers the capacity required to operate the tripping device was quite small. In the first breaker we made we were able to operate very satisfactorily at 500 volts, with a condenser of 1-microfarad capacity and at 1500 volts with a condenser of one-third of a microfarad capacity. A little later we used a stronger operating trip coil, which requires 2 or 3 microfarads at 500 volts and still later we have used a tripping device of very much the same pattern as that which the authors have described.

Another device that we have used to obtain quick operation is a relatively small choke coil, shunted by the trip coil. In this device the impulse current in the trip coil, when short circuit occurs operates the breaker enabling us to get very quick action.

We have had some of the condenser tripped apparatus in actual operation. We have had one installed at the plant of the Youghiogney Coal Company, operating in mine service, and the operation had been very satisfactory. In fact, most of our work has been done with the idea of developing something that could be used for such service and to protect generators and motors from short circuits on feeder circuits. We have tried to obtain something that could be sold at a moderate price, to be used for the protection of motors, etc., in industrial work.

We have also had another circuit breaker installed at the New River Coal Company. I saw a day's record of that system— I think it was last fall—which they say is quite an ordinary record: There were thirty shorts in one day, and in every case the breaker operated so that the motor or generator was protected. In this case we have a small choke coil in series with the machine in addition to the high-speed breakers.

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This is a rough summary of the work that we have done in connection with high-speed breakers. We have not reached as high speed as Messrs. Linebaugh and Burnham have obtained with their very special mechanical construction but the results have been very satisfactory with ordinary forms of breakers.

F. C. Hanker: The use of higher direct-current voltages on railway systems has increased the difficulties in connection with the supply of power, particularly with commutation during the short-circuit conditions. In the early days, somewhat similar conditions obtained on 600-volt railways and forced improvements in the commutating characteristics of direct-current generators and converters, so that the modern machines, with commutating poles, and, for more severe service, with pole-face windings, are very rugged and have given satisfactory service.

The advent of the higher voltages has made the problem of the designer more difficult and the resultant increase in flashing troubles has required concerted attention on improvements. The authors have described in detail the manner in which they have attacked the problem and outlined the various forms of breakers and barriers they have developed after extended experimental testing. This method of solving the problem is the logical one, as pointed out by the authors, when we take into consideration that the remedy must be applied to existing installations. In other words, it may be classed as an external attack.

In the form of breaker described in the paper as being installed in the stations of the present electrified zone of the C. M. & St. P., the tripping mechanism is complicated, and extremely heavy spring pressures for rapid opening are used. Such a circuit breaker will be subject to relatively rapid deterioration and, unless kept at the maximum point of efficiency, will slow up in its speed, then if it fails to open the circuit before the flashing starts, it will be too late to stop it.

Another method of solving the problem of flashing that sounded rather revolutionary when it was first proposed was suggested by Mr. N. W. Storer, who is well known for his work in the railway field. It differed radically from the previous methods using circuit opening or limiting schemes and attacked the cause of the flashing. It effectively suppresses it while all other schemes only minimize the possibility of flashing or reduce the resulting effect.

The flash suppressor consists of a circuit closer arranged to short-circuit the armature winding of the direct-current generator through collector rings. It is tripped in a manner similar to circuit breaker practise with the addition of the anticipatory feature developed in previous high-speed circuit breaker investigations. It is also feasible to supplement this method of tripping by taking advantage of the increase in field current under the short-circuit conditions for operating an

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auxiliary tripping coil. The moving parts are small and as it is only necessary to move them a relatively short distance the device can easily be closed in from 0.006 to 0.008 sec. without using abnormal spring pressures. Higher speed can be obtained, if necessary, without undue complication, but tests show conclusively that the flashing is suppressed effectively after intervals several times longer, so that it was not considered necessary to secure higher speed of operation.

The flash suppressor really performs a double function; first, short-circuiting the collector rings and thus simultaneously reducing the voltage between commutator bars practically to zero, or, in other words, all of the voltage generated in the winding is absorbed in that winding so that no difference of potential between commutator bars will remain; and second, in reducing the field flux. Short circuits such as are applied by a flashing suppressor, result in a heavy wattless current flowing through the armature winding. The magnetizing force of the armature ampere turns opposes that of the field winding and cuts down the voltage at the maximum safe rate without opening either the field or armature circuit. In this way, the stored energy not only of the line but of the machine itself is harmlessly short-circuited through the machine and dissipated in the armature and field windings.

During the studies made of this method of suppressing flashing numerous tests were made on different types of machines and it may be of interest to review some of the more representative oscillograms.

The first oscillogram Fig. 1 shows a direct-current short circuit limited by resistance that is typical of a standard direct-current generator. The armature current rises to about 13,000 amperes or about eight times full-load current. The actual current obtained was about 60 per cent of the theoretical current that would be obtained by dividing the terminal voltage by the circuit resistance. The initial rate is about 2500 amperes per 0.001 sec. and the current rose to practically its final value in the first 0.010 sec. The shunt field current undergoes an increase in value, due to the transformer action between the armature and field winding.

Fig. 2 shows the effect of the application of the flash suppressor 0.006 sec. after a short circuit was applied to the direct-current side. In this case, the short circuits were made without resistance or reactance of any kind in the circuit. The effect on the commutator was confined to a small explosive puff of very short duration which did no burning on either the brush holders or commutator surface. Many duplicate oscillograph records of combined short circuits were made which established that the suppression of the flashing was independent of the point on the alternating-current wave at which the short circuit occurred and that all armature positions give essentially the same effect. The longer delays resulted in a somewhat heavier

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